CHAPTER 3

DESIGN AND IMPLEMENTATION OF CONVENTIONAL CONTROL SCHEMES FOR INTEGRATING PROCESS WITH DEAD TIME

3.1 Introduction

In the last one and a half decades several modifications have been made on the smith predictor structures to improve the servo and regulatory responses. In this research work an attempt has been made to give an overall idea of controller design for integrating process with dead time. The survey explores the conventional controller design for the integrating processes with the dead time [1, 4-10, 15-19]. The analysis explores the concept of designing/tuning of the controller based on time domain analysis and frequency domain analysis. The simulation studies have been carried out and the results have been presented in this chapter.

3.2 Introduction to Design of Feedback Control Scheme Using Frequency Response Techniques.

3.2.1 Review of IMC

High performance is always the design target in industrial control applications. Internal model control (IMC) was proposed and aimed to realize this goal, and it has been under intensive
research and development in the last decade due to its simple yet effective framework for system design. The IMC structure has been formally introduced by Rivera et al [36, 40]. It uses the process model as the internal model to predict the process output. When the model is perfect with the process (i.e., $G(s) = \hat{G}(s)$), the IMC system becomes an open-loop system and the controller design and the stability analysis issues become trivial. When a model mismatch exists, by appropriately modifying the difference between the actual process and model of the process, the robustness can be obtained. The IMC enables the transient response and the robustness to be addressed independently. The advantages of IMC are exploited in many industrial applications. For open-loop stable processes, the IMC approach provides a very simple yet powerful parameterization of all stabilizing controllers. However, when open-loop unstable processes are considered, the original IMC structure cannot be directly used for control system implementation. This is because, under the perfect match between the process and the process model, the IMC system becomes an open-loop one and unstable processes are thus not stabilized at all. These unstable processes can be pre-stabilized using conventional feedback before the normal IMC structure is applied. M.Morari et al. [33] too have proposed using the IMC parameterization method to design the conventional single loop feedback controller for the unstable process. The IMC frame is abandoned in implementation in this situation. The design procedure is not so straightforward and some restrictions have to be imposed on the controller design to guarantee the internal stability. These methods tend to arrive at a high-order controller, which limits their practicality and popularity.
Figure 3.1 General block diagram of IMC

It is well known that an open-loop arrangement represents the best way to achieve the fast and accurate set-point tracking [34, 35]. For the open-loop scheme, the internal stability problem, which is complicated in feedback system, is trivial and the controller is easy to design. However, the disadvantages are the sensitivity of the performance to process/model mismatch and the inability to cope with unmeasured disturbances. To solve these problems, feedback is needed. Based on the idea of taking both advantages of open-loop scheme and feedback strategy, IMC structure is proposed [34]. The block diagram of the IMC loop is shown in Fig.3.1, where the process $G(s)$ is assumed to be stable, and $\hat{G}(s)$ is the process model, which is used as the internal model. Let $r$ and $d$ be the set point and load disturbance, respectively. It follows from

\[ y(s) = \frac{G(s)C(s)}{1+(G(s)-\hat{G}(s))C(s)} + \frac{1-\hat{G}(s)C(s)}{1+(G(s)-\hat{G}(s))C(s)}G(s)d(s) \]  

(3.1)

Which under the perfect match $G(s) = \hat{G}(s)$, is reduced to

\[ y(s) = G(s)C(s)r(s) + [1-\hat{G}(s)C(s)]G(s)d(s) \]  

(3.2)
Equation (3.2) shows that “perfect” control could be achieved by selecting $C(s) = \hat{G}^{-1}(s)$.

However, to ensure the internal stability, the controller $C(s)$ should be stable. The process model is then factorized as

$$\hat{G}(s) = \hat{G}^+(s)\hat{G}^-(s)$$  \hspace{1cm} (3.3)

Where $\hat{G}^+(s)$ contains all the time delays and unstable zeros. Consequently, $\hat{G}^-(s)$ is stable and does not involve predictors. The IMC controller is designed as

$$C(s) = \hat{G}(s)^{-1}F(s)$$  \hspace{1cm} (3.4)

Where $F(s)$ is a low-pass filter which is chosen such that $C(s)$ is bi-proper. With this primary controller, it follows from (3.2) that

$$y(s) = \hat{G}^+(s)F(s)r(s) + [1-\hat{G}(s)F(s)]G(s)d(s)$$  \hspace{1cm} (3.5)

In the case of no process-model mismatch, the closed-loop transfer function $H_{yr} = \hat{G}^+(s)F(s)$ from “$r$” to “$y$” only contains the part of process ($\hat{G}^+$). This part limits the achievable control performance, but these inherent limitations cannot be removed by any control system. Thus, this IMC controller will lead to a perfect controller [34].

If the model is perfect $G(s) = \hat{G}(s)$ and there is no disturbance ($d = 0$), the model output $\hat{y}$ and the process output $y$ are the same, which causes the feedback signal $(y - \hat{y})$ to be zero. Thus, when there is no uncertainty, the control system becomes an open loop. This shows very instructively that, for open-loop stable processes, feedback is only needed because of
uncertainty. It is the character of the IMC that makes it unable to control the open-loop unstable system. The reason is simple: if \( G(s) = \hat{G}(s) \) and \( d = 0 \), the feedback signal equals to zero, which means that the IMC system is open-loop and becomes unstable for an unstable process.

Tyreuset al.[1] too have used the same idea to tune the PID controller for low-order unstable processes. However, when high-order unstable processes are considered, high-order controllers are usually required with these methods. A simple and practical scheme to improve IMC in this regard is really needed.

### 3.2.2 CHIEN AND FRUEHAUF IMC MODEL

During the course of some recent plant wide control studies, an attempt has been made to tune a composition control loop in a fairly high-purity distillation column. These columns exhibit very large time constant for small changes around the set point and the response of the process looks almost like the response of a pure integrator [1 and 9]. In [1 and 9] the authors have proposed an internal model control (IMC) approach to find the settings for a PI controller. The original discussion of PI tuning parameters based on IMC is presented by Rivera et al.[36]. The process transfer function is assumed to be

\[
G(s) = \frac{Ke^{-\theta s}}{s}
\]  

(3.6)

Where \( K=0.0506 \) and \( \theta=6 \text{Sec.} \).
Where $K(0.0506)$ is the slope of the ramp change in the controlled variable for a step change in the manipulated variable (percent/minute) and $\theta$ (6Sec) is the dead time (typically from a composition analyzer).

In the Chien and Fruehauf procedure of tuning controller, a closed loop time constant $T_{cl}$ is specified. For the considered distillation column model, the reset time $T_i$ and the gain $K_p$ of a PI controller are calculated from the following equations:

$$K_p = \frac{2\tau_{cl} + \theta}{(\tau_{cl} + \theta)^2 K}$$  

(3.7)

$$T_i = 2\tau_{cl} + \theta$$  

(3.8)

If a closed loop time constant of 16 min is specified, the PI tuning parameters are $K_p = 1.55$ and $T_i = 38$ min. If a closed loop time constant of 6 min is specified, the PI tuning parameters are $K_p = 3.47$ and $T_i = 18$ min. However, if we use classical frequency response methods to calculate the maximum closed loop log modulus for these two sets of tuning constant, the following results are obtained.

For $\tau_{cl} = 16$ min, the calculated closed loop log modulus is $L_{cl}^{max} = +3.9$ dB  

(3.9)

For $\tau_{cl} = 6$ min, the calculated closed loop log modulus is $L_{cl}^{max} = +9.0$ dB  

(3.10)

The system with the closed loop time constant of 6 min will be very oscillatory. During our simulation studies, we first found the ultimate gain $K_u$ and ultimate frequency $\omega_u$ from the relay
feedback method as 6.1 and 0.25 rad/min and then using the ultimate gain and ultimate frequency, the ZN-PI setting were determined as 3.8 and 21min respectively. The resulting response is found to be too oscillatory. This time-domain response is confirmed by calculating the maximum closed loop log modulus with this setting $L_{cl}^{\max} +9.4 \text{ dB}$ [1].

Next the IMC controller design approach proposed in [1] has been followed. When a closed loop time constant of 16min is specified, the response is good. When a closed loop time constant of 6 min was specified, the response is very oscillatory as expected because these settings are about equivalent to the Ziegler-Nichols settings. This example (model of the distillation column) illustrates that the IMC approach requires some trial and error in order to specify a closed loop time constant that will give a reasonable closed loop damping coefficient.

3.2.3 Design of Feedback Control Scheme Using Frequency Response Technique

3.2.3.1 Z-N Method

Initially the ZN method of tuning PI controller based on the ultimate gain and ultimate period has been adapted, which results in the oscillatory response for the longer time intervals. In the closed loop identification the ultimate gain “$K_u$” and ultimate period “$P_u$” were obtained using the thumb rule and the controller parameter values were obtained as given below

$$K_p = \frac{K_u}{2.2} = 2.385$$  \hspace{1cm} (3.11)

$$T_i = 0.8 * P_u = 0.049$$  \hspace{1cm} (3.12)
The servo and regulatory performances have been found to be oscillatory.

3.2.3.2 IMC- Controller Design

As per the controller settings discussed in [1], we found the controller settings of PI controller by selecting the proper closed loop time constant which gives lesser overshoot and undershoot. The controller parameter values are derived from the formulae

\[ K_p = \frac{2\tau_{cl} + \theta}{(\tau_{cl} + \theta)^2 K} = 1.55 \]  \hspace{1cm} (3.13)

\[ T_i = 2\tau_{cl} + \theta = 38 \text{ min} \]  \hspace{1cm} (3.14)

results in the poor response for the integrating process with dead time. It should be noted that the value of the closed loop time constant. The closed loop time constant for which the system generates lesser oscillations have to be carefully selected.

3.3 Controller design using frequency response technique

Figure 3.2 gives a Nyquist plot of the process described by eq.3.6. Note that it starts (when \( \omega = 0 \)) at \(-90^0\) because of the integrator. Also as shown in Figure 3.2 are two plots of \( G(s)C(s) \) for a PI controller with two different values of reset time (20 and 52.5 min). Now the plots start at \(-180^0\) because of the double integrator. If the reset time is chosen to be too small, the \( G(s)C(s) \) curve will be very close or may even encircle the (-1,0) point no matter what value of gain is used. Therefore there is a minimum reset time below which stability and reasonable damping cannot be achieved. The damping of a closed loop system is directly related to the minimum distance
between the $G_M B$ curve and the (-1, 0) point. The parameter that indicates this minimum distance is the maximum closed loop log modulus, i.e., the peak in the log modulus plot of the closed loop servo transfer function.

$$L_{CL} = 20 \log \left| \frac{G_M B}{1 + G_M B} \right|$$  \hspace{1cm} (3.15)

It is important to also realize that even for reset times greater than the minimum there is a non-monotonic relationship between the maximum closed loop log modulus and the controller gain $K_p$. This is illustrated in Figure 3.3(b) and 3.4. When the gain is small, the low-frequency portion of the curve is close to the (-1, 0) point.

Figure 3.2 Nyquist plot with various integral time ($T_i$)
This gives a large resonant peak at low frequency. When the gain is large, the high frequency portion of the curve is close to the (-1,0) point. This gives a large resonant peak at high frequency. Thus there is an optimum controller gain that gives the smallest peak in the curve.

Another way to look at this system is to use the phase angle plot from an open loop Bode plot. The effect of changing reset time is shown in Fig. 3.3(b). The smaller the $T_i$, the lower the peak in the phase angle curve.

![Bode Diagram](image)

**Figure 3.3(a) Bode plot (Magnitude Vs Frequency)**

![Closed Loop Bode plot](image)

**Fig.3.3(b) Phase angle plot for open loop IPDT with PI control**
One way to find the maximum reset time is to specify what the maximum peak in the phase angle plot should be and then calculate the corresponding $\tau_{i\text{ min}}$. This can be done analytically for this simple process.

\[
G(s)C(s) = \frac{K_pe^{\phi_s}}{s} \frac{K_c(T_p s + 1)}{T_i s}
\]  

(3.16)

\[
\arg(G(s)C(s)) = -\pi - \omega \theta + \arctan(\omega \tau_i)
\]  

(3.17)

The peak in the phase angle curve will occur where the derivative with respect to frequency is zero.
\[
\frac{d}{d\omega} \arg(G(s)C(s)) = -\theta + \frac{\tau_i}{1 + \omega_p^2 \tau_i^2} = 0
\]

(3.18)

Where \( \omega_p \) = frequency where the peak in the phase angle curve occurs. Solving eq. (3.18) for \( \omega_p \) gives

\[
\omega_p = \frac{1}{\tau_i} \left( \frac{\tau_i - \theta}{\theta} \right)^{1/2}
\]

(3.19)

Substituting back into eq. (3.17) gives

\[
\arg(G(s)C(s))_{\theta \text{ peak}} = -\pi - \theta \left( \frac{T_i - \theta}{\theta} \right)^{1/2} + \arctan \left( \frac{T_i - \theta}{\theta} \right)^{1/2}
\]

(3.20)

Note that this equation can be rearranged so that the only variable is the \( T_i / \theta \) ratio.

\[
\arg(G(s)C(s))_{\theta \text{ peak}} = -\pi - \theta \left( \frac{T_i}{\theta} - 1 \right)^{1/2} + \arctan \left( \frac{T_i}{\theta} - 1 \right)^{1/2}
\]

(3.21)

Now the question is what maximum phase angle do we need to get a closed loop system that displays a reasonable amount of damping. The inference from the closed loop bode log modulus curve, the open loop phase angle must rise up to at least \(-128^\circ\) for \( K_p = 1.6 \). Therefore we specify that the peak phase angle should be \( 3.23 \text{ rad} \). Solving for the required \( T_i / \theta \) ratio gives

\[
T_i / \theta = 8.75
\]

(3.22)
Thus we can calculate the optimum reset time once the dead time is known. Now we want to find the value of the controller gain $K_p$ that gives the smallest value of the maximum closed loop log modulus. The maximum closed loop log modulus should be +2dB at this gain $K_p$, if the design procedure has been followed correctly. It is easy to vary $K_p$ over a range, till it reaches the closed loop log modulus peak value of +2dB (as shown in figure 3.3(a)). By looking at several numerical cases we found that the optimum can be simply expressed as a function of the dead time ($\theta$) and process gain ($K$).

$$K_p = \frac{0.487}{K\theta} \quad \text{(3.23)}$$

This unique relationship occurs because of shifting both the phase angle curves and the log modulus curves together along the frequency axis as the dead time varies. This relationship can probably be derived analytically if one can achieve through the complex algebra that gives the peak in the closed loop log modulus plot.

The summary of design procedure is as follows [1]:

1) To determine $K_p$ and $\theta$:

$$[\arg(G(S))]|_{\omega_u} = -\pi = -\omega_u \theta - \pi / 2 \quad \text{(3.24)}$$

$$|G_M|_{\omega_u} = \frac{1}{K_u} = \frac{K}{\omega_u} \quad \text{(3.25)}$$

Where $K_u$ and $\omega_u$ are ultimate gain and ultimate frequency.

2) To find the minimum reset time $T_i$:

$$\frac{T_i}{\theta} = 8.75 \quad \text{(3.26)}$$
3) To calculate the optimum controller gain $K_p$:

$$K_p = \frac{0.487}{K\theta} = 1.55$$  \hspace{1cm} (3.27)

An alternative way to present the above formulae so as to compare with Ziegler-Nichols formulae are:

$$T_i = 2.2P_u$$  \hspace{1cm} (3.28)

and

$$K_p = \frac{K_u}{3.22}$$  \hspace{1cm} (3.29)

Whereas Ziegler-Nichols formulae are:

$$T_i = \frac{P_u}{1.2}$$  \hspace{1cm} (3.30)

and

$$K_p = \frac{K_u}{2.2}$$  \hspace{1cm} (3.31)

### 3.4 Servo Response

The servo response of various control schemes are shown in the figure 3.5.

![Figure 3.5 Servo response of the process](image)

From Fig.3.5 it can be inferred that the optimum PI controller settings provide satisfactory servo response compared to ZN-PI and IMC-PI.
Table 3.1: PI controller settings for the integrating process with dead time.

<table>
<thead>
<tr>
<th>Tuning Method</th>
<th>K_p</th>
<th>T_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>IMC-PI</td>
<td>1.55</td>
<td>38.02</td>
</tr>
<tr>
<td>ZN-PI</td>
<td>2.385</td>
<td>20.4</td>
</tr>
<tr>
<td>Optimum PI</td>
<td>1.6</td>
<td>52.5</td>
</tr>
</tbody>
</table>

3.5 Controller Design based on the Performance Indices

Consider a unity-feedback control system with \( G(s) \) and \( C(s) \), where \( C(s) \) is a PID controller in parallel (non-interacting) form, whose expression is:

\[
C(s) = K_c \left( 1 + \frac{1}{T_i s} + T_d s \right)
\]  

(3.32)

For the transfer function (eq-3.32) to be proper, a high frequency pole has to be added in practical cases, i.e. the derivative action has to be filtered. The plant transfer function \( G(s) \) is assumed as:

\[
G(s) = \frac{K e^{-\delta s}}{s}
\]  

(3.33)

![Figure 3.6 A unity-feedback control system](image-url)
Note that transfer function (eq. 3.33) is capable of satisfactorily modeling the dynamics of a large number of integrating processes. The control scheme can be subjected to step changes both in the set-point signal \( r \) and in the load disturbance signal \( d \).

The tuning problem consists of selecting the values of \( K_p, T_i \) and \( T_d \) in order to fulfill some performance requirements in transient responses. The PD controller needs to be tuned in order to minimize an integral performance criterion such as the integral squared error (ISE). Time-moment weighted integral performance criteria can be minimized as well. Hence, the following performance indexes can be considered:

\[
J_n(\theta) = \int_0^\infty t^n [e(\theta, t)]^2 dt. n = 0, 1, 2, \ldots
\] (3.34)

where \([K_p, T_i, T_d]\) is the parameters vector to be selected to minimize (3.34) and \( e(t) = r(t) - y(t) \) is the system error. Note that \( J_n(\theta) \) is denoted as the ISE criterion, while \( J_1(\theta) \) and \( J_2(\theta) \) are known respectively as the ITSE and ITAE criteria. Note also that, owing to the presence of the integral part in the controller, the performance indexes are always finite.

Table 3.2: Tuning rules for optimal set-point response for integrating process

<table>
<thead>
<tr>
<th>PID parameter</th>
<th>ISE</th>
<th>ITSE</th>
<th>ITAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_p )</td>
<td>1.03/K( \theta )</td>
<td>0.96/K( \theta )</td>
<td>0.90/K( \theta )</td>
</tr>
<tr>
<td>( T_i )</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( T_d )</td>
<td>0.49( \theta )</td>
<td>0.45( \theta )</td>
<td>0.45( \theta )</td>
</tr>
</tbody>
</table>
Table 3.3: Tuning rules for optimal load disturbance rejection for integrating process

<table>
<thead>
<tr>
<th>PID parameter</th>
<th>ISE</th>
<th>ITSE</th>
<th>ITAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>1.37/$K\theta$</td>
<td>1.36/$K\theta$</td>
<td>1.34/$K\theta$</td>
</tr>
<tr>
<td>$T_i$</td>
<td>1.49$\theta$</td>
<td>1.66$\theta$</td>
<td>1.83$\theta$</td>
</tr>
<tr>
<td>$T_d$</td>
<td>0.59$\theta$</td>
<td>0.53$\theta$</td>
<td>0.49$\theta$</td>
</tr>
</tbody>
</table>

3.6 INTEGRATING PROCESS WITH DEAD TIME

As an illustrative example regarding integrating processes with dead time, consider the transfer function

$$G(s) = \frac{0.0506e^{-6s}}{s}$$ (3.35)

Table 3.4: PID parameter values for set-point response of the integrating process example

<table>
<thead>
<tr>
<th>PID parameter</th>
<th>ISE</th>
<th>ITSE</th>
<th>ITAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>3.394</td>
<td>3.164</td>
<td>3.96</td>
</tr>
<tr>
<td>$T_i$</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$T_d$</td>
<td>3.94</td>
<td>3.70</td>
<td>3.70</td>
</tr>
</tbody>
</table>

Table 3.5: PID parameter values for load disturbance response of the integrating process

<table>
<thead>
<tr>
<th>PID parameter</th>
<th>ISE</th>
<th>ITSE</th>
<th>ITAE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>4.51</td>
<td>4.48</td>
<td>4.41</td>
</tr>
<tr>
<td>$T_i$</td>
<td>8.94</td>
<td>9.96</td>
<td>10.98</td>
</tr>
<tr>
<td>$T_d$</td>
<td>3.54</td>
<td>3.18</td>
<td>3.94</td>
</tr>
</tbody>
</table>
Applying the proposed optimal tuning methodology the PID setting have been determined and the resulting values if the PID parameters are reported in Tables 3.4 and 3.5 for the set-point following and load disturbance rejection case, respectively. The servo and regulatory response of the integrating process with dead time are shown in Fig.3.7 and Fig.3.8 respectively.

![Servo response Vs Time](image)

Figure 3.7 Set-point response of integrating process with time delay

The comparison of the step response of the controllers tuned based on ZN-PI, ISE criterion tuning, ITSE criterion tuning and ITAE criterion tuning are shown in Fig.3.7. It is observed that the PI/PID controller tuned based on the performance index gives better result than the ZN-PI Method
Fig. 3.8 reveals that the performance index based controller gives better response when compared to the ZN-PI controller.

From the presented results it turns out that using the ITAE criterion is more convenient for set-point following since a smaller overshoot and a shorter settling time is achieved. The same applies to the load disturbance rejection, where again the ITAE criterion guarantees the shortest settling time.
3.7 Controller Tuning Based On Equating Coefficient Method

The most widely used controller in industry is the Proportional-Integral-Derivative (PID) controllers. This is due to their simple structure and ease of use in addition to good robustness and wide range of applicability. This thesis deals with a simple method of designing various PI, PD and PID controllers for pure integrating systems with dead time. The method used is based on matching the corresponding coefficients of $s$, $s^2$, $s^3$ in the numerator and that in the denominator of the closed loop transfer function for a servo problem. The performance of the PID controller is compared with the controller designed by Tyreus et al.[1], A.Visioli et al. [6] with and without parameter uncertainty in the dead time of the process.

3.8 Dead Time Compensator for Integrating Processes with Dead Time.

A dead time compensator for controlling processes with integral action and long dead-time has been proposed by Matausek, M. Ret al.[37] and tuning formulas have been derived. If the velocity gain and dead time are estimated experimentally, only one parameter, the time constant defining the speed of the closed-loop setpoint response, has to be tuned manually. The same setpoint response is obtained as in the modified smith predictor, while the load disturbance rejection is much faster.

The tuning formulae [37] reduce the number of adjustable parameters to three, the velocity gain, the dead-time, and the time constant defining the speed of the closed loop
setpoint response. The dead time compensator proposed is a further development of the modified smith predictor. By adding a derivative action, the same setpoint response is obtained, while the load disturbance rejection is considerably faster.

The dead time compensator proposed, has a simple structure and can easily be tuned manually. The proposed DTC provides considerably faster load disturbance rejection while preserving the same setpoint. Thus the modified smith predictor appears as a particular case of the proposed dead time compensator.

The advantage of the new design is that the setpoint response is decoupled from the load response and hence can be independently optimized. It is easy to see that the new Smith predictor structure gives the designer more freedom to choose the closed-loop poles. Hence, there is a significant improvement in performance.

3.9 Controller Tuning Methods for Integrating Process with Dead Time

3.9.1 Tuning Methodology – Equating Coefficient

An integrating process is represented by $\frac{Ke^{-qs}}{s}$. The closed loop transfer function relating the output($y$) to the set point ($y_r$) is given by:

$$\frac{y(q)}{y_r(q)} = \frac{(K_1q + K_2 + K_3q^2)\exp(-q)}{[q^2 + (K_1q + K_2q + K_3q^2)\exp(-q)]}$$

(3.36)
Optimal Robust $H^\infty$ Controller for Unstable Processes with Dead Time

Where

$$K_1 = K_c K_p T_d$$  
(3.37)

$$K_2 = \frac{K_i}{T_i T_d}$$  
(3.38)

$$K_3 = \frac{K_i}{T_i T_d}$$  
(3.39)

$$q = \theta s$$  
(3.40)

Here “s” is the laplace operator. Using Pade’s approximation for exp (-q) as \(\frac{(1-0.5q)}{(1+0.5q)}\) in the denominator of eq. (3.36). The resulting equation can be written as

$$\frac{y(q)}{y_r(q)} = \frac{[(K_i q + K_2 + K_3 q^2)(1+0.5q)e^q]}{[q^2 (1+0.5q) + (K_i q + K_2 + K_3 q^2) (1-0.5q)]}$$  
(3.41)

exp(-q) in the numerator is removed for further study, since this term only shifts the corresponding time axis. Since the objective of the control system is to make \(y\) follow \(y_r\), the corresponding coefficients of \(q\), \(q^2\) and \(q^3\) of the numerator with that of the denominator are equated. Since the presence of the integral mode makes the offset zero, the constant term in the numerator and that in the denominator is the same. By equating the corresponding coefficient of \(q\), \(q^2\) and \(q^3\) of the numerator with that of the denominator, the following equations are obtained.
K₂ = 0 \quad (3.42)

K₁ = 1 \quad (3.43)

K₃ = 0.5 \quad (3.44)

Using the definitions of these constants [eq. (3.37) to eq. (3.39)], the controller settings are obtained which is the setting of a PD controller, A. Visoli et al. [6].

\[ K_c K_p T_d = 1 \quad (3.45) \]

\[ \frac{T_i}{T_d} = \infty \quad (3.46) \]

\[ \frac{\theta}{T_d} = 0.5 \quad (3.47) \]

As we already know the value of Kₚ and Tₜ we can find other parameters required to tune the controller.

Kₖ = 3.2938 \quad (3.48)

Kᵢ = 0 \quad (3.49)

K₅ = 3 \quad (3.50)

Where \( K_i = \frac{1}{T_i} \) and \( K_d = T_d \).

Since with a PI or a PID controller, the closed loop response shows some overshoot for the servo response, the value of \( \frac{y(q)}{y_r(q)} \) can be allowed to be more than one.
At \( q=0 \), \( y \) is automatically equal to \( y_r \), because of the presence of the integral action. Therefore, the corresponding coefficients of “\( q \)” of the numerator are equated to \( \alpha \) times that of the denominator. Here the value of \( \alpha \) is greater than one and this parameter is considered as a tuning parameter. The following set of linear algebraic equations is obtained as:

\[
\begin{align*}
(1-\alpha)K_1 + 0.5(1+\alpha)K_2 & = 0 \\
0.5(1+\alpha)K_1 + (1-\alpha)K_3 & = \alpha \\
(1+\alpha)K_3 & = \alpha
\end{align*}
\]  

For \( \alpha=1 \), as stated earlier, it results in PD type control law. By solving the above equations the PID controller parameters are obtained.

\[
K_cK_pT_d = \frac{4\alpha^2}{(1+\alpha)^2}
\]  

\[
\frac{T_i}{T_d} = \frac{0.5(\alpha+1)}{\alpha-1}
\]  

\[
\frac{\theta}{T_d} = \frac{0.25(\alpha+1)}{\alpha}
\]

Similarly by considering only PI controller mode in eq. (3.36), the following equations for the PI controller settings are obtained.

\[
K_cK_pT_d = \frac{2\alpha}{(\alpha+1)}
\]

\[
\frac{T_i}{T_d} = \frac{0.5(\alpha+1)}{(\alpha-1)}
\]
Where “α” is a tuning parameter and should be greater than one. Similar to the IMC method, care should be taken in selecting this tuning parameter.

Assuming α = 1.25 in eq. (3.51) to eq. (3.53), the following relations are obtained.

\[ K_c K_p T_d = 1.2346 \]  
\[ \frac{T_i}{T_d} = 4.5 \]  
\[ \frac{\theta}{T_d} = 0.45 \]

Substituting the values of \(K_p\) and \(T_d\) in the above equations:

\[ K_c = 4.0666 \]  
\[ K_i = 0.037 \]  
\[ K_d = 3.7 \]

where \(K_i = \frac{1}{T_i}\) and \(K_d = T_d\).

All the methods reported in this thesis are valid for \(T_d > 0\). When \(T_d = 0\), the transfer function model is a pure integrator. However, the controller gives an oscillatory response when the uncertainty in the time delay is +33%. Therefore the present method is extended using two tuning parameters \(\alpha_1\) and \(\alpha_3\). Writing \(\exp(-q)\) in the denominator of eq.(3.36) as \(\frac{\exp(-0.5q)}{\exp(0.5q)}\), the following equation for the closed loop transfer function is obtained as:

\[
\frac{y(q)}{y(q)} = \frac{\left[\left(K_1q + K_2 + K_3q^2\right)e^{0.5q}e^{-q}\right]}{\left[q^2e^{0.5d} + \left(K_1q + K_2 + K_3q^2\right)e^{-0.5q}\right]} 
\]  
\[ (3.64)\]
The numerator and denominator terms using the Taylor series expansion for $e^{0.5q}$ and $e^{-0.5q}$ are considered. The coefficient of $q$ in the numerator is equated $\alpha_1$ times that of denominator of the closed loop transfer function. The coefficients of $q^2$ and $q^3$ of the numerator are equated to $\alpha_2$ times of that of the denominator. The following set of linear algebraic equations is obtained:

\[ (1-\alpha_1)K_1 + 0.5(1+\alpha_1)K_2 = 0 \]  \hspace{1cm} (3.65)

\[ 0.5(1+\alpha_2)K_1 + 0.125(1-\alpha_2)K_2 + (1-\alpha_2)K_3 = \alpha_2 \]  \hspace{1cm} (3.66)

\[ 0.125(1-\alpha_2)K_1 + 0.0208(1+\alpha_2)K_2 + 0.5(1+\alpha_2)K_3 = 0.5\alpha_2 \]  \hspace{1cm} (3.67)

By simulation it has been found that $\alpha_2 = 0.6\alpha_1$ and $\alpha_1 = 1.5$ give best result. Thereby the present method has ultimately no tuning parameters. Solving these equations, $K_1$, $K_2$ and $K_3$ are obtained. Using these definitions of $K_1$, $K_2$ and $K_3$, PID controller settings are obtained as:

\[ K_cK_pT_d = 0.8956 \]  \hspace{1cm} (3.68)

\[ \frac{T_i}{T_d} = 3.5 \]  \hspace{1cm} (3.69)

\[ \frac{\theta}{T_d} = 0.5 \]  \hspace{1cm} (3.70)

Substituting the values of $K_p$ and $T_d$ in the above equations:

\[ K_c = 3.9499 \]  \hspace{1cm} (3.71)

\[ K_i = 0.0666 \]  \hspace{1cm} (3.72)

\[ K_d = 3 \]  \hspace{1cm} (3.73)

where $K_i = \frac{1}{T_i}$ and $K_d = T_d$. 
The PID controller settings proposed by A. Visioli et al. [6] are:

\[ K_c K_p T_d = 1.37 \]  \hspace{1cm} (3.74)

\[ \frac{T_i}{T_d} = 1.49 \]  \hspace{1cm} (3.75)

\[ \frac{\theta}{T_d} = 0.59 \]  \hspace{1cm} (3.76)

Substituting the values of \( K_p \) and \( T_d \) in the above equations:

\[ K_c = 4.5125 \]  \hspace{1cm} (3.77)

\[ K_i = 0.11186 \]  \hspace{1cm} (3.78)

\[ K_d = 3.54 \]  \hspace{1cm} (3.79)

where \( K_i = \frac{1}{T_i} \) and \( K_d = T_d \).

The PID settings obtained by Tyreus and Luyben [1] are:

\[ K_c = 3.5639 \]  \hspace{1cm} (3.80)

\[ K_i = 0.0177 \]  \hspace{1cm} (3.81)

\[ K_d = 3.561 \]  \hspace{1cm} (3.82)

where \( K_i = \frac{1}{T_i} \) and \( K_d = T_d \).

With a first order filter time constant as 0.382

The robustness of the proposed controller is studied by using about 33.3% perturbation in \( \tau_d \) from the nominal value in the simulation (\( T_d = 8 \)) whereas the controller settings are those calculated for the process with nominal time delay (\( T_d = 6 \)).
3.9.2 Dead time compensator

![Diagram of Dead Time Compensator]

Fig. 3.9 General Structure of the dead time compensator

The structure of the proposed DTC for controlling a process with an integrator and long dead-time is shown in Fig. 3.9, where

\[ F(s) = \frac{K_0(T_0s + 1)}{T_r s + 1}, \quad T_r = \frac{T_d}{10} \quad (3.84) \]

When \( T_d = 0 \) (\( F(s) = K_0 \)) the modified Smith predictor structure is obtained. For this case, supposing that the essential character of the process \( G_p(s)e^{-\theta s} \) with an integrator can be approximated with the model \( \frac{Ke^{\theta s}}{s} \), tuning formulas are derived in for adjusting controller parameters \( K_0 \) and \( K_r \). If the velocity gain \( K \) and the dead-time \( \tau \) are estimated experimentally, only one parameter, the time constant \( T_r \) defining the speed of the closed-loop setpoint response, has to be tuned manually.
The modification proposed in the present method is to add the derivative action. Following the general idea usually applied when deriving the tuning formulas for the adjustment of the controller parameters, the model is assumed to be a perfect representation of the unknown plant \( G(s)e^{-\theta s} \); namely, it is assumed that

\[
G(s) = \frac{K}{s}, \quad \theta = \tau \quad (3.85)
\]

Then, the set point load disturbance is given by

\[
Y(s) = H_d(s)R(s) + H_d(s)D(s) \quad (3.86)
\]

Where

\[
H_i(s) = \frac{KK_se^{-ts}}{s + KK_r} \quad (3.87)
\]

\[
H_d(s) = \frac{K[s + KK_r(1-e^{-ts})]e^{-ts}}{(s + KK_r)(s + \frac{K_dK(t_d s + 1)e^{-ts}}{(T_d s + 1)})} \quad (3.88)
\]

From eq.(3.87) and eq.(3.88) that the stability of the proposed dead time compensator depends on the roots of the characteristic equation. \((s+KK_r)(s + \frac{K_dK(T_d s + 1)}{(T_d s + 1)}e^{-ts}) = 0 \quad (3.89)\)

Thus, the derivation of tuning formulas for the gain \( K_0 \) and the derivative time constant \( T_d \) can be based on the analysis of the following characteristic equation:

\[
1 + W(s) = 0 \quad (3.90)
\]

Where

\[
W(s) = \frac{K_0K(T_d s + 1)}{s(T_d s + 1)}e^{-ts} \quad (3.91)
\]
Introducing the parameter $\Phi_{pm}$ i.e. the phase margin of the closed loop system with the Eq.(3.3.8) as loop transfer function we obtain

$$\Phi_{pm} = \pi + \arg\{W(j\omega_1)\} \quad (3.92)$$

$$|W(j\omega_1)| = 1 \quad (3.93)$$

Choosing the derivative time constant $T_d$ proportional to the dead time $\tau$ we have

$$T_d = \alpha \tau \quad 0 \leq \alpha < 1. \quad (3.94)$$

Then by ignoring the influence of the low-pass filter time constant $T_f$ (since $T_f = \frac{T_d}{10}$) and using the approximation $\arctan(\omega_1) = \omega_1$ from eq. (3.93) we obtain

$$\omega_1 = \frac{\pi}{2} - \phi_{pm} \quad (3.95)$$

From eq.(3.92)

$$\omega_1 = \frac{KK_0}{\sqrt{1-(KK_0\alpha\tau)^2}} \quad (3.95)$$

Where it is supposed that eq. (3.96) holds true

$$0 \leq KK_0\alpha\tau < 1 \quad (3.96)$$

From eq. (3.94) and eq. (3.95) the next formula for the gain $K_0$ is obtained

$$K_0 = \frac{\frac{\pi}{2} - \phi_{pm} \quad (3.97)}{K\tau \sqrt{(1-\alpha)^2 + \left(\frac{\pi}{2} - \phi_{pm}\right)^2 \alpha^2}}$$

eq.(3.97) implies that
\[ KK_0 \alpha \tau = \frac{\alpha (\frac{\pi}{2} - \phi_{pm})}{\sqrt{(1 - \alpha)^2 + (\frac{\pi}{2} - \phi_{pm})^2 \alpha^2}} \]  

Consequently eq.(3.96) is also fulfilled

This completes the design of the proposed dead time compensator. Using eq.(3.93) to eq.(3.97) it follows that the dead time compensator has five adjustable parameters namely \( K, \tau, K_r, \alpha, \Phi_{pm} \).

Now an efficient tuning of the proposed controller will be derived, reducing the number of adjustable parameters to the smallest set.

The tuning formula for the main controller gain \( K_r \) is given by

\[ K_r = \frac{1}{KT_r} \]  

There are several other conventional controller design techniques for the unstable processes are also discussed in [69-71, 75].
3.10 Simulation Results

Fig. 3.10: Servo response of the plant $G(s) = \frac{0.0506e^{-6s}}{s}$

Fig. 3.10 shows the comparison of the various control schemes for the integrating process with dead time. The PD controller design based on the Visioli’s approach gives the least settling time when compared to the other conventional methods.
Fig 3.11: Regulatory response of the plant $G(s) = \frac{0.0506e^{6s}}{s}$

The regulatory responses of the conventional controllers for the process with dead time are shown in the above figure 3.11. It is observed that the PI controller based on the Padmasree-Chidambaram method gives the fast recovery from load disturbance.
Fig 3.12 PI controller response of the plant $G(s) = \frac{0.0506e^{-6s}}{s}$ using Zeigler Nichols tuning method

Controller settings:

\[ K_p = 3.97 \]
\[ T_i = 0.05 \]

From Fig.3.12 it is observed that the proposed PI controller based on the ZN-PI gives the settling time of around 170 Sec for the unstable process with dead time.
Fig 3.13 PID controller response of the plant $G(s) = \frac{0.0506e^{6s}}{s}$ using ZN-PID tuning method

Controller settings:

$K_p = 3.96$
$T_i = 0.083$
$T_D = 3$

From Fig.3.13 it is observed that the proposed PID controller based on the ZN-PID method improved the settling time of the process. The settling time is around 145 Sec.
Fig. 3.14. Servo response of the plant $G(s) = \frac{0.0506e^{-6s}}{s}$ in the presence of uncertainty in dead time.

The conventional control scheme (Visoli-PI) fails to generate the bounded output in the presence of 33\% uncertainty in dead time.
Fig.3.15. Regulatory response of the plant \( G(s) = \frac{0.0506e^{6s}}{s} \) in the presence of uncertainty in dead time.

The conventional controller design (Visioli-PI) fails to generate the bounded output in the presence of 33% uncertainty in dead time.
Fig 3.16 Servo response of the plant $KG_p = \frac{0.0506e^{-6s}}{s}$ in the presence of uncertainty in dead time.

The conventional control scheme (Luyben-PI) gives oscillatory response in the presence of 33% uncertainty in dead time.
Fig. 3.17. Regulatory response of the plant $KG_p = \frac{0.0506e^{6s}}{s}$ in the presence of uncertainty in dead time.

The conventional control scheme (Luyben-PI) gives oscillatory response in the presence of 33% uncertainty in the dead time for the regulatory response.
The conventional control scheme (Chidambaram-PI) fails to maintain the stability in the presence of 33% uncertainty in dead time.
3.19 Regulatory response of the plant $KG_p = \frac{0.0506e^{-6s}}{s}$ in the presence of uncertainty in dead time.

The conventional control scheme (Chidambaram-PI) shows the oscillatory regulatory response in the presence of the uncertainty in dead time.
Optimal Robust $H^\infty$ Controller for Unstable Processes with Dead Time

Fig 3.20 Servo response of the plant $G(s) = \frac{0.0506e^{-6s}}{s}$ using PI controller (Padmasree-Chidambaram Method) in the presence of uncertainty in dead time.
Fig. 3.21. Regulatory response of the plant $G(s) = \frac{0.0506e^{-6s}}{s}$ using PI controller (Padmasree-Chidambaran Method). In the presence of uncertainty in dead time.
The controller effort made by Vissioli-PD controller is very less to eliminate the disturbance, when compared to the other conventional controller designs.
The controller effort made by Padmasree-PI controller is very less to eliminate the disturbance, when compared to the other conventional controller designs.
The Equating coefficient method fails to maintain the stability of the unstable process with dead.
Fig.3.25 Regulatory response of the plant $G(s) = \frac{0.0506e^{-6s}}{s}$ (Chidambaram method)

The Equating coefficient method fails to recover from the load disturbance
Fig. 3.26. Servo response of the dead time compensator by Matausek for the plant

\[ G(s) = \frac{0.0506e^{-6s}}{s} \]

Tuning and Filter Parameters:

\[ K_r = 19.76 \]

\[ F(s) = 0.99 \]

Settling time = 18 sec
Fig 3.27: Manipulated variable of the DTC for the plant $G(s) = \frac{0.0506e^{-6s}}{s}$
Fig 3.28: Servo and regulatory response of the DTC for the plant \( G(s) = \frac{0.0506e^{-6s}}{s} \) with error in estimating dead time.

The settling time of around 22 sec is obtained with the filter parameters of \( K_r = 9.88 \) and \( F(s) = 0.99 \).
3.11 Chapter Conclusion

A proposed design procedure permits the calculation of proper controller settings for the unstable process with dead time. The basic criterion is that the maximum closed loop log modulus should be +2dB. This is essentially equivalent to specifying a closed loop damping coefficient of about 0.4 and then finding the best settings that will minimize the closed loop time constant i.e. maximize closed loop resonant frequency. Unlike the IMC approach in which the closed loop time constant must be assumed and then the results tested, the proposed procedure involves no trial and error. These settings have been tested on a wide variety of processes and have worked much better than the classical Ziegler-Nichols settings.

New tuning method, based on the minimization of integral criteria, for integrating plus time delay processes has been presented. Analytical functions are given for the determination of the PID parameters as well as for the determination of the value of the adopted objective functions, which is useful for performance assessment of pre-existing controllers. Also the PID controller based on the equation coefficient method is also tuned. Results show the effectiveness of the methodology and how the use of the ITAE criterion is generally the most convenient to be employed to assure good setpoint responses and load disturbance rejection. Dead Time Compensator design also shows the good response for the setpoint tracking and load disturbance rejection for the nominal plant and fails to stabilize the system under perturbations. This limitation can be overcome by designing the robust controller for the integrating process with dead time.

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