PREFACE

The standard way of treating problems in general theory of relativity is to consider Einstein field equations in a local coordinate basis adapted to the problems with which one is working. In recent times it has proved advantageous to choose a suitable basis of four linearly independent vectors, to project the relevant quantities on to the chosen basis and consider the equations satisfied by them. This is tetrad formalism - and in general relativity certain types of calculations are simplified if one deals with a tetrad formalism. An important example is the Newman-Penrose formalism.

On the other hand, certain physical problems in general relativity are often conveniently described by using a formalism adopted to the geometry of the particular situation. One such formalism is the extension of Newman-Penrose formalism given by Geroch, Held and Penrose. This formalism is now known as GHP formalism or compacted spin coefficient formalism. This formalism is clearly more concise and efficient than the widely known NP formalism. The present thesis entitled "A study of the compacted spin coefficient formalism in general relativity" is devoted to the study of GHP formalism and its applications in general theory of relativity. It consists of five chapters and an appendix.

Chapter I deals with a study of GHP formalism. The salient features of this formalism are mentioned and a complete set of field equations, commutator relations and Bianchi identities has been given (as far as I know, this set is not available in literature).

In order to obtain the information about the structure of the gravitational field, a study of the null congruences has been made in Chapter II. The compacted spin coefficient formalism has been used for this study. This investigation is an important activity as it brings out the geometrical meaning of the scalars characterizing the gravitational field. The contents of this Chapter have already been appeared in Proc. Indian Acad. Sci 85 A (1977) 546-551.

Non null electromagnetic fields via the GHP formalism is the subject of study of Chapter III. The Maxwell's equations for an arbitrary type elec-
tromagnetic field as well non null and null electromagnetic fields have been translated into the language of GHP formalism. The propagation equations for shear, expansion and twist of the null congruences have been obtained and a coupling between twist and expansion of the congruences has been established. The behaviour of the modified Lie derivative operator on the electromagnetic field bivector, Ricci tensor and metric tensor has been investigated.

The non-local part of the gravitational field in general relativity is described by the 10 components of the Weyl conformal curvature tensor. For this field, Lanczos found a potential $\Lambda_{abc}$ - now known as Lanczos potential. Chapter IV is devoted to the study of this tensor. The method of general observers has been considered and the kinematical quantities such as expansion, shear and twist etc., and the equations satisfied by them have been written in terms of the NP formalism. The tensorial versions of the earlier results of Novello and Velloso about the Lanczos potential for the perfect fluid space-times have been written in terms of the spin coefficient. A structural link between the spin coefficients and the Lanczos scalars has been established and a potential for the Gödel solution is obtained. The Weyl-Lanczos equations are translated into GHP formalism and a potential for a Petrov type D space-time is found. These results are then applied to a Kerr black hole. The contents of this Chapter has been accepted for presentation at the 21st meeting of the Indian Association for General Relativity and Gravitation, to be held at Nagpur(India), Jan. 2001.

Once a tetrad frame $\{l^a, n^a, m^a, \bar{m}^a\}$ is chosen we can subject the frame to a Lorentz transformation at some point and extend it continuously through all of space-time. Corresponding to six parameters of the group of Lorentz transformation, we have six degrees of freedom to rotate a chosen tetrad. The transformation laws are (a) null rotation about $l^a$ (b) null rotation which leaves the direction of $l^a$ and $n^a$ unchanged, but rotate $m^a$ (and $\bar{m}^a$) in $m^a - \bar{m}^a$ plane (c) null rotation about $n^a$ (d) reflection in $l^a - n^a$ plane (e) reflection in $m^a - \bar{m}^a$ plane (f) improper complex Lorentz transformation. The effects of these transformation laws on the scalars (spin coefficients, GHP derivative operators, the components of the Weyl and Ricci tensors), used to describe the gravitational field, have been examined in Chapter V. Some of the applications of these transformation laws have also been mentioned here. The contents of this Chapter have been presented at the Conference on Recent Devolopments in Relativity and Allied Topics held at Aligarh, August
In different parts of the thesis some of the results need more explanation. Such explanations and the equations required to derive these results have been mentioned in the Appendix along with some other related results. For example, GHP field equations, commutator relations, Bianchi identities, GHP versions of the Weyl-Lanczos equations and Lanczos differential gauge conditions have been written for different types of Petrov classification of the gravitational fields. A comparison between the electromagnetic and gravitational theories is also given here.

The thesis ends up with a list of references which by no means is exhaustive on the subject. Only the work referred to in the thesis has been included in the list.

Mathematical relations obtained in the thesis have been numbered serially in each Chapter and so are the Theorems. Thus equation (8) refers to equation (8) in the current Chapter. If equation (8) of Chapter I is used in any subsequent Chapter it will be represented by equation (8 - I).