CHAPTER 3

Z-SOURCE INVERTER

3.1 INTRODUCTION

In power electronics, the word ‘inverter’ denotes a class of power conversion or power conditioning circuits that operates from a DC voltage source or a DC current source and converts it into AC voltage or current of desired magnitude and frequency. Inverters are used widely in industrial applications such as adjustable speed drives (ASD), induction heating, uninterruptible power supplies (UPS), static var compensators, active filters, flexible AC transmission system (FACTS), voltage compensators, etc. Inverters can be broadly classified into two types: Voltage Source Inverters and Current Source Inverters. Each type can use controlled turn-on and turn-off power devices (e.g., Power MOSFET, IGBT, MCT, and GTO).

3.1.1 Voltage Source Inverter

A voltage source inverter is one in which the DC input voltage is essentially constant and independent of the load current drawn. The inverter specifies the load voltage while the drawn current shape is determined by the load. This inverter should have a stiff DC voltage source at the input. The DC voltage may be fixed or variable, and may be obtained from a utility line, a controlled rectifier, a battery, fuel cell stack or solar photovoltaic array. A large capacitor can be connected at the input terminals to make the input DC voltage constant. A variable output voltage can be obtained by varying the
input DC voltage and maintaining the desired voltage at the inverter output. On the other hand, if the DC input voltage is fixed and is not controllable, then the variable output voltage is achieved by pulse width modulation control within the inverter. Figure 3.1 shows the topology of three phase voltage source inverter.

The AC output voltage obtained from this inverter is limited below and cannot exceed the DC input voltage. Therefore the voltage source inverter operates only in buck mode (step down) for DC to AC conversion. For applications where over drive is desirable and the available DC voltage is limited, an additional DC to DC boost converter is needed to obtain a desired AC output. The additional power converter stage increases system cost and lowers efficiency. In addition, the upper and lower devices of any phase leg cannot be gated on simultaneously, either by purpose or by EMI noise. Otherwise a shoot through problem occurs and destroys the devices. Dead time to prevent the simultaneous conduction of upper and lower devices has to be provided.

![Figure 3.1 Three phase voltage source inverter](image-url)
3.1.2 Current Source Inverter

A current source inverter is fed from a constant current source. Load current remains constant irrespective of the load on the inverter. The load voltage shape is determined by the load impedance. When a voltage source has a large inductance in series with it, it functions as a current source. The large inductance maintains current source constant. Figure 3.2 shows the traditional current source inverter structure.

![Current Source Inverter Diagram]

**Figure 3.2 Three phase current source inverter**

The AC output voltage of the current source inverter is greater than the original DC voltage that feeds the input inductor. Therefore the current source inverter is a boost inverter for DC to AC power conversion. For applications where a wide voltage range is desirable, an additional DC to DC buck converter is needed. Again it increases cost and lower efficiency. At least one of the upper devices and one of the lower devices need to be gated on and maintained on at any time. Otherwise, an open circuit of the input inductor would occur and destroy the devices. The open circuit problem by EMI noise misgating is a major concern of the inverter reliability. Hence,
overlap time should be provided in current source inverter to avoid the open circuit problem.

Thus, both the voltage source inverter and the current source inverters have the following theoretical limitations:

1. They are either a buck or boost converter and cannot be a buck-boost converter. That is, their obtainable output voltage range is limited to either greater or smaller than the input voltage. This requires an additional boost or buck DC to DC converter for applications exceeding available voltage range.

2. Their main circuit cannot be interchanged. In other words, neither the voltage source inverter main circuit can be used for the current source converter nor the vice versa.

3. They are susceptible to EMI noise in terms of reliability. In addition, it is required to provide the dead time in voltage source inverters and overlap time in current source inverter for safe commutation which causes waveform distortion.

The above limitations make these inverters hard to use for some applications without additional circuitry. Z-source inverter is the new topology introduced into power inverter category by Fang Zheng Peng (2003) which overcomes all the above limitations. This inverter can boost the output voltage to the desired level by introducing a shoot-through operation mode, which is forbidden in voltage source inverters. With this unique feature, the Z-source inverter provides a simple, cheap and single stage approach for power conditioning applications. In addition, it renders a high reliable system because the inverter can handle momentary shoot-through caused by EMI without interrupting the operation.
3.2 OPERATION OF Z-SOURCE INVERTER

The topology of the three phase Z-source inverter is shown in Figure 3.3. This inverter topology employs a two port network consisting of two inductors $L_1$ and $L_2$ and two capacitors $C_1$ and $C_2$ connected in X shape. This impedance network is coupled between the inverter bridge to the input DC source. As inductors and capacitors are used in the DC link, it acts as a constant high impedance voltage source. This unique impedance network allows the Z-source inverter to buck and boost its output voltage, and provides it with unique features that cannot be achieved with conventional voltage source or current source inverters.

![Z-source inverter topology](image.png)

**Figure 3.3 Z-source inverter topology**

The operating principle of the conventional three phase voltage source inverter is explained with six active and two zero vectors. The DC input voltage is impressed across the load when any one of the six active vectors is applied. The zero vector is produced when the upper or lower three devices are turned on at the same time and the output voltage is zero. In three phase Z-source inverter, one extra zero state is applied by gating on both the upper and lower devices of any one phase leg, combinations of any two phase legs, or all three phase legs. Thus there are seven different ways to generate
this extra zero state. This additional zero state named as shoot-through state is forbidden in the traditional voltage source inverter, because it would cause a short circuit. The output voltage across the load remains zero in the shoot-through state. Thus the effect of shoot-through state is same as traditional zero state. In Z-source inverter the part of zero state or entire zero state is converted in to shoot-through state. This shoot-through zero states provide the required boost feature to the inverter. When the input voltage is large enough to produce the desired AC voltage, the shoot-through zero state is not required and the Z-source inverter operates like conventional voltage source inverter. Table 3.1 lists the permissible switching states of the three phase Z-source inverter.

<table>
<thead>
<tr>
<th>State</th>
<th>Output Voltage</th>
<th>S&lt;sub&gt;ap&lt;/sub&gt;</th>
<th>S&lt;sub&gt;an&lt;/sub&gt;</th>
<th>S&lt;sub&gt;bp&lt;/sub&gt;</th>
<th>S&lt;sub&gt;bn&lt;/sub&gt;</th>
<th>S&lt;sub&gt;cp&lt;/sub&gt;</th>
<th>S&lt;sub&gt;cn&lt;/sub&gt;</th>
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<td>Finite</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Active A2 [110]</td>
<td>Finite</td>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
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<td>1</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
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<tr>
<td>Active A4 [011]</td>
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<td>1</td>
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<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
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<tr>
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<td>0</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Zero Z1 [000]</td>
<td>0 V</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
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<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>Shoot-through ST1</td>
<td>0 V</td>
<td>1</td>
<td>1</td>
<td>S&lt;sub&gt;bp&lt;/sub&gt;</td>
<td>S&lt;sub&gt;bp&lt;/sub&gt;</td>
<td>S&lt;sub&gt;cp&lt;/sub&gt;</td>
<td>S&lt;sub&gt;cp&lt;/sub&gt;</td>
</tr>
<tr>
<td>Shoot-through ST2</td>
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<td>S&lt;sub&gt;ap&lt;/sub&gt;</td>
<td>S&lt;sub&gt;ap&lt;/sub&gt;</td>
<td>1</td>
<td>1</td>
<td>S&lt;sub&gt;cp&lt;/sub&gt;</td>
<td>S&lt;sub&gt;cp&lt;/sub&gt;</td>
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<tr>
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<td>S&lt;sub&gt;ap&lt;/sub&gt;</td>
<td>S&lt;sub&gt;ap&lt;/sub&gt;</td>
<td>S&lt;sub&gt;bp&lt;/sub&gt;</td>
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<td>1</td>
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<tr>
<td>Shoot-through ST4</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>S&lt;sub&gt;cp&lt;/sub&gt;</td>
<td>S&lt;sub&gt;cp&lt;/sub&gt;</td>
<td>S&lt;sub&gt;cp&lt;/sub&gt;</td>
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<tr>
<td>Shoot-through ST5</td>
<td>0 V</td>
<td>1</td>
<td>1</td>
<td>S&lt;sub&gt;bp&lt;/sub&gt;</td>
<td>S&lt;sub&gt;bp&lt;/sub&gt;</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
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<td>S&lt;sub&gt;ap&lt;/sub&gt;</td>
<td>S&lt;sub&gt;ap&lt;/sub&gt;</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Shoot-through ST7</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
There are 3 modes of operation explaining the operation of a Z-source inverter.

**Mode-1: Active state**

The inverter bridge is operating in one of the six active vectors, thus acting as a current source viewed from the Z-source network. Figure 3.4 shows the equivalent circuit of this mode.

![Figure 3.4 Mode 1- Active state](image)

Since the circuit is symmetrical, both inductors have an identical current value. During this mode the DC source voltage appears across the inductor and capacitor. Capacitor is charged and energy flows to the load via the inductor.

**Mode-2: Zero state**

The inverter bridge is operating in one of the two zero vectors and shorting through either the upper or lower three device. During this mode the
bridge can be viewed as an open circuit, shown on Figure 3.5. The input DC voltage source appears across the inductor and capacitor, except that no current flows to the load from the DC source. The input diode D conducts and carries the inductor current, which contributes to the harmonic reduction of the line current.

![Figure 3.5 Mode 2-Zero state](image)

**Mode-3: Shoot-through state**

The inverter bridge is operating in one of the seven shoot-through states. The bridge is viewed as a short circuit from the DC link of the inverter as shown in Figure 3.6. During this mode, no voltage appears across the load like in the zero state operation, the DC value of the capacitor is boosted to the required value according to the shoot-through duty cycle. Also, in this mode the capacitors are connected in parallel with inductors. Thus the charge is transferred from the capacitor to inductor causing the inductor currents to ramp up. This makes the input diode reverse biased, and hence turned off, separating the DC link voltage from the input DC source. This is the shoot-
through mode to be used in every switching cycle during the traditional zero vector period to boost the voltage whenever the photovoltaic module is unable to provide the required voltage or during any voltage dips due to the changing solar irradiance and temperature. Depending on how much a voltage boost is needed, the shoot-through interval \( T_o \) or its duty cycle \( T_{o}/T \) is determined. It can be seen that the shoot-through interval is only a fraction of the switching cycle.

\[ \text{Figure 3.6 Mode 3- Shoot-through state} \]

### 3.3 STEADY STATE ANALYSIS

The analysis of the Z-source inverter in steady state and the determination of the conversion ratio can be done with the equivalent circuit of the Z-source inverter shown in Figure 3.7. From the Z-source network point of view, when in a shoot-through state during time interval \( T_o \), the inverter side of the Z-source network is shorted. Alternatively, when in a non-shoot-through active or zero state during the time interval \( T_i \), current flows from the Z-source network through the inverter topology to the
connected AC load. The inverter side of the Z-source network can now be represented by an equivalent current source. In the equivalent circuit, when the parallel switch $S_2$ is on, the Z-source network is short circuited and the load sees zero voltage. Similarly, when switch $S_2$ is off, the Z-source network sees the load.

![Equivalent circuit of Z-source inverter](image)

**Figure 3.7 Equivalent circuit of Z-source inverter**

To make the analysis simple, assume the two inductors in the Z-source network are identical, i.e $L_1 = L_2 = L$. Also the two capacitors are identical, i.e $C_1 = C_2 = C$. Accordingly the capacitor and inductor voltages of the Z-source network are expressed in Equations (3.1) and (3.2).

\[
V_{c_1} = V_{c_2} = V_c \tag{3.1}
\]

\[
V_{l_1} = V_{l_2} = V_L \tag{3.2}
\]

Equation (3.3) through Equation (3.5) are obtained when the inverter is in the shoot-through state for an interval of $T_o$. 
\[ V_L = V_C \]  \hspace{1cm} (3.3)

Voltage across the impedance network is given in Equation (3.4)

\[ V_d = 2V_C \]  \hspace{1cm} (3.4)

\[ V_i = 0 \]  \hspace{1cm} (3.5)

Similarly, if the inverter in is the non-shoot-through for an interval of \( T_i \), during the switching cycle \( T \) and the corresponding equations are given from Equation (3.6) to Equation (3.8).

\[ V_L = V_s - V_C \]  \hspace{1cm} (3.6)

\[ V_d = V_s \]  \hspace{1cm} (3.7)

\[ V_i = V_C - V_L = 2V_C - V_s \]  \hspace{1cm} (3.8)

The average value of the voltage across an inductor for a switching period (\( T \)) is zero in steady state, so from Equations (3.3) to (3.8),

\[ V_L = \frac{1}{T} \int_{0}^{T} V_L(t)dt = \frac{T s V_C + T_i (V_s - V_C)}{T} = 0 \]  \hspace{1cm} (3.9)

From the Equation (3.9), the ratio between the capacitor voltage and input voltage can be derived and is given in Equation (3.10).

\[ \frac{V_C}{V_s} = \frac{T_i}{T_1 - T_s} = \frac{1 - D_o}{1 - 2D_o} \]  \hspace{1cm} (3.10)

where \( D_o = \frac{T_o}{T} \) is the shoot-through duty cycle, \( V_C \) is the steady state value of the capacitor voltage, \( V_s \) is the input DC voltage.
Similarly, the average DC link voltage across the inverter bridge can be determined and is given in Equation (3.11).

\[
V_i = \frac{1}{T} \int_0^T v_i(t)dt = \frac{T}{T_0} \left( \frac{T_0 + T_0(2V_c - V_s)}{T} \right) = \frac{T}{T - T_0} V_s = V_c
\]  
(3.11)

The peak value of the pulsating DC link voltage across the inverter bridge is given in Equation (3.12). Equation (3.13) represents the boost factor resulting from the shoot-through state.

\[
\hat{v}_i = 2V_c - V_s = \left( \frac{T}{T - T_0} \right) V_s = BV_s
\]  
(3.12)

\[
B = \frac{T}{T_0 - T_0} = \frac{1}{1 - \frac{T_0}{T}} = \frac{1}{1 - 2D_o}
\]  
(3.13)

Since \(D_o\) is between zero and 0.5, \(B\) can be any value between one and infinity.

Equations (3.14) and (3.15) are derived from Equation (3.13).

\[
\frac{T_o}{T} = \frac{B - 1}{2B}
\]  
(3.14)

\[
1 - \frac{T_o}{T} = \frac{B + 1}{2B}
\]  
(3.15)

Now, the capacitor voltage can be derived using Equations (3.10) and (3.15) and is given in Equation (3.16).

\[
V_c = \left( \frac{B + 1}{2B} \right) BV_s = \left( \frac{B + 1}{2} \right) V_s
\]  
(3.16)
Now, peak DC link voltage is expressed as given in Equation (3.17).

\[ \hat{V}_i = B \cdot V_s = B \left( \frac{2V_c}{B+1} \right) = V_c \left( \frac{2B}{B+1} \right) \]  \hspace{1cm} (3.17)

The output peak phase voltage of the Z-source inverter can be expressed as,

\[ \hat{V}_{ac} = M \frac{\hat{V}_i}{2} = MB \frac{V_s}{2} \]  \hspace{1cm} (3.18)

where \( M \) is the modulation index.

For the traditional voltage source inverter, the well known equation for output peak phase voltage is \( \hat{V}_{ac} = M \frac{V_s}{2} \). Equation (3.18) has an additional multiplication factor of \( B \) compared to the voltage conversion ratio of voltage source inverter which gives the boosting capability to the Z-source inverter. The boost factor can be controlled by duty cycle of the shoot-through state over the non-shoot-through states of the inverter. The shoot-through state does not affect the PWM control of the inverter, because it produces zero voltage only across the load like during zero states.

### 3.4 MODELING OF Z-SOURCE INVERTER

The analysis, design and implementation of any power electronic system without modeling and simulation is highly laborious, time-consuming and therefore expensive. Modeling is the representation of a physical system using mathematical tools. In engineering, it is desired to model the important dominant behavior of the system, while neglecting other insignificant phenomena. The resulting simplified model gives a physical insight into the dynamics of the system and helps in designing the system to operate in a given specified manner.
In general, the simplified models of power converters are obtained by making assumptions and approximations. A common assumption taken to model a power converter is ignoring the switching ripple. The power converters are usually operated at high frequency switching, so the switching ripple is very low and the switching frequency is much higher than the natural frequencies of the converter filtering elements. Hence, a possible simplification of a power converter model could be ignoring the switching ripple and averaging the circuit waveforms over the switching period.

For power converter, various modeling techniques and procedures have been developed in literature, including the current injected approach, circuit averaging and the state space averaging method. Although given method may be preferred to express the results in a specific form, the end results of nearly all methods are equivalent. It is found in the literature that circuit averaging and small-signal linearization are the key steps in modeling power converters. The state-space averaging method is used to derive the small-signal averaged equations of PWM switching converters. The benefits of state-space averaging techniques are providing the general unified treatment of the state-space approach and giving equivalent linear circuit model as the end result.

Small signal analysis is the study of deviations from an operating point for a system subjected to small disturbances. The assumption made by this method is that the disturbances are so small that the deviation of the system can be described linearly. Whilst the small signal model is strictly only accurate for infinitesimal disturbances, it can be used to predict the behavior of the systems subjected to large non- infinitesimal disturbances. Naturally if the disturbances are large and the system is more non-linear, then the result of the system is more inaccurate. However, the small signal model is easier to manipulate and for many applications this tradeoff between model
accuracy and model form is worthwhile. Small signal analysis has commonly appeared in electrical and electronic systems literature in various forms. In particular for power electronics, linearized state space averaging, linearized dynamic phasors, Newton time domain shooting methods and Newton harmonic domain methods are all based around small signal models. In this work, a small signal model for the Z-source inverter is derived based on the state space averaging technique and linearization of the state variables around their steady state values.

3.4.1 State-Space Averaging Technique

The state-space description is a canonical form for writing the differential equations that describe a system. Generally, for a linear network, the derivatives of the state variables are expressed as linear combinations of the system independent inputs and the state variables themselves. The state variables of a power converter circuit are the independent inductor currents and capacitor voltages. At any point in time, the values of the state variables depend on the previous history of the system, rather than on the present values of the system inputs. To solve the differential equations of the system, the initial values of the state variables need to be specified. In other words, the values of all the state variables at a given time \( t_o \) and the system inputs for \( t \geq t_o \) be known, then the system state equations can be solved to find the system waveforms at any future time (Erickson and Dragan Maksimovic 2001).

Generally, the state equations of a system can be written in the compact matrix form of

\[
K \frac{dx(t)}{dt} = Ax(t) + Bu(t) \\
y(t) = Cx(t) + Du(t)
\] (3.19)
Here, \( x(t) \) is the state vector containing all the state variables, that is
the inductor currents and capacitor voltages. The vector \( u(t) \) is the input
vector containing independent inputs to the system, such as the input voltage
source, \( K \) is a matrix containing the values of inductances and capacitances,
such that \( Kdx(t)/dt \) is a vector containing the inductor voltages and capacitor
currents in a power circuit. Equation (3.19) states that the inductor voltages
and capacitor currents in a circuit can be expressed as linear combinations of
the state variables and the independent inputs. The matrices \( A \) and \( B \) contain
constants of proportionality. It is also desired to compute other circuit
waveforms that do not coincide with the elements of the state vector \( x(t) \) or
the input vector \( u(t) \). These other signals are dependent waveforms that can be
expressed as linear combinations of the elements of the state vector and input
vector. The vector \( y(t) \) is called the output vector. In the state Equation (3.19),
the elements of \( y(t) \) are expressed as a linear combination of the elements of
the \( x(t) \) and \( u(t) \) vectors. The matrices \( C \) and \( D \) contain constants of
proportionality.

The power converter to be modeled is assumed to be operating in
continuous conduction mode having two subintervals during a switching
period. During the first subinterval, when the switches are in position 1, the
converter is reduced to a linear circuit with the following state equations:

\[
K \frac{dx(t)}{dt} = A_1x(t) + B_1u(t) \\
y(t) = C_1x(t) + D_1u(t)
\]

(3.20)

During the second subinterval, with the switches in position 2, the
converter is reduced to another linear circuit whose state equations are

\[
K \frac{dx(t)}{dt} = A_2x(t) + B_2u(t) \\
y(t) = C_2x(t) + D_2u(t)
\]

(3.21)
Equations (3.20) and (3.21) constitute the state space model of the power converter. Using this model, the converter can be averaged over the switching cycle provided that the natural frequencies of the converter are much lower than the switching frequency. This is called the state space averaging and the results of it are the state space equations of the equilibrium and the AC small signal model. The state space model that describes the converter in equilibrium is

\[ 0 = AX + BU \]
\[ Y = CX + DU \]  \hspace{1cm} (3.22)

where the averaged matrices are

\[ A = D_o A_1 + D_o \ ' A_2 \]
\[ B = D_o B_1 + D_o \ ' B_2 \]
\[ C = D_o C_1 + D_o \ ' C_2 \]
\[ D = D_o D_1 + D_o \ ' D_2 \]  \hspace{1cm} (3.23)

The equilibrium DC components are

\[ X = \text{Equilibrium (DC) state vector} \]
\[ U = \text{Equilibrium (DC) input vector} \]
\[ Y = \text{Equilibrium (DC) state vector} \]
\[ D_o = \text{Equilibrium (DC) state vector} \]
\[ D_o \ ' = 1 - D_o \]

Quantities defined in Equation (3.23) represent the equilibrium values of the averaged vector. Equation (3.22) can be solved to find the equilibrium state and output vectors and is given in Equation (3.24).

\[ X = A^{-1} BU \]
\[ Y = (-CA^{-1} B + D)U \]  \hspace{1cm} (3.24)
The state equations of the small signal AC model are

\[
K \frac{d\hat{x}(t)}{dt} = A\hat{x}(t) + B\hat{u}(t) + \{(A_1 - A_2)X + (B_1 - B_2)U\}\hat{d}(t)
\]
\[
\hat{y}(t) = C\hat{x}(t) + D\hat{u}(t) + \{(C_1 - C_2)X + (D_1 - D_2)U\}\hat{d}(t)
\]

(3.25)

where the quantities \( \hat{x}(t), \hat{u}(t), \hat{y}(t), \) and \( \hat{d}(t) \) in Equation (3.25) are small AC variations about the equilibrium solution defined by Equations (3.22) to (3.24).

3.4.2 Small-signal Model of Z-source Inverter

The equivalent circuit of the Z-source inverter has been obtained in Figure 3.7. During the active state, energy transfer occurs from the DC source as well as the inductors in impedance network to the load. During the shoot-through state, the load and source sides are decoupled and the inductors are charged by the capacitors.

The circuit modeling using state-space modeling and analysis begin with the following assumptions:

1. The input voltage \( V_S \) is an independent voltage source. The two switches \( S_1 \) and \( S_2 \) are ideal. So the voltage drop across the switches is zero.

2. The inductors \( (L_1 \) and \( L_2 \)) and capacitors \( (C_1 \) and \( C_2 \)) are ideal elements and lossless.

3. Z-source inverter is operating in continuous conduction mode.

4. On state resistance of switch \( S_2 \) is neglected because its value is much smaller than that of load.

5. The load current is continuous because of an inductive load. The load impedance is \( Z_o = R_o + jL_o \).
From the equivalent circuit of the Z-source inverter shown in Figure 3.7, the following state variables are defined: capacitor voltages, inductor currents and load current. Equation (3.26) represents the state variables in vector form.

\[
x(t) = \begin{bmatrix}
i_{L1}(t) \\
i_{L2}(t) \\
v_{C1}(t) \\
v_{C2}(t) \\
i_{o}(t)
\end{bmatrix}
\] (3.26)

For derivation, in Z-source inverter operation two different operating modes are identified from the equivalent circuit (Jingbo Liu et al 2007). In model 1, switch S\(_1\) is open and S\(_2\) is closed as shown in Figure 3.8. Thus energy transferred from source to load equals to zero because the load side and source side are essentially decoupled by the shoot-through states. The duty ratio of switch S\(_2\) is defined as the shoot-through duty ratio \(D_o\). So the duty ratio of the switch S\(_1\) is defined as \(D'_o = 1 - D_o\).

![Figure 3.8 Model 1: S\(_1\) open and S\(_2\) closed](image)
In mode 2, the switch $S_1$ is closed and $S_2$ is open as shown in Figure 3.9. In this mode, real energy is transferred between the source and load.

![Circuit Diagram](image)

**Figure 3.9 Mode 2: $S_1$ closed and $S_2$ open**

The circuit equations in mode 1 can be written in the state space form $\dot{\mathbf{x}} = A \mathbf{x} + B \mathbf{u}$, and is given in Equation (3.27).

$$
\begin{bmatrix}
L_1 & 0 & 0 & 0 & 0 \\
0 & L_2 & 0 & 0 & 0 \\
0 & 0 & C_1 & 0 & 0 \\
0 & 0 & 0 & C_2 & 0 \\
0 & 0 & 0 & 0 & L_o
\end{bmatrix}
\begin{bmatrix}
\frac{d}{dt} v_{c_1}(t) \\
\frac{d}{dt} v_{c_2}(t) \\
\frac{d}{dt} i_o(t)
\end{bmatrix}
= 
\begin{bmatrix}
0 & 0 & 1 & 0 & 0 & i_{L_1}(t) \\
0 & 0 & 0 & 1 & 0 & i_{L_2}(t) \\
-1 & 0 & 0 & 0 & 0 & v_{c_1}(t) \\
0 & -1 & 0 & 0 & 0 & v_{c_2}(t) \\
0 & 0 & 0 & 0 & -R_o & i_o(t)
\end{bmatrix}
$$

(3.27)

where $K = 
\begin{bmatrix}
L_1 & 0 & 0 & 0 & 0 \\
0 & L_2 & 0 & 0 & 0 \\
0 & 0 & C_1 & 0 & 0 \\
0 & 0 & 0 & C_2 & 0 \\
0 & 0 & 0 & 0 & L_o
\end{bmatrix}$
Similarly, the state space representation in mode 2 can be written in the form
\[ K\dot{x} = A_2 x + B_2 u, \]
and is given in Equation (3.28).

\[
\begin{bmatrix}
L_1 & 0 & 0 & 0 & 0 \\
0 & L_2 & 0 & 0 & 0 \\
0 & 0 & C_1 & 0 & 0 \\
0 & 0 & 0 & C_2 & 0 \\
0 & 0 & 0 & 0 & L_o
\end{bmatrix}
\begin{bmatrix}
i_{L_1}(t) \\
i_{L_2}(t) \\
v_{c_1}(t) \\
v_{c_2}(t) \\
i_o(t)
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & -1 \\
1 & 0 & 0 & 0 & -1 \\
0 & 0 & 1 & 1 & -R_o
\end{bmatrix}
\begin{bmatrix}
i_{L_1}(t) \\
i_{L_2}(t) \\
v_{c_1}(t) \\
v_{c_2}(t) \\
i_o(t)
\end{bmatrix}
+ \begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
-1
\end{bmatrix}v_i(t)
\]

(3.28)

where

\[
A_2 = \begin{bmatrix}
0 & 0 & 0 & -1 & 0 \\
0 & 0 & -1 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 \\
1 & 0 & 0 & 0 & -1 \\
0 & 0 & 1 & 1 & -R_o
\end{bmatrix}
\quad \text{and} \quad
B_2 = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
-1
\end{bmatrix}
\]

Equations (3.27) and (3.28) constitute the state space model of the simplified Z-source inverter. These equations give the relation between the input and state variables for both the shoot-through states and active states given in Figure 3.8 and Figure 3.9, respectively. The state space model of the Z-source inverter can be used for state space averaging and to find the state space equations of the equilibrium and the AC small-signal model.

The dynamic behavior of the state variables is found out by perturbing the input voltage and shoot-through duty ratio of switch S_2 around
their steady state values and their expressions are given in Equations (3.29) and (3.30).

\[ v_s(t) = V_s + \hat{v}_s(t) \]  \hspace{1cm} (3.29)

\[ d_o(t) = D_o + \hat{d}_o(t) \]  \hspace{1cm} (3.30)

where \( V_s \) and \( D_o \) are the steady state values and \( \hat{v}_s \) and \( \hat{d}_o \) are the perturbed values of the input voltage and shoot-through duty ratio.

The resulting small signal perturbations in state variables is given in Equation (3.31).

\[ x = X + \hat{x} \]  \hspace{1cm} (3.31)

where \( x \) represents the state variables and \( X \) represents the equilibrium values of the state variables.

Now, the state-space averaging method is applied, the averaged matrix \( A \) is

\[ A = D_o \cdot A_1 + D_o \cdot A_2 \]  \hspace{1cm} (3.32)

Similarly, the averaged matrix \( B \) is

\[ B = D_o \cdot B_1 + D_o \cdot B_2 \]  \hspace{1cm} (3.33)

In addition, DC steady state equations at the operating point where the perturbations applied are

\[
\begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & D_o & -D_o' & 0 & I_{L1} \\
0 & 0 & -D_o' & D_o & 0 & I_{L2} \\
0 & -D_o & D_o' & 0 & 0 & -D_o' \ V_{c1} \\
0 & D_o' & -D_o & 0 & 0 & -D_o' \ V_{c2} \\
0 & 0 & 0 & D_o' & D_o' & -R_o - I_o & \ V_s
\end{bmatrix} +
\begin{bmatrix}
D_o' \\
D_o' \\
0 \\
0 \\
0
\end{bmatrix}
\]  \hspace{1cm} (3.34)
Combining Equations (3.32) through (3.34), the small signal equation is obtained and is given in Equation (3.35).

\[ K\hat{X} = A\hat{X} + B\hat{U} + (A_1 - A_2)X + (B_1 - B_2)U \]  
(3.35)

\[
\begin{bmatrix}
    L_1 & 0 & 0 & 0 & 0 \\
    0 & L_2 & 0 & 0 & 0 \\
    0 & 0 & C_1 & 0 & 0 \\
    0 & 0 & 0 & C_2 & 0 \\
    0 & 0 & 0 & 0 & L_o \\
\end{bmatrix}
\begin{bmatrix}
    i_{l1}(t) \\
    i_{l2}(t) \\
    v_{c1}(t) \\
    v_{c2}(t) \\
    i_o(t) \\
\end{bmatrix} =
\begin{bmatrix}
    0 & 0 & D_o & -D_o' & 0 \\
    0 & 0 & -D_o' & D_o & 0 \\
    -D_o & D_o' & 0 & 0 & -D_o' \\
    D_o' & -D_o & 0 & 0 & -D_o' \\
    0 & 0 & D_o' & D_o' & -R_o \\
\end{bmatrix}
\begin{bmatrix}
    \hat{i}_{l1}(t) \\
    \hat{i}_{l2}(t) \\
    \hat{v}_{c1}(t) \\
    \hat{v}_{c2}(t) \\
    \hat{i}_o(t) \\
\end{bmatrix}
\]

\[ +
\begin{bmatrix}
    D_o' \\
    D_o' \\
    0 \\
    0 \\
    -D_o' \\
\end{bmatrix}
\begin{bmatrix}
    V_{c1} + V_{c2} - V_s \\
    -I_{l1} - I_{l2} + I_o \hat{d}_o(t) \\
    0 \\
    -I_{l1} - I_{l2} + I_o \\
    -V_{c1} - V_{c2} + V_s \\
\end{bmatrix}
\]

(3.36)

where \( I_{l1}, I_{l2}, V_{c1}, V_{c2}, \) and \( V_s \) are steady state values at the operating points when small signal perturbations occur.

The state equations of the small signal AC model can be determined after linearizing the averaged model and subtracting the equilibrium values of the state variables. The state equations of the small signal AC model of the Z-source inverter can be written by taking Laplace transformation on Equation (3.36). The following are the equations

\[ sL_1\hat{i}_{l1}(s) = D_o\hat{v}_{c1}(s) - D_o'\hat{v}_{c2}(s) + D_o'\hat{v}_s(s) + (V_{c1} + V_{c2} - V_s)\hat{d}_o(s) \]
(3.37)

\[ sL_2\hat{i}_{l2}(s) = -D_o'\hat{v}_{c1}(s) + D_o\hat{v}_{c2}(s) + D_o'\hat{v}_s(s) + (V_{c1} + V_{c2} - V_s)\hat{d}_o(s) \]
(3.38)

\[ sC_1\hat{v}_{c1}(s) = -D_o\hat{i}_{l1}(s) + D_o\hat{i}_{l2}(s) + (-I_{l1} - I_{l2} + I_o)\hat{d}_o(s) - D_o'\hat{i}_o(s) \]
(3.39)
\[
sC_2 \hat{v}_{c2}(s) = D_o' \hat{i}_{l1}(s) - D_o' \hat{i}_{l2}(s) + (-I_{l1} - I_{l2} + I_o) \hat{d}_o(s) - D_o' \hat{j}_o(s) \quad (3.40)
\]

\[
sL_o \hat{i}_o(s) = D_o' \hat{v}_{c1}(s) + D_o' \hat{v}_{c2}(s) - D_o' \hat{j}_i(s) + (-V_{c1} - V_{c2} + V_o) \hat{d}_o(s) - R_o \hat{j}_o(s) \quad (3.41)
\]

The Z-source network is symmetric, that is

\[
L_1 = L_2 = L
\]

and

\[
C_1 = C_2 = C
\]

Also, it is assumed that Z-source inverter is operating in continuous conduction mode and \(D_o + D_o' = 1\).

From Equations (3.37) and (3.38), the following equation is obtained.

\[
\hat{i}_{l1}(s) - \hat{i}_{l2}(s) = \frac{1}{sL} [\hat{v}_{c1}(s) - \hat{v}_{c2}(s)] \quad (3.42)
\]

Similarly, combining equations (3.39) and (3.40), the following equation is obtained.

\[
sC[\hat{v}_{c1}(s) - \hat{v}_{c2}(s)] = -\hat{i}_{l1}(s) + \hat{i}_{l2}(s) \quad (3.43)
\]

From Equations (3.42) and (3.43),

\[
sC[\hat{v}_{c1}(s) - \hat{v}_{c2}(s)] + \frac{1}{sL} [\hat{v}_{c1}(s) - \hat{v}_{c2}(s)] = 0 \quad (3.44)
\]
Furthermore, from Equation (3.41),

\[(1 + s^2 LC) [\hat{v}_{c1} (s) - \hat{v}_{c2} (s)] = 0 \]  

(3.45)

Equation (3.45) indicates that at any non-oscillatory frequency, that is at \( \omega \neq \frac{1}{\sqrt{LC}}, \)

\[\hat{v}_{c1} (s) = \hat{v}_{c2} (s) = \hat{v}_c (s) \]  

(3.46)

Substitute Equation (3.46) in Equation (3.42), we obtain the Equation (3.47).

\[\hat{i}_{l1} (s) = \hat{i}_{l2} (s) = \hat{i}_l (s) \]  

(3.47)

So all the small signal expressions are simplified as

\[sL\hat{i}_l (s) = (D_o - D_o')\hat{v}_c (s) + D_o'\hat{v}_s (s) + (2V_c - V_s)\hat{d}_o (s) \]  

(3.48)

\[sC\hat{v}_c (s) = (D_o - D_o')\hat{i}_l (s) + D_o'\hat{i}_{l2} (s) + (-2I_l + I_o)\hat{d}_o (s) - D_o'\hat{\dot{i}_o} (s) \]  

(3.49)

\[sL_o\hat{\dot{i}_o} (s) = 2D_o'\hat{v}_c (s) - R_o \hat{\dot{i}_o} (s) - D_o'\hat{v}_s (s) + (-2V_c + V_s)\hat{d}_o (s) \]  

(3.50)

The DC steady state values also satisfy \( I_{l1} = I_{l2} = I_l \) and \( V_{c1} = V_{c2} = V_c \) because of Z-network symmetry.

Also from Equation (3.34), the steady state values are obtained and are given from Equation (3.51) to Equation (3.53).

\[V_c = \left( \frac{1 - D_o}{1 - 2D_o} \right) V_s \]  

(3.51)
\[ I_L = \left( \frac{1 - D_o}{1 - 2D_o} \right) I_o \]  
(3.52)

and

\[ I_o = \frac{V_c}{R_o} \]  
(3.53)

In small signal modeling and analysis of Z-source inverter, two sources of perturbations are required, namely the shoot-through duty ratio \( \hat{d}_o \) and the input voltage perturbations \( \hat{v}_s \). Generally, the response of one state variable to multiple small signal perturbations can be expressed as a linear combination of the variable response to each individual perturbation. Hence the small signal expression for the Z-source network capacitor voltage is expressed as

\[ \hat{v}_c (s) = G_{vs} (s) \hat{v}_s (s) + G_{vd} (s) \hat{d}_o (s) \]  
(3.54)

and the inductor current small signal expression is

\[ \hat{i}_L (s) = G_{is} (s) \hat{v}_s (s) + G_{id} (s) \hat{d}_o (s) \]  
(3.55)

The transfer functions \( G_{vs} (s) \) and \( G_{vd} (s) \) are defined as

\[ G_{vs} (s) = \frac{\hat{v}_c (s)}{\hat{v}_s (s)} \bigg|_{\hat{d}_o (s) = 0} \text{ is the input-to-capacitor voltage transfer function} \]

\[ G_{vd} (s) = \frac{\hat{v}_c (s)}{\hat{d}_o (s)} \bigg|_{\hat{v}_s (s) = 0} \text{ is the control to-capacitor voltage transfer function} \]
From the simplified small signal expressions specified in Equations (3.48) through (3.50), the above transfer functions are derives as

\[
G_{\text{vd}}(s) = \frac{k_1 s + k_2}{k_3 s^3 + k_4 s^2 + k_5 s + k_6}
\]

(3.56)

and

\[
G_{\text{vo}}(s) = \frac{k_7 s^2 + k_8 s + k_9}{k_3 s^3 + k_4 s^2 + k_5 s + k_6}
\]

(3.57)

where the coefficients $k_1$ through $k_9$ are

\[
k_1 = L_o D_o' (D_o' - D_o) + LD_o'^2
\]

\[
k_2 = R_o D_o' (D_o' - D_o)
\]

\[
k_3 = L_o L C
\]

\[
k_4 = R_o L C
\]

\[
k_5 = L_o (D_o - D_o')^2 + 2LD_o'^2
\]

\[
k_6 = R_o (D_o - D_o')^2
\]

\[
k_7 = (-2I_L + I_o)L_o L
\]

\[
k_8 = (-2I_L + I_o)R_o L + (D_o' - D_o)(2V_C - V_S)L_o - D_o'L(V_S - 2V_C)
\]

\[
k_9 = (D_o' - D_o)(2V_C - V_S)R_o
\]
Similarly, for the inductor current, the transfer functions $G_{ua}(s)$ and $G_{id}(s)$ are defined as

\[
G_{ua}(s) = \frac{i_L(s)}{v_s(s)} \bigg|_{d_s(s)=0}
\]

is the input-to-inductor current transfer function

\[
G_{id}(s) = \frac{i_L(s)}{d_s(s)} \bigg|_{d_s(s)=0}
\]

is the control-to-inductor current transfer function

These equations are derived as

\[
G_{ua}(s) = \frac{k_{10} s^3 + k_{11} s^2 + k_{12} s + k_{13}}{sL(k_3 s^3 + k_4 s^2 + k_5 s + k_6)}
\]  \hspace{1cm} (3.58)

and

\[
G_{id}(s) = \frac{k_{14} s^3 + k_{15} s^2 + k_{16} s + k_{17}}{sL(k_3 s^3 + k_4 s^2 + k_5 s + k_6)}
\]  \hspace{1cm} (3.59)

where the coefficients $k_{10}$ through $k_{17}$ are

\[
k_{10} = D_o' L_o LC
\]

\[
k_{11} = D_o' R_o LC
\]

\[
k_{12} = LD_o^{-1}
\]

\[
k_{13} = 0
\]

\[
k_{14} = (2V_c - V_s)L_o LC
\]

\[
k_{15} = (D_o - D_o')(2I_L + I_o)L_o L + (2V_c - V_s)R_o LC
\]
\[ k_{16} = (D_o - D_o')(\text{-}2I_L + I_o)R_oL + (2V_c - V_s)D_o'L \]

\[ k_{17} = 0 \]

Using Equations (3.54) to (3.59), the mathematical model for Z-source inverter is developed in MatLab environment.

### 3.4.3 Mathematical Model Verification

The effectiveness of the mathematical model derived in the previous section can be verified by simulating and comparing the dynamics of the small signal circuit with the dynamics of the actual switching circuit. The state variables taken here for simulation and comparison are the capacitor voltage and inductor current. The circuit parameters given in Table 3.2 are used for verification purposes.

**Table 3.2 Simulation parameters for small signal model verification**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input voltage ( V_S )</td>
<td>42V</td>
</tr>
<tr>
<td>Z-network inductance ( L_1, L_2 )</td>
<td>3mH</td>
</tr>
<tr>
<td>Z-network capacitance ( C_1, C_2 )</td>
<td>1000μF</td>
</tr>
<tr>
<td>Load resistance ( R_o )</td>
<td>25Ω</td>
</tr>
<tr>
<td>Load inductance ( L_o )</td>
<td>10mH</td>
</tr>
<tr>
<td>Switching frequency ( f_s )</td>
<td>10kHz</td>
</tr>
<tr>
<td>Shoot-through duty ratio ( D_o )</td>
<td>0.27</td>
</tr>
</tbody>
</table>

The simulation results of the capacitor voltage waveforms obtained from the circuit model and small signal model are shown in Figure 3.10. Figure 3.11 shows the simulation results of the Z-network inductor current waveforms by the circuit model and small signal model.
Figure 3.10  Simulation results of Z-network capacitor voltage by circuit model and small signal model
**Figure 3.11** Simulation results of Z-network inductor current by circuit model and small signal model

The transient response of the Z-network capacitor voltage and inductor current waveforms obtained from both the small signal model and the actual switching circuit model are shown in Figure 3.12 and Figure 3.13 when there is a step increase of duty cycle from 0.27 to 0.34 at 0.1s. Also Figure 3.14 and Figure 3.15 show the capacitor voltage and inductor current waveforms obtained from both small signal and switching circuit when there is a step increase of input voltage from 42V to 50V.
Figure 3.12 Transients of capacitor voltage due to change in duty ratio

Figure 3.13 Transients of inductor current due to change in duty ratio
Figure 3.14 Transients of capacitor voltage due to change in input voltage

Figure 3.15 Transients of inductor current due to change in input voltage
The simulation results obtained from both the small signal model and the actual circuit model show that the responses by two models agree with each other very well. The corresponding experimental results are shown in Figure 3.16 through Figure 3.19. The state space averaged small signal model derived is proven to be accurate enough to prognosticate the transient responses as well as steady state values of the Z-source inverter.

![Capacitor Voltage and Duty Ratio Signal](image)

**Figure 3.16** Experimental result of capacitor voltage due to change in duty ratio

![Inductor Current Signal](image)

(2A/div, 1ms/div)

**Figure 3.17** Experimental result of inductor current due to change in duty ratio
Figure 3.18 Experimental result of capacitor voltage due to change in input voltage

(2A/div, 1ms/div)

Figure 3.19 Experimental result of inductor current due to change in input voltage

3.4.4 Design Oriented Analysis Based on Small Signal Model

Design oriented methods based on the small signal model is a useful tool to designers, helping them clearly understand the transient response and the limits of the system. Specifically, parametric sweep of bode
plots in frequency domain is helpful to optimize component parameters according to overall system specifications. The bode plots of the various transfer functions derived for the Z-source inverter are shown in Figure 3.20 through Figure 3.27.

For the Z-source inverter, the values of impedance network components $L$ and $C$ can be properly computed with the help of salient features of bode plots based on the transfer functions. The frequency response of input to capacitor voltage transfer function $G_{is}$ for different values of capacitances $C$ if inductance $L$ is chosen as 3mH is shown in Figure 3.20. From the bode plot, it is shown that the quality factor $Q$ of $G_{is}$ is increased if $C$ is increased, causing a sharper change of phase in the neighborhood of the corner frequency. The bode plot of the same transfer function $G_{vs}$ with different values of inductances $L$ if capacitance $C$ is selected as 1300uF is shown in Figure 3.21. Here the quality factor $Q$ of $G_{vs}$ is decreased with increase of inductance $L$. Also, it is observed from these two bode plots that increase either in $L$ or $C$ will decrease the resonant frequency. Similarly, the frequency responses of input-to-inductor current transfer function $G_{is}$ for different values of $C$ and $L$ are shown in Figure 3.22 and Figure 3.23.

The important parameter for controlling the boosting operation of Z-source inverter is the shoot-through duty cycle. The parametric sweep of shoot-through duty cycle provides useful perceptiveness into the system frequency responses at different control values. Frequency responses of input to capacitor voltage transfer function $G_{vs}$, control-to-capacitor voltage transfer function $G_{vd}$, input-to-inductor current transfer function $G_{is}$, and input-to-inductor current transfer function $G_{id}$ with different shoot-through duty cycles are shown in Figure 3.24 through Figure 3.27.
Figure 3.20  Bode plot of input-to-capacitor voltage transfer function $G_{sc}$, with different capacitances

Figure 3.21  Bode plot of input-to-capacitor voltage transfer function $G_{sc}$, with different inductances
Figure 3.22 Bode plot of input-to-inductor current transfer function $G_{iL}$, with different capacitances

Figure 3.23 Bode plot of input-to-inductor current transfer function $G_{iL}$, with different inductances
Figure 3.24 Bode plot of input-to-capacitor voltage transfer function $G_{v_{oc}}$, with different shoot-through duty ratio

Figure 3.25 Bode plot of control-to-capacitor voltage transfer function $G_{v_{oc}}$, with different shoot-through duty ratio
Figure 3.26  Bode plot of input-to-inductor current transfer function $G_{iL}$, with different shoot-through duty ratio

Figure 3.27  Bode plot of input-to-inductor current transfer function $G_{id}$, with different shoot-through duty ratio
The shoot-through duty cycle $D_o$ will change the boost factor or the gain of Z-source inverter as analyzed in previous section. The results show that variation of $D_o$ results in change of resonant frequency as well as change of the quality factor $Q$.

The selection of Z-network inductances and capacitances can be compared with the time domain transient simulations of Z-source inverter. Figure 3.28 shows simulation results of capacitor voltage and inductor current of Z-source inverter.

The values of $L$ and $C$ chosen are:

(a) $L = 2\text{mH}$ and $C = 1300\mu\text{F}$
(b) $L = 1\text{mH}$ and $C = 1000\mu\text{F}$
(c) $L = 3\text{mH}$ and $C = 1300\mu\text{F}$
(d) $L = 3\text{mH}$ and $C = 1000\mu\text{F}$

The time response of capacitor voltage in transient state and steady state with different values of capacitances $C$ if inductance $L$ is selected as 3mH is shown in Figure 3.29. Similarly, the Figure 3.30 shows time response of inductor current in transient state and steady state with different values of inductances $L$ if capacitance $C$ is selected as 1000μF. From the results, if capacitance is increased, the capacitor voltage overshoot is increased. Also it increases the settling time. But in the steady state high capacitance produces reduced ripple. Similarly, smaller inductance causes large overshoot in the transient state and high ripple in the steady state.
Figure 3.28 Simulation results of capacitor voltage and inductor current 
for different $L$ and $C$

(a)

Figure 3.29 Simulation results of capacitor voltage with $L = 3\text{mH}$ and 
different $C$
Figure 3.30 Simulation result inductor current with $C = 1000\mu \text{F}$ and different $L$

The time domain simulation results shown in Figure 3.28 through Figure 3.30 confirm that the selection of inductance $L$ as 3mH and capacitance $C$ as 1000$\mu \text{F}$ is reasonable, verifying the conclusions obtained from the parametric sweeps in the frequency domain.

3.5 SUMMARY

$Z$-source inverter is the emerging converter topology in power electronics systems. The working principles, modes of operation and steady state analysis have been presented in this chapter. In order to study the dynamics introduced by the capacitors and inductors in the impedance network, the development of accurate mathematical models is necessary. The state-space averaging technique has been used to derive the small-signal model of $Z$-source inverter. Based on the mathematical model, the inductor current and capacitor voltage transfer functions are attained. Both small signal
model and actual circuit model have been simulated using MatLab/Simulink software. In addition, the design oriented analysis has been provided based on frequency domain and time domain.