CHAPTER 4

ANALYTICAL INVESTIGATION ON FERROCEMENT LAMINATES AND FLEXURAL STRENGTHENING OF R.C BEAMS WITH FERROCEMENT LAMINATES

4.1 GENERAL

This chapter describes the analytical investigation on flexural behaviour of ferrocement laminates based on plastic and elastic approach (ACI 549-R1997, Naaman, 2000) and flexural strengthening of reinforced concrete beams with ferrocement laminates (IS 456 2000).

4.2 ANALYTICAL INVESTIGATION ON FERROCEMENT LAMINATES

The development of an analytical model to compare the experimentally obtained ultimate moment of resistance of the ferrocement laminates reinforced separately with weld mesh and chicken mesh in flexure is discussed. The methods adopted were based on plastic and elastic analysis approach.

4.2.1 Terminology (Naaman, AE, 2000)

4.2.1.1 Volume fraction of reinforcement

The volume fraction of reinforcement is the ratio of volume of reinforcement to the volume of composite.
When the square or rectangular wire mesh is used throughout the depth of a ferrocement element, the volume fraction of reinforcement can be calculated from the equation 4.2:

\[ V_r = \frac{\text{V}_{\text{reinforcement}}}{\text{V}_{\text{composite}}} \]  

(4.1)

\[ V_{\text{reinforcement}} - \text{volume of reinforcement} \]
\[ V_{\text{composite}} - \text{volume of composite} \]

\[ V_r = \frac{N_1 \pi d_w^2}{4h} \left( \frac{1}{D_L} + \frac{1}{D_T} \right) \]  

(4.2)

\[ D_L - \text{center to center distance between longitudinal wires} \]
\[ D_T - \text{center to center distance between transverse wires} \]
\[ h - \text{thickness of ferrocement element} \]
\[ N - \text{number of layers of mesh} \]
\[ d_w - \text{diameter of mesh wire} \]

For other type of meshes, (hexagonal or chicken wire, expanded metal, fibre reinforced plastic-FRP or irregular meshes) equation 4.3 is recommended for use.

\[ V_r = \frac{NW_r}{h \gamma_r} \]  

(4.3)

\[ W_r - \text{unit weight of reinforcing mesh (kg/m}^2) \]
\[ \gamma_r - \text{density of reinforcement material (kg/m}^3) \]

Using the above expression, the volume fraction of weld mesh and chicken mesh was determined.
4.2.1.2 Effective area of reinforcement

The effective area of reinforcement, resisting tension and associated with a typical mesh layer \( i \) of a ferrocement section subjected to direct tension or bending is given in equation (4.4).

\[
A_{ri} = \eta \times V_{ri} \times A_c
\]

where,
- \( V_{ri} \) - total volume fraction of reinforcement due to mesh layer \( i \)
- \( A_c \) - cross sectional area of the ferrocement composite section
- \( \eta \) - efficiency factor of reinforcement
- \( A_{ri} \) - effective area of reinforcement

4.2.1.3 Efficiency factor of reinforcement (\( \eta \))

Efficiency factor of reinforcement is the equivalent cross sectional area of reinforcement in the direction of loading considered.

4.3 ANALYTICAL METHOD FOR THE PRESENT STUDY

4.3.1 Plastic analysis approach

A simple method, based on the concept of plastic analysis is proposed herein to calculate the moment capacity of ferrocement laminates reinforced separately with weld mesh and chicken mesh. The following assumptions were made in calculating the ultimate moment capacity of ferrocement laminates in flexure:

- Plane sections normal to the axis remained plane even after bending.
- The material was homogeneous.
- The tensile strength of mortar was neglected.
The compressive stress in the mortar was represented by a rectangular stress block.

- The maximum compressive strain was 0.003.
- The distribution of reinforcement was uniform throughout.
- A perfect bond exists between the wire meshes and matrix.

The idealized stress strain block for the ferrocement laminate at ultimate under bending was considered, as shown in Figure 4.1. (Naaman, AE 2000).

The efficiency factor $\eta$ should be taken for the direction of mesh resisting the load. The depth of plastic neutral axis ‘$a$’ was determined from the consideration of equilibrium of forces in the horizontal direction. From the stress strain block in Figure 4.1, the equations for compressive and tensile forces were obtained.

![Figure 4.1 Stress strain block for ferrocement laminate (plastic method)](image)

Total compression $(C) = \text{Total tension (T)}$

Total compressive force $= 0.85 \times f'c \times b_f \times a$ \hspace{1cm} (4.5)

Total tensile force $= \sigma_c \times b_f \times (h - a)$ \hspace{1cm} (4.6)

Total force = Compressive force + Tensile force
The neutral axis depth ‘a’ was determined by solving the equation 4.8.

\[ a = \frac{\eta X h}{0.85 + \eta X} \]  

(4.8)

where \( X = \text{factor} = \frac{v_r \sigma_{ry}}{f'_{c}} \)  

(4.9)

\[ \sigma_c = \eta \times X \times f'_{c} \]  

(4.10)

The tensile stress of the composite \( \sigma_c \) was determined by assuming that all the steel layers are in the plastic range.

- \( V_r \) - volume fraction of mesh reinforcement
- \( \sigma_{ry} \) - yield stress of mesh reinforcement
- \( f'_{c} \) - compressive stress of mortar
- \( \eta \) - efficiency factor of reinforcement
- \( b_f \) - breadth of ferrocement laminate
- \( h \) - thickness of ferrocement element
- \( \sigma_c \) - tensile stress of the composite

\[ C = 0.85 \times f'_{c} \times b_f \times \frac{\eta X h}{0.85 + \eta X} \]  

(4.11)

\[ T = \eta \times f'_{c} \times X \times b_f \times \left( h - \frac{\eta X h}{0.85 + \eta X} \right) \]  

(4.12)

Then the ultimate moment due to compressive force \( (M_c) \) is given by taking moment about the line of action as,
\[ M_c = 0.425 \times f' \times b_f \times h \times \frac{nXh}{0.85 + nX} \]  
\( (4.13) \)

The ultimate moment due to tensile force \( (M_t) \) is given by,

\[ M_t = 0.5n \times f' \times X \times b_f \times h \times \left( h - \frac{nXh}{0.85 + nX} \right) \]  
\( (4.14) \)

4.3.2 Elastic analysis approach

The idealised stress strain block for the ferrocement composite section and reinforcement mesh at ultimate under bending was considered as shown in Figure 4.2.(ACI 549-R,1997). The following assumptions were made:

- Strain in reinforcement and mortar should be assumed directly proportional to the distance from the neutral axis.
- Maximum strain at extreme mortar compression fiber should be assumed equal to 0.003.
- Tensile strength of mortar shall be neglected.
- Relationship between mortar compressive stress distribution and mortar strain may be considered as satisfactory by the use of the equivalent rectangular concrete stress distribution.
- Stress in reinforcement below specified yield strength should be taken as effective modulus of reinforcing system times steel strain.
In this method, the experimentally obtained ultimate moment capacity was compared with an elastic approach. The tension reinforcement mesh is assumed to take the total tensile force and the tensile strength of the mortar is neglected. The distance from the extreme compression fiber to the neutral axis ‘a’ was obtained by the tedious trial and error computation. This was begun by assuming the value for ‘a’. If this estimated distance from the extreme compression fiber to neutral axis is correct, then the summation of all compressive forces should equal the summation of all the tensile forces. This was done to check the accuracy of the assumed distance. After a number of trials, the value of ‘a’ for which the equilibrium is satisfactory was arrived. Then, the ultimate moment capacity about the neutral axis was determined for forces in the section.

The compressive force and tensile force are expressed in equation 4.18 and 4.20.

\[ C = C_m + \sum_{i} C_{ri} \]  \hspace{1cm} (4.15)

\[ C_m = 0.85 f' \times b_f \times \beta \times a \]  \hspace{1cm} (4.16)

\[ C_{ri} = \sum_{i} \sigma_{cij} \times A_n \]  \hspace{1cm} (4.17)
\[ C = 0.85 \times f'c \times b_f \times \beta \times a + \sum_{i=1}^{N} \sigma_{ryi} \times A_{ri} \quad (4.18) \]

\[ T = \sum_{i=1}^{N} T_{ri} \quad (4.19) \]

\[ T = \sum_{i=1}^{N} \sigma_{ryi} \times A_{ri} \quad (4.20) \]

\( C \) - total compressive force \((C_m + C_{ri})\)

\( C_m \) - compressive force in mortar

\( C_{ri} \) - compressive force in mesh layer ‘i’

\( T \) - total tensile force \((T_{r2}+T_{r3}+T_{r4})\)

\( \beta \) - factor defining depth of rectangular stress block

\( \sigma_{ryi} \) - yield stress at layer ‘i’ of reinforcement

\( A_{ri} \) - area of reinforcement at layer ‘i’

\( h_i \) - distance between the extreme concrete compressive fiber to the centroid of steel in layer ‘i’

Total compression = Total tension

Total compressive force + total tensile force = 0

\[ C_m + \sum_{i}^{N} C_{ni} + \sum_{i}^{N} T_{ri} = 0 \quad (4.21) \]

The depth of neutral axis for the ferrocement laminate was calculated, by solving the equilibrium of internal forces (Equation 4.21). The ultimate moment carrying capacity of a ferrocement laminate shown in Figure 4.2 is obtained by summing the moments of all internal forces about the neutral axis of the laminate as given in equation 4.18.
\[ M_u = 0.85 f'_c b_f \beta a \left( a - \frac{\beta a}{2} \right) + \sum_{i}^N \sigma_{ult} A_i (h_i - a) \]  

(4.22)

where,

\( N \) - number of layers of mesh reinforcement

\( f'a \) - equivalent depth of rectangular stress block

4.4 ANALYTICAL INVESTIGATION ON FLEXURAL STRENGTHENING OF R.C BEAMS

The development of an analytical model for beams strengthened with ferrocement laminates in conventional mortar and optimum mortar has been explained. The analytical procedure based on the equilibrium of forces is developed to compare the experimental ultimate moment capacity of the beam strengthened with ferrocement laminates. The stress strain diagram of strengthened beam is shown in Figure 4.3.

4.4.1 Flexural Strengthening of Beams

To increase the flexural strength, the ferrocement laminates are attached to the tension side of the reinforced concrete beams. The following assumptions are made in calculating the ultimate moment capacity of the strengthened beam.

- The strain in reinforcement and concrete is directly proportional to the distance from the neutral axis.
- The plane sections before loading remain plane and after loading.
- There is no relative slip between ferrocement laminate and the concrete.
- The maximum compressive strain in concrete is 0.0035 in bending.
- The tensile strength of concrete is neglected.
- The mesh reinforcement in ferrocement laminate has linear elastic stress strain relationship to failure.
- Shear deformation is small.

\[ C_c = 0.36 f_{ck} \times a \times b \]  \hspace{1cm} (4.23)

\[ C_s = f_{sc} \times A_{s,c} \]  \hspace{1cm} (4.24)

where

\[ C_c \] - compressive force in concrete

\[ f_{ck} \] - characteristic compressive strength of concrete

\[ a \] - depth of neutral axis

**Figure 4.3 Stress strain diagram of strengthened beam**
Considering the effect of strengthening of the beam by ferrocement laminate along with tensile force $T_s$, an additional tensile force due to mortar $T_m$ and tensile force due to mesh $T_{wm}$ will also act. The value of tensile force on mortar is 10% of the compressive strength of the mortar (Azad A Mohammed et al 2010). The tensile strength of mesh is $(\sigma_{ry} \times A_i)$. 

Total force due to compression $= \text{Total force due to tension}$

$$C_c + C_s = T_s + T_{wm} + T_m \quad (4.25)$$

$$C_c + C_s + T_s + T_{wm} + T_m = 0 \quad (4.26)$$

$$\begin{align*}
    0.36 \times f_{ck} \times a \times b + f_{wc} \times A_{rc} + 0.87 \times f_s \times A_n + \sum_n \sigma_{ri} \times A_i + 0.1 f_{sc} (A_s - A_n) & = 0 \\
    (4.27)
\end{align*}$$

The depth of neutral axis for the beam strengthened with ferrocement laminate was calculated, by solving the equilibrium of internal forces (Equation 4.27).
The moment of resistance of a section shown in Figure 4.3 is obtained by summing the moments of all internal forces about mid depth of the beam.

Moment of resistance due to R.C beam is expressed in equation

\[ 4.28. M.R_{(R.C.\ beam)} = C_c \left[ \frac{H}{2} - 0.42a \right] + C_s \left[ \frac{H}{2} - d' \right] + T_s \left[ \frac{H}{2} - d' \right] \] (4.28)

Moment of resistance due to ferrocement laminate is given in equation 4.29.

\[ M.R_{(Ferrocement \ laminate)} = T_{wm} \times \sum_{i}^{N} \left[ h_i + \frac{H}{2} \right] + T_m \left[ \frac{H}{2} + \frac{h}{2} \right] \] (4.29)

M. R = M. R due to R.C beam + M.R due to ferrocement laminate, are shown in equation 4.30.

\[ M.R = C_c \left[ \frac{H}{2} - 0.42a \right] + C_s \left[ \frac{H}{2} - d' \right] + T_s \left[ \frac{H}{2} - d' \right] + \sum_{i}^{N} \left[ h_{li} + \frac{H}{2} \right] \times T_{wm} + T_m \times \left[ \frac{H}{2} + \frac{h}{2} \right] \] (4.30)

where,

- \( H \) - depth of the beam,
- \( h_i \) - distance between the extreme concrete compressive fiber to the centroid of steel in layer \( i \)
- \( h \) - depth of laminate
- \( d' \) - effective cover of beam