CHAPTER VII

SHORTEST HYPERPATH IN INTUITIONISTIC FUZZY GRAPHS

Hypergraph is a common name for various combinatorial structures that generalize graphs. Besides the most known undirected hypergraphs or simply hypergraphs, a relevant role is played by directed hypergraphs, a generalization of directed graphs, which find applications in several areas of computer science and mathematics for representing implicative structures. Directed hypergraph is an extension of directed graphs, and have often been used in several areas as a modeling and algorithmic tool. A technical as well as historical introduction to directed hypergraphs has been given by Gallo et al [37]. Hyperpaths in hypergraphs are a nontrivial extension of directed paths whose expressive power allows us to deal with more complex situations. One of the most classical problems extended in the analysis of networks is the shortest path problem. Traditionally the shortest path problem was a single objective problem with the objective being minimizing total distance or travel time. Nevertheless, due to the multiobjective nature of many transportation and routing problems, a single objective function is not sufficient to completely characterize some real world problems. In a road network, the fastest path may be too costly or the cheapest path may be too long. Therefore, the
decision maker must choose a solution among the paths. Any time a structure is represented by means of a hypergraph, it may be relevant to find hyperpaths that connect nodes or sets of nodes, or minimum hyperpaths, where the minimalty is defined on the basis of a weight to hyperpath. Instead of assigning weights to hyperpath, we use intuitionistic fuzzy numbers for modeling the problem and finding shortest hyperpath in IFHGs. The fuzzy shortest path problem was first analyzed by Dubois and Prade [33] by assigning fuzzy number, instead of a real number, to each edge.

In this chapter, a new method namely, the score based method is proposed for finding shortest hyperpaths in IFDHGs with intuitionistic fuzzy weights of hyper-edges. The computation procedure of this method is comparatively easier.

7.1 BASIC CONCEPTS

In this section, the definitions of intuitionistic fuzzy hypergraph, directed intuitionistic fuzzy hypergraph, triangular intuitionistic fuzzy number, score, ranking of intuitionistic fuzzy numbers are given. These are the basic concepts required for designing the algorithm to find the shortest hyperpath in IFDHGs.

**Definition 7.1**

Let $V$ be a non-empty set and let $A = \{(v, \mu_A(v), \nu_A(v)) | v \in V\}$ be an IFS,
then the pair \((\mu_A(v), \nu_A(v))\) is called as an \textit{intuitionistic fuzzy number} (IFN), denoted by \((\langle a, b, c \rangle, \langle e, f, g \rangle)\), where \(\langle a, b, c \rangle \in F(I), \langle e, f, g \rangle \in F(I), I = [0, 1]\), \(0 \leq c + g \leq 1\).

\textbf{Definition 7.2} \cite{72}

A \textit{triangular intuitionistic fuzzy number} (TriIFN) \(A\), is denoted by \(A = \{ (\mu_A(v), \nu_A(v)) | v \in V \}\), where \(\mu_A(v)\) and \(\nu_A(v)\) are triangular fuzzy numbers with \(\nu_A(v) \leq \mu_A(v)\).

So, a TriIFN \(A\) is given by \(A = (\langle a, b, c \rangle, \langle e, f, g \rangle)\) with \((\langle e, f, g \rangle \leq \langle a, b, c \rangle^c)\). That is, either \(e \geq b\) and \(f \geq c\) or \(f \leq a\) and \(g \leq b\) are membership and nonmembership fuzzy numbers of \(A\). The diagrammatic representation of a triangular intuitionistic fuzzy number \(A = (\langle a, b, c \rangle, \langle e, f, g \rangle)\) with \(e \geq b\) and \(f \geq c\) is shown in Figure 7.42.

![Figure 7.42: Triangular intuitionistic fuzzy number \(A = (\langle a, b, c \rangle, \langle e, f, g \rangle)\)](image)

\textbf{Definition 7.3}

Let \(A = (\langle a_1, b_1, c_1 \rangle, \langle e_1, f_1, g_1 \rangle)\) and \(B = (\langle a_2, b_2, c_2 \rangle, \langle e_2, f_2, g_2 \rangle)\) be two Tri-
IFNs. The addition of two TriIFN, denoted by \( A + B \), is defined as 
\[
A + B = (\langle a_1 + a_2, b_1 + b_2, c_1 + c_2 \rangle, \langle e_1 + e_2, f_1 + f_2, g_1 + g_2 \rangle).
\]

**Definition 7.4**

Let \( \tilde{A}^I = \{(a, b, c), (e, f, g)\} \) be a TriIFN, then the score of \( \tilde{A}^I \) is an IFS whose membership and non-membership values are given respectively as 
\[
S(\tilde{A}^I_\mu) = \frac{a + 2b + c}{4} \quad \text{and} \quad S(\tilde{A}^I_\nu) = \frac{e + 2f + g}{4}.
\]

**Definition 7.5**

The accuracy of a TriIFN \( \tilde{A}^I \) is defined as 
\[
Acc(A) = \frac{1}{2} \left( S(\tilde{A}^I_\mu + S(\tilde{A}^I_\nu)) \right)
\]

### 7.2 Minimum Arc Length of an Intuitionistic Fuzzy Hyperpath

In this section, as discussed earlier, the arc length in a hypernetwork is considered to be a TriIFN. The algorithm given in this section is based on [51]. Let \( L_i \) denotes arc length of the \( i^{th} \) hyperpath.

**Algorithm**

**Step 1.** Compute the lengths of all possible hyperpaths \( L_i \) for \( i = 1, 2, 3..., n \), where \( L_i = (\langle a'_i, b'_i, c'_i \rangle, \langle e'_i, f'_i, g'_i \rangle) \).

**Step 2.** Initialize \( L_{\text{min}} = (\langle a, b, c \rangle, \langle e, f, g \rangle) = L_1 = (\langle a'_1, b'_1, c'_1 \rangle, \langle e'_1, f'_1, g'_1 \rangle) \).
Step 3. Set \( i = 2 \)

Step 4. Compute the membership values \( \langle a, b, c \rangle \) as

\[
a = \min(a, a_i')
\]

\[
b = \begin{cases} 
  b & \text{if } b \leq a_i' \\
  \frac{bb_i'-aa_i'}{(b+b_i')-(a+a_i')} & \text{if } b > a_i'
\end{cases}
\]

\[
c = \min(c, b_i')
\]

and nonmembership values \( \langle e, f, g \rangle \) as

\[
e = \min(e, e_i')
\]

\[
f = \begin{cases} 
  f & \text{if } f \leq e_i' \\
  \frac{ff_i'-ee_i'}{(f+f_i')-(e+e_i')} & \text{if } f > e_i'
\end{cases}
\]

\[
g = \min(g, f_i')
\]

Step 5. Set \( L_{\text{min}} = (\langle a, b, c \rangle, \langle e, f, g \rangle) \) as calculated in step 4

Step 6. \( i = i + 1 \)

Step 7. if \( i < n + 1 \), go to step 3, otherwise stop the procedure.

Example 7.6

Consider a network with a triangular intuitionistic fuzzy arc length shown in Figure 7.43.
Figure 7.43: Intuitionistic fuzzy hypernetwork

**Step 1.** There are six possible paths \( n = 6 \) from source node 1 to destination node 8, as given below:

Path (1): \( 1 \to 2 \to 6 \to 8 \); \( L_1 = (\langle 10, 15, 18 \rangle, \langle 13, 21, 25 \rangle) \)

Path (2): \( 1 \to 3 \to 6 \to 8 \); \( L_2 = (\langle 13, 18, 23 \rangle, \langle 18, 27, 32 \rangle) \)

Path (3): \( 1 \to 3 \to 7 \to 8 \); \( L_3 = (\langle 16, 21, 26 \rangle, \langle 22, 31, 36 \rangle) \)

Path (4): \( 1 \to 4 \to 7 \to 8 \); \( L_4 = (\langle 5, 9, 13 \rangle, \langle 12, 19, 24 \rangle) \)

Path (5): \( 1 \to 5 \to 7 \to 8 \); \( L_5 = (\langle 6, 10, 13 \rangle, \langle 15, 21, 26 \rangle) \)

Path (6): \( 1 \to 5 \to 8 \); \( L_6 = (\langle 10, 14, 17 \rangle, \langle 16, 20, 25 \rangle) \)

**Step 2.** Initialize \( L_{min} = (\langle a, b, c \rangle, \langle e, f, g \rangle) = L_1 = (\langle a_1', b_1', c_1' \rangle, \langle e_1', f_1', g_1' \rangle) = (\langle 10, 15, 18 \rangle, \langle 13, 21, 25 \rangle) \)

**Step 3.** Initialize \( i = 2 \);

**Step 4.** Let \( L_{min} = (\langle 10, 15, 18 \rangle, \langle 13, 21, 25 \rangle) \) and \( L_2 = (\langle a_2', b_2', c_2' \rangle, \langle e_2', f_2', g_2' \rangle) = (\langle 13, 18, 23 \rangle, \langle 18, 27, 32 \rangle) \). Compute the membership values \( \langle a, b, c \rangle \) as
a = \min(a, a'_2) = \min(10, 13) = 10
\begin{align*}
b &= \begin{cases}
\frac{(15 \times 18) - (13 \times 10)}{(15 + 18) - (13 + 10)} = 14 & \text{since } b > a'_2 \\
\end{cases}
c &= \min(c, b'_2) = \min(18, 23) = 18
\end{align*}
and nonmembership values \langle e, f, g \rangle as
\begin{align*}
e &= \min(e, e'_2) = \min(13, 18) = 13
f &= \begin{cases}
\frac{(27 \times 21) - (13 \times 18)}{(27 + 21) - (13 + 18)} = 19.59 & \text{since } f > e'_2 \\
\end{cases}
g &= \min(g, f'_3) = \min(25, 27) = 25
\end{align*}

**Step 5.** Set \( L_{\text{min}} = (\langle 10, 14, 18 \rangle, \langle 13, 19.59, 25 \rangle) \)

**Step 6.** \( i = i + 1 = 3 \)

**Step 7.** if \( i < n + 1 (= 6) \), go to step 4.

**Step 4.** Let \( L_{\text{min}} = (\langle 10, 14, 18 \rangle, \langle 13, 19.59, 25 \rangle) \) and \( L_3 = (\langle a'_3, b'_3, c'_3 \rangle, \langle e'_3, f'_3, g'_3 \rangle) = (\langle 16, 21, 26 \rangle, \langle 22, 31, 36 \rangle) \). Compute the membership values \langle a, b, c \rangle as
\begin{align*}
a &= \min(a, a'_3) = \min(10, 16) = 10
b &= 14 & \text{since } b < a'_3 
c &= \min(c, b'_3) = \min(18, 21) = 18
\end{align*}
and nonmembership values \langle e, f, g \rangle as
\begin{align*}
e &= \min(e, e'_3) = \min(13, 22) = 13
f &= 19.59 & \text{since } f < e'_3 
g &= \min(g, f'_3) = \min(25, 31) = 25
\end{align*}
Step 5. Set $L_{\min} = (\langle 10, 14, 18 \rangle, \langle 13, 19.59, 25 \rangle)$. Repeat the procedure until $i = 5$.

Finally, we get the minimum of arc lengths of intuitionistic fuzzy hyperpath as $L_{\min} = (\langle 5, 7.65, 9 \rangle, \langle 12, 15.55, 19 \rangle)$

### 7.3 AN ALGORITHM FOR SEARCHING THE SHORTEST HYPERPATH

The algorithm, in section 7.2, gives us the minimum arc length from source node to destination node of the intuitionistic fuzzy hyperpath. But we aim at determining an intuitionistic fuzzy shortest hyperpath to traverse from source to destination. To achieve this, there are methods, namely similarity measure method and Euclidean distance method. These methods need the arc length of intuitionistic fuzzy hyperpath to be known. But the newly proposed method based on scores of IFNs, called as, score-based method does not need the same to be known. The output of the score-based method is compared with that of Euclidean distance method.
7.4 SCORE-BASED METHOD

The steps involved in finding IF shortest hyperpath using score-based method is given below.

**Step 1.** Consider *all possible paths* from source node to destination node.

**Step 2.** Find the *scores* of the paths.

**Step 3.** Find their *accuracy*.

**Step 4.** Obtain the *shortest hyperpath* with the lowest accuracy.

**Example 7.7**

Consider the intuitionistic fuzzy hypernetwork given in Figure 7.43. The intuitionistic fuzzy shortest hyperpath in this hypernetwork is identified using score-based method.

<table>
<thead>
<tr>
<th>Path</th>
<th>Score</th>
<th>Accuracy</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_1$</td>
<td>$\langle 14.5, 20 \rangle$</td>
<td>17.25</td>
<td>4</td>
</tr>
<tr>
<td>$P_2$</td>
<td>$\langle 18, 26 \rangle$</td>
<td>22</td>
<td>5</td>
</tr>
<tr>
<td>$P_3$</td>
<td>$\langle 21, 30 \rangle$</td>
<td>25.5</td>
<td>6</td>
</tr>
<tr>
<td>$P_4$</td>
<td>$\langle 9, 18.5 \rangle$</td>
<td>13.75</td>
<td>1</td>
</tr>
<tr>
<td>$P_5$</td>
<td>$\langle 9.75, 20.75 \rangle$</td>
<td>15.25</td>
<td>2</td>
</tr>
<tr>
<td>$P_6$</td>
<td>$\langle 13.75, 20.25 \rangle$</td>
<td>17</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 7.1 Scores and Accuracy
From Table 7.1, the path $P_4: 1 \rightarrow 4 \rightarrow 7 \rightarrow 8$ with least accuracy has been identified as intuitionistic fuzzy shortest hyperpath.

### 7.5 COMPARISON BETWEEN SCORE BASED METHOD AND EUCLIDEAN DISTANCE METHOD

The steps involved in finding IF shortest hyperpath using Euclidean distance method is given below

**Step 1.** Find out all possible hyperpaths from source node $s$ to destination node $d$ and compute the arc lengths of corresponding hyperpath $L_i$, $i = 1, 2, 3, \ldots n$.

**Step 2.** Compute $L_{min}$ by using an intuitionistic fuzzy shortest hyperpath procedure.

**Step 3.** Find the Euclidean distance $d_i$ for $i = 1, 2, 3 \ldots n$ between all possible hyperpaths and $L_{min}$.

**Step 4.** Decide the shortest hyperpath with the lowest Euclidean distance.

For the network given in Figure 7.43, the algorithm is executed as follows:

**Step 1.** Path (1):$1 \rightarrow 2 \rightarrow 6 \rightarrow 8$ $L_1 = (\langle 10, 15, 18 \rangle, \langle 13, 21, 25 \rangle)$

Path (2):$1 \rightarrow 3 \rightarrow 6 \rightarrow 8$ $L_2 = (\langle 13, 18, 23 \rangle, \langle 18, 27, 32 \rangle)$

Path (3):$1 \rightarrow 3 \rightarrow 7 \rightarrow 8$ $L_3 = (\langle 16, 21, 26 \rangle, \langle 22, 31, 36 \rangle)$

Path (4):$1 \rightarrow 4 \rightarrow 7 \rightarrow 8$ $L_4 = (\langle 5, 9, 13 \rangle, \langle 12, 19, 24 \rangle)$

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Path (5): 1 → 5 → 7 → 8 \( L_5 = (\langle 6, 10, 13 \rangle, \langle 15, 21, 26 \rangle) \)

Path (6): 1 → 5 → 8 \( L_6 = (\langle 10, 14, 17 \rangle, \langle 16, 20, 25 \rangle) \)

**Step 2.** \( L_{\text{min}} = (\langle 5, 7.65, 9 \rangle, \langle 12, 15.55, 19 \rangle) \)

**Step 3.** Calculate Euclidean distance between all hyperpath lengths \( P_i (i = 1, 2, ..., 5) \) and minimum arc lengths.

\[
d(P_1, L_{\text{min}}) = \left( \sqrt{(10 - 5)^2 + (15 - 7.65)^2 + (18 - 9)^2} \right), \\
\left( \sqrt{(13 - 12)^2 + (21 - 15.55)^2 + (25 - 19)^2} \right) = (12.65, 8.17) \\
d(P_2, L_{\text{min}}) = (19.16, 18.33) \\
d(P_3, L_{\text{min}}) = (24.25, 25.05) \\
d(P_4, L_{\text{min}}) = (4.22, 6.07) \\
d(P_5, L_{\text{min}}) = (4.74, 9.36) \\
d(P_6, L_{\text{min}}) = (11.37, 8.47) \\
\]

**Step 4.** Decide the shortest hyperpath with the path having lowest membership and non-membership values by examining the Euclidean distance \( d \) between \( L_{\text{min}} \) and \( d_i \) for \( i = 1, 2, ..., 6 \). Hence it is concluded that path (4) 1 → 4 → 7 → 8 has the least shortest hyperpath.