Part III

Probing Non-Standard Physics with Events at IceCube
Chapter 4

Introduction

Neutrinos produced via decay of pions are expected to roughly carry the flavour ratio $(\nu_e : \nu_\mu : \nu_\tau =) 1 : 2 : 0$ at the source. Standard neutrino oscillations in vacuum massage this ratio during propagation to $1 : 1 : 1$ \cite{57, 58} at the detector, if we assume $\theta_{13} = 0$ and $\theta_{23} = \pi/4$ consistent with current data \cite{59, 60, 61}. As we will show in the following section, standard flavour oscillations over Mega-parsec distances make the neutrino spectra of every flavour nearly identical in shape. Therefore, if for any reason the astrophysics in the source leads to a ratio different from $1 : 2 : 0$ or spectral shapes for flavours which differ widely from each other, standard oscillations still massage them into identical shapes and magnitudes which are within a factor of roughly 2 of each other by the time they reach the earth.

4.1 Spectral Averaging due to Oscillations

Fig. 4.1 (in arbitrary units, and without normalisation) shows the spectra\footnote{The spectra shown here is unrealistic and chosen only to demonstrate the effect of standard oscillations on even such widely differing flavour fluxes.} of two flavours in a single source AGN, intentionally chosen to be significantly differing in shape and magnitude, and the resulting diffuse fluxes from all such sources for the same flavours as seen at earth after standard propagation using the procedure described above. It is evident that not only do oscillations tend to bring widely differing magnitudes close (to within a factor of 2) to each other, but they wash out even large differences in spectral shapes that may originate in a particular source, perhaps due to conventional physics, as e.g. in \cite{63}. We have checked that this conclusion holds in general, and a common intermediate shape is assumed by both fluxes at earth detectors. These conclusions are no longer true if in the propagation equation, the oscillation probability is modified by new physics in an energy-dependant manner, as we demonstrate in the examples below.
Figure 4.1: The even-ing out of possible spectral distortions present at source due to oscillations over large distances. The green and deep red lines represent assumed spectra from a single (hypothetical) extra-galactic source for the flavours $\nu_e$ and $\nu_\mu$ respectively at various stages: (clockwise, starting from the top-left) at the source, at earth from the single source evened out by standard oscillation and finally, the corresponding diffuse fluxes (from similar sources) at earth after integrating over source distribution and oscillations.
Chapter 5

Effect of Non-Standard Physics on Neutrino Propagation

A series of papers [64, 65, 66, 67, 68, 69, 70, 71, 72, 73] over the past few years have demonstrated that if the flavour ratios $\nu_e^d : \nu_\mu^d : \nu_\tau^d$ detected by extant and upcoming neutrino telescopes were to deviate significantly from this democratic prediction, then important conclusions about physics beyond the Standard Model and neutrino oscillation parameters may consequently be inferred. In addition, deviations of these measured ratios have been shown to be sensitive to neutrino oscillation parameters [74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 73] (e.g. the mixing angles and the Dirac CP violating phase).

In this chapter we study the spectral distortions in the diffuse (i.e. integrated over source distribution and redshift) UHE neutrino flux as a probe for the effects of new physics. For specificity, we focus on AGN fluxes, and use, as a convenient benchmark, the well-known upper bounds first derived by Waxman and Bahcall (WB) [85] and later by Mannheim, Protheroe and Rachen (MPR) [86] on such fluxes for both neutron-transparent and neutron-opaque sources (or, equivalently, sources that are optically thin and optically thick, respectively, to the emission of neutrons). In particular we focus on the upper bounds to the diffuse neutrino flux from hadronic photoproduction in AGN’s derived in [86] using the experimental upper limit on cosmic ray protons. All distortions in the fluxes are, as would be expected, transmitted to the upper bounds, thus providing a convenient way of representing and studying them.

As demonstrated in Sec. 4.1, the usual (SM) neutrino oscillations not only tend to equilibrate widely differing source flux magnitudes between flavours, but also massage them into a common spectral shape, as one would intuitively expect. Thus observed relative spectral distortions among flavours are a probe of new physics present in the propagation equation. To demonstrate our approach we then focus on specific cases of non-standard physics, viz. a) Decay of neutrinos, b) The effect of variation of $\theta_{13}$ and the presence of a non-zero CP-violating phase ($\delta_{\text{CP}} \neq 0$), and c) Lorentz violation (LV) in the
neutrino sector. The effect of decoherence among the neutrino flavours during propagation and that of the presence of pseudo-Dirac neutrinos is also briefly discussed. Our method can straightforwardly be applied to other new physics scenarios and our results translated into bounds on muon track versus shower event rates\footnote{These count the sum of a) neutral current (NC) events of all flavours, b) electron neutrino charged current (CC) events at all energies, and c) $\nu_\tau$ induced CC events at energies below $\leq 1$ PeV ($10^6$ GeV), whereas muon track events arise from $\nu_\mu$ induced muons born in CC interactions.} for UHE experiments.

In our calculations throughout this chapter we use the following values of the neutrino mixing parameters\cite{9}:  
\[
\begin{align*}
\Delta m^2_{21} &= 7.65 \times 10^{-5} \text{ eV}^2 \\
\Delta m^2_{31} &= \pm 2.40 \times 10^{-3} \text{ eV}^2 \\
\sin^2(\theta_{12}) &= 0.321, \quad \sin^2(\theta_{23}) = 0.47, \quad \sin^2(\theta_{13}) = 0.003.
\end{align*}
\]

The chapter is organized as follows. Section 5.1 shows the modification of these fluxes due to decay of the heavier neutrinos, and its effect on the number of detectable events at a large volume detector like the IceCube. We examine the effect of variation of $\theta_{13}$ and the CP violating phase $\delta_{\text{CP}}$ in Section 5.2. We look at the effect of Lorentz-symmetry violation in Section 5.3 and finish with brief investigations of the effects of pseudo-Dirac neutrinos and decoherence in the last two sections.

5.1 Effect of neutrino decay

5.1.1 Introduction to neutrino decay

Bounds on the life-times of neutrinos are obtained primarily from observations of solar\cite{8} and atmospheric neutrinos. Observations from solar neutrinos lead to  
\[
\frac{\tau_2}{m_2} \geq 10^{-4} \text{ s/eV} \tag{5.1}
\]
while, if the neutrino spectrum is normal, the data on atmospheric neutrinos constrain the life-time of the heaviest neutrino  
\[
\frac{\tau_3}{m_3} \geq 10^{-10} \text{ s/eV}. \tag{5.2}
\]

In the following, we treat the lightest neutrino as stable in view of the fact that its decay would be kinematically forbidden, and consider the decay of the heavier neutrinos to invisible daughters like sterile neutrinos, unparticle states, or Majorons. Neutrinos may decay via many possible channels. Of these, radiative two-body decay modes are\cite{9}:
strongly constrained by photon appearance searches \[89\] to have very long lifetimes, as are three-body decays of the form $\nu \rightarrow \nu \nu \bar{\nu}$ which are constrained \[90\] by bounds on anomalous $Z\nu\bar{\nu}$ couplings \[91\]. Decay channels of the form

$$\nu_i \rightarrow \nu_j + X \quad (5.3)$$

$$\nu \rightarrow X \quad (5.4)$$

where $\nu_i$ represents a neutrino mass eigenstate and $X$ represents a very light or massless invisible particle, e.g. a Majoron, are much more weakly constrained, however and are therefore the basis of our consideration in this section. When considering decays via the channel in Eq. (5.3) we assume that the daughter neutrino produced is significantly reduced in energy and does not contribute to the diffuse flux in the energy range relevant for our purpose (1000 GeV to $10^{11}$ GeV). A detailed study of the various possible scenarios for neutrino decay is made in \[115\].

Prior to proceeding, we would like to discuss cosmological observations of high precision which might be able to constrain models of decay via channels as in Eq. (5.3) in the future. These constraints are based on the determination of the neutrino mass scale as discussed in \[93\], or from the cosmic microwave background as discussed in \[111\]. Such observations would serve to push the lower bound of neutrino decay lifetimes by several orders of magnitude compared to those discussed here. However, these predictions are dependent upon the number of neutrinos that free-stream and assume couplings of similar nature and strength for all the species of the neutrino family. As discussed in \[114\] and \[94\] these assumptions must await confirmation and rely on future data. Hence, “fast” neutrino decay scenarios are not ruled out within the scope of current theory and experiment, though they are disfavoured. Further the decay of neutrinos via Eq. (5.4) and in the cases where the decay, both via Eq. (5.3) and Eq. (5.4) happen due to unparticle scenarios are not covered by such constraints and the purely phenomenological and general study of neutrino decay in the life-times discussed here would still be interesting and relevant for future neutrino detectors.

5.1.2 Effect of neutrino decay on the flavour fluxes

A flux of neutrinos of mass $m_i$, rest-frame lifetime $\tau_i$, energy $E$ propagating over a distance $L$ will undergo a depletion due to decay given (in natural units with $c = 1$) by a factor of

$$\exp(-t/\gamma\tau) = \exp\left(-\frac{L}{E} \times \frac{m_i}{\tau_i}\right)$$

where $t$ is the time in the earth’s (or observer’s) frame and $\gamma = E/m_i$ is the Lorentz boost factor. This enters the oscillation probability and introduces a dependence on the lifetime and the energy that significantly alters the flavour spectrum. Including the decay factor,
the probability of a neutrino flavour $\nu_\alpha$ oscillating into another $\nu_\beta$ becomes

$$P_{\alpha\beta}(E) = \sum_i |U_{\beta i}|^2 |U_{\alpha i}|^2 e^{-L/\tau_i(E)}, \alpha \neq \beta,$$

(5.5)

which modifies the flux at detector from a single source to

$$\phi_{\nu_\alpha}(E) = \sum_{i\beta} \phi_{\nu_\beta}^{\text{source}}(E) |U_{\beta i}|^2 |U_{\alpha i}|^2 e^{-L/\tau_i(E)}.$$  

(5.6)

We use the simplifying assumption $\tau_2/m_2 = \tau_3/m_3 = \tau/m$ for calculations involving the normal hierarchy (i.e. $m_3^2 - m_1^2 = \Delta m^2_{31} > 0$) and similarly, $\tau_1/m_1 = \tau_2/m_2 = \tau/m$ for those with inverted hierarchy (i.e. $\Delta m^2_{31} < 0$), but our conclusions hold irrespective of this. The total flux decreases as per Eq. (5.6), which is expected for decays along the lines of Eq. (5.4) and, within the limitations of the assumption made in Sec. 5.1.1, also for Eq. (5.3).

The assumption of complete decay leads to (energy independent) flux changes from the expected $\nu_e^d : \nu_\mu^d : \nu_\tau^d = 1 : 1 : 1$ to significantly altered values depending on whether the neutrino mass hierarchy is normal or inverted as discussed in [65]. From Fig. 5.1 we note that the range of energies covered by UHE AGN fluxes spans about six to seven orders of magnitude, from about $10^3$ GeV to $10^{10}$ GeV. For the “no decay” case, the lowest energy neutrinos in this range should arrive relatively intact, i.e. $L/E \simeq \tau/m \simeq 10^4$ sec/eV. In obtaining the last number we have assumed a generic neutrino mass of 0.05 eV and an average $L$ of 100 Mpc. On the other hand, if there is complete decay, only the highest energy neutrinos arrive intact, and one obtains i.e. $L/E \simeq \tau/m \leq 10^{-3}$ sec/eV. Thus, a study of the relative spectral features and differences of flavour fluxes at earth allows us to study the unexplored range $10^{-3} < \tau/m < 10^3$ via decays induced by lifetimes in this range (we have referred to this case as “incomplete decay” in what follows).

To calculate the MPR-like bounds with neutrino decay we use the procedure of Sec. 1.3 but replace the standard neutrino oscillation probability by $P_{\alpha\beta}$ given in Eq. (5.5) with $E$ replaced by $E/(1+z)$ to account for red-shifting. Since, unlike standard oscillations, $P_{\alpha\beta}$ has an energy dependence that does not just average out, the diffuse flux obtained with decay effects differ considerably from the MPR bounds in shape as well as magnitude. Fig. 5.1 shows the effect for both normal and inverted hierarchies with a lifetime of $\tau_2/m_2 = \tau_3/m_3 = 0.1$ s/eV. We note that the effect of decay in altering the diffuse flux spectrum is especially strong in the case of inverted hierarchy.

Fig. 5.2 shows how the diffuse flux spectral shapes change as the lifetimes of the two heavier mass-eigenstates are varied between $10^{-3}$ s/eV and 1 s/eV. From the figure it is clear that this ($10^{-3}$ s/eV – 1 s/eV) is the range of life-times that can be probed by ultra-high-energy detectors looking for spectral distortions in the diffuse fluxes of the three flavours. For lifetimes above 1 s/eV the spectral shapes start to converge and become
Figure 5.1: Modification of MPR bound for incomplete decay with normal hierarchy (left) and inverted hierarchy (right), and life-time $\tau_2/m_2 = \tau_3/m_3 = 0.1\ s/eV$. The $\nu_\mu$ and $\nu_e$ fluxes shown are from optically thick (in thick) and optically thin sources (thinner). Similarly the gray lines indicate the $\nu_e$, $\nu_\mu$, or $\nu_\tau$ undistorted flux modified only by neutrino oscillation, for both optically thick and thin sources. Sensitivity thresholds and energy ranges of relevant experiments, viz., AMANDA and IceCube, and ANITA [92] are indicated. $I(E)$ denotes the diffuse flux spectrum of flavours at earth, obtained as described in the text.

As is also the case for complete decays, the results are very different for the two possible hierarchies. This is because the mass eigenstate $m_1$ contains a large proportion of $\nu_e$, whereas the state $m_3$ is, to a very large extent, just an equal mixture of $\nu_\mu$ and $\nu_\tau$ with a tiny admixture of $\nu_e$. Therefore decay in the inverted hierarchy case would lead to a disappearance of the eigenstate with high content of $\nu_e$ and, hence, to its strong depletion against the other two flavours. In the normal hierarchy case, in comparison, the mass eigenstate with the high content of $\nu_e$ is also the lightest, and decay of the heavier states consequently leads to a depletion of $\nu_\mu$ and $\nu_\tau$. Thus incomplete decay to the lowest mass eigenstate with a normal hierarchy (i.e. $m_1$) would lead to considerably more shower events than anticipated with an inverted hierarchy.

While assessing the results presented here, it must be borne in mind that observation of a significant amount of $\bar{\nu}_e$ from supernova SN1987A possibly imposes lower limits on decay lifetimes of the heavier neutrinos for the inverted hierarchy scenario that are much higher than those considered here [95, 96]. This observation, of a flux of $\bar{\nu}_e$ roughly in keeping with standard predictions constrains its “lifetime” $\tau/m > 10^5$, i.e., higher than what would give observable results with the methods described here. Despite the uncertainties involved with neutrino production from supernovae and the fact that the
Figure 5.2: Modification of MPR bound for incomplete decay with normal hierarchy (left) and inverted hierarchy (right), and life-times varying from $\tau/m = 0.001 \text{ s/eV}$ to $1.0 \text{ s/eV}$. The $\nu_\mu$ and $\nu_e$ fluxes shown are from optically thick sources. The gray lines indicate the $\nu_e$, $\nu_\mu$, or $\nu_\tau$ undistorted flux modified only by neutrino oscillation. Similar effects are seen with fluxes from optically thin sources as well.
total signal from SN1987A was only a handful of events, the results for decay with inverted hierarchy must be judged keeping this in view.

5.1.3 Modification of total UHE events due to decay

The effect of decay as seen in the diffuse fluxes in Fig. 5.1 above must also translate to modifications in the shower and muon event rates observable at UHE detectors. In this section we demonstrate this by a sample calculation. We calculate the event-rates induced by the three flavours of high-energy cosmic neutrinos after decay using a simplified version of the procedure in Ref. [116] and compare it to those predicted by standard physics.

Events at the IceCube will be classified primarily into showers and muon-tracks. Shower events are generated due to the charged current (CC) interactions of \( \nu_e \) and \( \nu_\tau \) below the energy of 1.6 PeV and neutral current (NC) interactions of all the three flavours. For energies greater than 1.6 PeV, CC interactions of the \( \nu_\tau \) have their own characteristic signatures in the form of double-bangs, lollipops, earth-skimming events, etc. [97, 98]. Muon-tracks are generated due to the \( \nu_\mu \) induced CC events.

\( \nu_e \) induced events

In the standard model \( \nu_e \) interacts with nucleons via CC and NC interactions leading to electromagnetic and hadronic showers.

In the CC events, the shower energy is equal to the initial neutrino energy \( E_\nu \), that is, the total energy of the two final state particles (an electron and a scattered quark). The event rate for \( \nu_e N \rightarrow e^- \chi \), with \( \chi \) being a final state quark, is given by

\[
\text{Rate} = \int_{E_{th}}^{\infty} dE_\nu \int_0^1 dy \, N_A L \frac{d\sigma_{CC}}{dy} A \mathcal{F}(E_\nu) \\
= N_A V \int_{E_{th}}^{\infty} dE_\nu \, \sigma_{CC}(E_\nu) \mathcal{F}(E_\nu)
\]  

(5.7)

(5.8)

where

- \( E_\nu \): the incident neutrino energy
- \( E_{th} \): detection threshold for shower events
- \( y \): the inelasticity parameter defined as \( y \equiv 1 - \frac{E_{e,\mu,\tau}}{E_\nu} \)
- \( A, L, V \): the area, length and volume of the detector respectively
- \( \mathcal{F}(E_\nu) \): the flux spectrum of neutrinos in GeV\(^{-1}\)cm\(^{-2}\)s\(^{-1}\)
It is assumed that the electron range is short enough such that the effective volume of
the detector is identical to the instrumental volume. Using standard tabulated values of
the cross-section $\sigma_{CC}$\,[99, 100] it is straightforward to evaluate the integral in Eq. (5.8) to
obtain the event rate. The event rate for anti-neutrino process $\bar{\nu}_e N \rightarrow e^+ \chi$ is calculated
similarly.

For the NC events, the final state neutrino develops into missing energy, so that the
rate is given by

$$\text{Rate} = \int_{E_{th}}^{\infty} \int_{E_{th}/E_\nu}^{1} dy N_A L \frac{d\sigma_{NC}}{dy} A F (E_\nu)$$

To simplify Eq. (5.9) we use the approximation

$$\frac{d\sigma}{dy} \approx \sigma \delta (y - \langle y \rangle)$$

where $\langle y \rangle$ is the mean inelasticity parameter. Thus, we have

$$\text{Rate} = N_A V \int_{E_{th}}^{\infty} dE_\nu \sigma_{NC}(E_\nu) F(E_\nu),$$

$E_{th}$ is an effective threshold energy at which the curves defined by $y = E_{th}/E_\nu$ and $y = \langle y \rangle$
intersect.

$\nu_\mu$ induced events

The muon track event is calculated by

$$\int_{E_{th}}^{\infty} dE_\nu N_A \int_{0}^{1 - \frac{E_{th}}{E_\nu}} dy R(E_\nu(1 - y), E_{th}) \frac{d\sigma_{CC}}{dy} S(E_\nu) A F (E_\nu),$$

where,

$$R(x, y) = \frac{1}{b} \ln \left( \frac{a + bx}{a + by} \right)$$

with $a = 2.0 \times 10^{-3}$ GeV cm$^{-1}$ and $b = 3.9 \times 10^{-6}$ GeV cm$^{-1}$. $S(E_\nu)$ represents the
shadowing effect by the earth [99, 100].

Approximating using Eq. (5.10) gives

$$\text{Rate} = \int_{E_{th}}^{\infty} dE_\nu N_A R(E_\nu(1 - \langle y \rangle), E_{th}) \sigma_{CC}(E_\nu) S(E_\nu) A F (E_\nu)$$

with $E'_{th}$ being determined similarly as for the $\nu_e$ induced events.
Using the procedure described above, we calculate the total shower and muon-track detector events (for $\bar{\nu} + \nu$) for the inverted hierarchy scenario with a life-time of $1.0 \text{ s/eV}$ depicted in Fig. 5.2 (top-right) and compare it to the events expected from standard physics. The results are tabulated in Table 5.1 where we show event rates for UHE detectors, like the IceCube, over a 10 year period integrated over solid angle. The difference between the ratio of muon-track to shower events due to standard oscillation and that after considering neutrino decay are shown in Fig. 5.3.

<table>
<thead>
<tr>
<th>Energy [GeV]</th>
<th>Shower</th>
<th>Muon Track</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^3 - 10^4$</td>
<td>7</td>
<td>10</td>
</tr>
<tr>
<td>$10^4 - 10^5$</td>
<td>42</td>
<td>96</td>
</tr>
<tr>
<td>$10^5 - 10^6$</td>
<td>145</td>
<td>325</td>
</tr>
<tr>
<td>$10^6 - 10^7$</td>
<td>129</td>
<td>297</td>
</tr>
<tr>
<td>$10^7 - 10^8$</td>
<td>64</td>
<td>85</td>
</tr>
<tr>
<td>$10^8 - 10^9$</td>
<td>21</td>
<td>16</td>
</tr>
<tr>
<td>$10^9 - 10^{10}$</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>$10^{10} - 10^{11}$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5.1: Total shower and muon-track detector events (for $\bar{\nu} + \nu$) over 10 years, and integrated over solid angle for the inverted hierarchy scenario with a life-time of $\tau/m = 1.0 \text{ s/eV}$ depicted in Fig. 3.

The disappearance of a majority of shower events (due to the depletion of the $\nu_e$ flux compared to that of $\nu_\mu$) at lower energies, and their reappearance at higher energies is a distinctive feature. It indicates the presence of new physics (like incomplete decay) as opposed to spectral distortions originating in the source, or the appearance of a new class of sources. In the latter case, a corresponding depletion and subsequent enhancement is expected in muon events. By contrast, in the case of incomplete decay the fluxes return to the democratic ratio at higher energies where the neutrinos do not decay.

### 5.2 Effect of non-zero CP violating phase and $\theta_{13}$ variation on neutrino decay

As described in Sec. 5.1 the calculation for the effect of decay of heavier neutrinos on the diffuse flux spectrum was done keeping the CP violating phase $\delta_{CP} = 0$ and $\theta_{13}$ at the $3\sigma$ best fit value which is close to zero. In this section we look at how our conclusions are affected if we change these parameters significantly. In Sec. 5.2.1 we look at how
Figure 5.3: The ratio (R) of muon-track events to shower events with inverted hierarchy and life-time $\tau/m = 1.0 \text{ s/eV}$ as shown in Table 1. The ratio expected due to standard physics is shown in brown, while the modified ratio due to the effects of decay is shown in light red. At energies greater than $10^8 \text{ GeV}$, $R$ due to standard physics and that after considering decay become equal.
changing $\theta_{13}$ from 0 to the CHOOZ maximum affects the decay effected diffuse fluxes, while in Sec. 5.2.2 we examine the consequences of a non-zero CP violating phase in the same context.

### 5.2.1 Variation of $\theta_{13}$

Observations at CHOOZ [101] constrain the maximum value of $\theta_{13}$ (90% confidence level) such that

$$\sin^2 (2\theta_{13}^{\text{max}}) = 0.10.$$  

Therefore, we have for $\theta_{13}$ the following experimentally allowed range of values

$$0 \leq \theta_{13} \leq 9.1^\circ$$

We allow $\theta_{13}$ to vary within this range and study its effect on the results of Sec. 5.1. The results are represented in Fig. 5.4. It is clear that the effect of varying $\theta_{13}$ is significant. However, given the strong difference in the diffuse flux spectra for inverted and normal hierarchies, variation of $\theta_{13}$ over the entire range would not affect our qualitative conclusions in Sec. 5.1 regarding differentiating between the two.

### 5.2.2 Non-zero CP violating phase.

The CP violation phase in the three family neutrino mixing matrix is as yet not experimentally determined. Neutrino telescopes probing ultra-high energies might be able to
improve upon our present knowledge of this parameter (see [82], for example). Here we look at how the presence of a non-zero CP violating phase, $\delta_{CP}$, in the mixing matrix could affect results obtained in Sec. 5.1.

$\delta_{CP}$ enters the oscillation probability via the mixing matrix as the product $\sin(\theta_{13}) \cdot \exp(\pm i\delta_{CP})$. Therefore, a non-zero CP violating phase does not affect any of our calculations if $\theta_{13} = 0$ and its effect is imperceptible even when the $3\sigma$ best-fit value of $\theta_{13}$ is used as is the case in Sec. 5.1. For the remainder of this section we keep $\theta_{13}$ at the CHOOZ maximum and vary the CPV phase from 0 to $\pi$. Fig. 5.5 shows the result on the $\nu_\mu$ flavour for decay in the case of a normal hierarchy for diffuse flux from optically thick sources. In the same way Fig. 5.6 shows the effect of a non-zero CP violating phase on decay with both the normal and inverted hierarchy. The effect of CP violation is quite small on the diffuse flux with inverted hierarchy as compared to that with normal hierarchy.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure5.5.png}
\caption{Effect of CP violation on the diffuse flux of the $\nu_\mu$ flavour obtained by considering decay with normal hierarchy and life-time of $\tau/m = 0.1$ s/eV. The variation in the flux as the CP violating phase is varied between $0 - \pi$ is shown as the shaded region.}
\end{figure}

To summarise, it is clear from the discussion in Sec. 5.1 and 5.2 that future neutrino detectors capable of distinguishing between flavours should be able to probe and potentially provide stronger bounds on decay lifetimes of heavier neutrinos. If the neutrinos decay with a lifetime within the ranges discussed here, then they would also be able to distinguish between the two hierarchies due to the strongly different diffuse flux spectra.
the two hierarchies lead to for the flavours $\nu_e$ and $\nu_\mu$, notwithstanding the effect of a non-zero CP violating phase or the uncertainty over the value of $\theta_{13}$.

5.3 Effect of Lorentz symmetry violation

Low energy phenomenology can be affected by Lorentz symmetry violating effects originating at very high energies. Typically such effects originate at energies close to the Planck scale. They may appear in certain theories which are low energy limits of string theory [102, 103], or could possibly signal the breakdown of the CPT theorem [104]. Additionally, if quantum gravity demands a fundamental length scale, leading to a breakdown of special relativity, or loop quantum gravity [105, 106, 107, 112, 108, 109] leads to discrete space-time, one expects tiny LV effects to percolate to lower energies. UHE neutrinos, with their high energies and long oscillation baselines present a unique opportunity for testing these theories. Their effects in the context of flavour flux ratios have been discussed in [70]. They may arise, for example, due to a vector or tensor field forming a condensate and getting a vacuum expectation value, thereafter behaving like a background field. The effective contribution of such background fields can then be handled in the low energy theory using standard model extensions [103]. It has been shown [104] that although CPT symmetry violation implies Lorentz violation, Lorentz violation does not necessarily require or imply the violation of CPT symmetry. In this section we focus on the modification of the propagation of neutrinos due to Lorentz symmetry violating effects along the lines discussed in Ref. [110]. Since the effects of Lorentz-violation and CPT violation are understandably tiny at low energies, it is difficult to explore their phenomenological signatures using low energy probes, in colliders for example. Since they

![Diagram](image-url)
originate in extremely energetic cosmological accelerators and propagate over cosmic distances, ultra-high energy neutrinos provide the perfect laboratory for constraining and, possibly, determining Lorentz-violating parameters.

5.3.1 Modification of neutrino transition probabilities due to LV effects

As an example, we will study, for the simplification that it provides, a two-flavour scenario with massive neutrinos and consider the modification of the transition probability from one flavour to the other by Lorentz-violation due to an effective standard model extension. Our focus is on LV from off-diagonal terms in the effective hamiltonian describing the propagation of the neutrinos [70].

We consider an effective Hamiltonian describing neutrino propagation

$$H^{\text{eff}}_{\alpha\beta} = |\vec{p}| \delta_{\alpha\beta} + \frac{1}{2} |\vec{p}| \left[ \tilde{m}^2 + 2 (a^\mu p_\mu) \right]_{\alpha\beta}$$

(5.15)

where $\tilde{m}$ is related to the neutrino mass and $a$ is a real CPT and Lorentz violating parameter. In the two neutrino mass basis this gives

$$H^{\text{eff}} = \begin{pmatrix} m^2_1 & a \sqrt{\frac{m^2_1}{2E}} \\ a \sqrt{\frac{m^2_2}{2E}} & m^2_2 \end{pmatrix}$$

(5.16)

With the mixing angle between the two flavours $\theta_{23} = \pi/4$, this modifies the probability of transition from one flavour to another during propagation to

$$P [\nu_\mu \rightarrow \nu_\tau] = \frac{1}{4} \left( 1 - \frac{a^2}{\Omega^2} - \frac{\omega^2}{\Omega^2} \cos (2\Omega L) \right)$$

(5.17)

where $\omega = \frac{\Delta m^2}{4E}$ and $\Omega = \sqrt{\omega^2 + a^2}$.

5.3.2 Effect of Lorentz violation on neutrino flavour fluxes

To calculate the diffuse fluxes of the two neutrino flavours we use Eq. (5.17) instead of the standard oscillation probability and integrate over the red-shift $z$. The probability above contributes a $z$ dependent term through its dependence on energy. Further the cos $(2\Omega L)$ term averages out and consequently does not contribute.

The results of including Lorentz violation in the propagation phenomenology of neutrinos are shown in Fig. [5.7]. It is clear from these plots that the strong departure of diffuse spectral shapes of $\nu_\mu$ and $\nu_\tau$ from the symmetry expected under standard oscillation
Figure 5.7: Effect of Lorentz violation on the $\nu_\mu - \nu_\tau$ diffuse flux with various values of the lorentz violating parameter $a$ (in GeV). Clockwise from top-left (i) $a = 0$, (ii) $a = 10^{-30}$, (iii) $a = 10^{-28}$, (iv) $a = 10^{-26}$. The plots show how an increase in the LV parameter results in depletion of the $\nu_\tau$ flux at progressively lower energies. For the Auger experiment, sensitivities for $\nu_\tau$ detection using the most pessimistic systematics (top line) and the most optimistic systematics (bottom line) are indicated [113].
phenomenology with $\theta_{23} = 45^\circ$ is a unique signature of Lorentz-violation. This would lead
to a significant decrease in the signature $\nu_\tau$ events at high energies, like “double-bang”,
“lollipop” and “earth-skimming” events as compared to muon-track events. Differences
in shape between the two flavours can be seen for $a < 10^{-30}$ GeV. We have used the case
where $a$ is independent of energy, however if the parameter $a \propto E^n$ the results would
be qualitatively similar to that obtained here but involve significantly different ranges of
values for the parameter as expected.

5.3.3 Detectability of Lorentz-violation

Unlike in neutrino decay, the effect of Lorentz violation is seen in the deviation of the flux
spectra of both the $\nu_\mu$ and, more strikingly, the $\nu_\tau$ flavour, from the standard fluxes toward
the higher end of the spectrum. This makes it especially interesting for probe by detectors,
such as ANITA and the Pierre Auger Observatory \cite{113, 117} having sensitivity to $\nu_\tau$ in
the energy range $10^8 - 10^{11}$ GeV. While Auger can separate out the $\nu_\tau$ events, ANITA
detects the sum of all three flavours. As is clear from the experimental thresholds shown
in Fig. 5.7, should even tiny Lorentz-violation effects exist, both these experiments will, in
principle, be able to detect it via lack of characteristic $\tau$ events expected at these energies
from standard physics. As they collect more data in the future, expectedly bringing the
corresponding thresholds down, the ability of such experiments to detect tiny LV effects
will be gradually enhanced.

5.4 Pseudo-Dirac neutrinos

Masses for neutrinos can be generated by extending the Standard model to include right-

handed sterile neutrinos to the particle spectrum. The generic mass term for neutrinos
becomes

$$\mathcal{L} = -\frac{1}{2} \overline{\Psi} C M \Psi + h.c.,$$  \hspace{1cm} (5.18)

where considering 3 right-handed neutrinos in the spectrum

$$\Psi = \begin{pmatrix} \nu_{eL}, \nu_{\mu L}, \nu_{\tau L}, (\nu_{1R})^C, (\nu_{2R})^C, (\nu_{3R})^C \end{pmatrix},$$

and $\nu^C = C \overline{\nu}^T$, $C$ being the charge conjugation operator.

The mass matrix $M$ is of the form

$$M = \begin{pmatrix} m_L & m_D^T \\ m_D & m_R \end{pmatrix},$$  \hspace{1cm} (5.19)

and for $m_L = m_R = 0$ reduces to neutrino states with Dirac mass. In this case the six
neutrinos decompose into three active-sterile pairs of neutrinos degenerate in mass with maximal mixing angle $\theta = \pi/4$ for each pair. Due to the mass degeneracy within the neutrinos in such a pair, an active neutrino cannot oscillate into a sterile neutrino from the same pair.

Instead, neutrinos may be pseudo-Dirac states [68] where $m_L$ and $m_R$ are tiny but non-zero, i.e. $m_L, m_R \ll m_D$. This lifts the degeneracy in mass within an active-sterile pair, and gives a mixing angle $\theta \approx \pi/4$ between its members. The result of the lifting of this degeneracy is to enable oscillation among species that was not possible in the pure Dirac neutrino case.

The presence of non-zero $m_L, m_R$ changes the probability of transition of one active state to another during propagation. The expression for the probability for neutrinos propagating over cosmological distances (after various phase factors involving terms like $\Delta m^2/\ell$ average out) is [68]

\[
P_{\alpha\beta} = \sum_{j=1}^{3} |U_{\alpha j}|^2 |U_{\beta j}|^2 \cos^2 \left( \frac{\Delta m^2_j L}{4E_\nu} \right),
\]

where $\Delta m^2_j = (m^+_j)^2 - (m^-_j)^2$ is the mass squared difference between the active and sterile states in the $j$th pair.

There has been a recent study [118] that explores the pseudo-Dirac scenario at neutrino telescopes using the ratio of shower to muon-track events. Here, we look at distortion of spectral shape from the standard diffuse flux due to the modification of the oscillation probability to Eq. (5.20). We use Eq. (5.20) instead of the standard oscillation probability, otherwise following the same procedure used to derive the standard MPR flux (the base flux in our plots). The results are shown in Fig. 5.8 which shows a decrease in the affected flux at lower energies and rise at the higher end of the spectrum to merge with the standard flux. However, the decrease is only to about half the base flux, and the rise at higher energies is not steep. Therefore, it would be very difficult to detect such an effect in future detector experiments.

### 5.5 Effect of decoherence during neutrino propagation

Quantum decoherence arises at the Planck scale in theories where CPT invariance is broken independently of Lorentz symmetry due to loss of unitarity and serves to modify the time evolution of the density matrix [70, 69]. Though not expected in a majority of string theories, a certain class of string theories called noncritical string theories may allow for decoherence.
Figure 5.8: Effect of pseudo-Dirac (PD) neutrinos on the $\nu_\mu$ diffuse flux with $\Delta m^2 = 10^{-14}$ eV$^2$.

In the context of neutrino oscillation, decoherence serves to modify the transition probabilities among the three flavours. While a general treatment discussing how this happens for the three family case is complicated, we work under the simplifying conditions assumed in [70, see Sec IV.B] to arrive at the transition probability

$$ P [\nu_\mu \rightarrow \nu_q] = \frac{1}{3} + \frac{1}{6} e^{-2\delta L} \left[ 3 \left( U_{\mu 1}^2 - U_{\mu 2}^2 \right) \left( U_{q 1}^2 - U_{q 2}^2 \right) + \left( U_{\mu 1}^2 + U_{\mu 2}^2 - 2 U_{\mu 3}^2 \right) \left( U_{q 1}^2 + U_{q 2}^2 - 2 U_{q 3}^2 \right) \right], \quad (5.21) $$

where $\delta$ is the only decoherence parameter. This leads to a flavour composition at the detector given by

$$ R_{\nu_e} = P [\nu_e \rightarrow \nu_e] \frac{\Phi_{\nu_e}}{\Phi_{\text{TOT}}} + P [\nu_\mu \rightarrow \nu_e] \frac{\Phi_{\nu_\mu}}{\Phi_{\text{TOT}}} + P [\nu_\tau \rightarrow \nu_e] \frac{\Phi_{\nu_\tau}}{\Phi_{\text{TOT}}}, \quad (5.22a) $$
$$ R_{\nu_\mu} = P [\nu_e \rightarrow \nu_\mu] \frac{\Phi_{\nu_e}}{\Phi_{\text{TOT}}} + P [\nu_\mu \rightarrow \nu_\mu] \frac{\Phi_{\nu_\mu}}{\Phi_{\text{TOT}}} + P [\nu_\tau \rightarrow \nu_\mu] \frac{\Phi_{\nu_\tau}}{\Phi_{\text{TOT}}}, \quad (5.22b) $$
$$ R_{\nu_\tau} = P [\nu_e \rightarrow \nu_\tau] \frac{\Phi_{\nu_e}}{\Phi_{\text{TOT}}} + P [\nu_\mu \rightarrow \nu_\tau] \frac{\Phi_{\nu_\mu}}{\Phi_{\text{TOT}}} + P [\nu_\tau \rightarrow \nu_\tau] \frac{\Phi_{\nu_\tau}}{\Phi_{\text{TOT}}}, \quad (5.22c) $$

where $\Phi_e/\Phi_{\text{TOT}}$, etc. are flux composition ratios at source.
Atmospheric IceCube /LParen1/Tilde2012/RParen1
AMANDA /LParen1Down/RParen1
AMANDA /LParen1Up/RParen1
ANITA

$10^3$ $10^4$ ... cm$^{-2}$ sr$^{-1}$ Ne flux with decoherence
$\nu_{\mu}$, $\nu_{\tau}$ flux with decoherence
Fluxes without decoherence

$\nu_{\tau}$ is almost invisible even if purposes, for a certain range of values of the decoherence parameter. However the effect due to dominance of decoherence might happen within the energy range relevant for our at source, the transition from the flux due to dominance of standard oscillation to that the effect of decoherence is to reduce the flavour ratios to 1 : 1 : 1 irrespective of ratios reduced to 0 parameter is shown in Fig. 5.9. For our calculation, we have chosen the parameter $\delta = \alpha E^2$ and $\alpha = 10^{-40} \text{GeV}^{-1}$. A base flux composition of 0 : 1 : 0 corresponding to $\bar{\nu}$ (left) and 1 : 1 : 0 corresponding to $\nu$ (right) from pion decay is used for the calculation. It is clear from the figure that (anti-)neutrinos from pion decay are not useful probes for decoherence.

We use the flavour ratios given by Eq. (5.22) to calculate the diffuse flux spectra of each flavour arriving at the detector. The effect of decoherence is to bring the flavour fluxes close to the ratio 1 : 1 : 1. If we use the standard flux from AGN’s (1 : 2 : 0 at source) then standard neutrino oscillation already brings the ratio to the above value as discussed in Sec. 4.1 and this makes it difficult to distinguish between the effects of decoherence and standard oscillation. However, if we have detection capabilities that can distinguish between neutrinos and anti-neutrinos, it might be worth investigating decoherence using the differences in flavour spectral shapes. As discussed earlier pion decays in the source via $\pi^+ \rightarrow \nu_\mu \mu^+$ and subsequently, $\mu^+ \rightarrow e^+ \bar{\nu}_\mu \nu_e$ contribute to a flavour spectral ratio of 1 : 1 : 0 for $\nu$ and 0 : 1 : 0 for $\bar{\nu}$. Due to standard oscillation these flavour ratios are reduced to 0.78 : 0.61 : 0.61 and 0.22 : 0.39 : 0.39 at the detector respectively. Since the effect of decoherence is to reduce the flavour ratios to 1 : 1 : 1 irrespective of ratios at source, the transition from the flux due to dominance of standard oscillation to that due to dominance of decoherence might happen within the energy range relevant for our purposes, for a certain range of values of the decoherence parameter. However the effect is almost invisible even if $\nu$ and $\bar{\nu}$ fluxes are used as probes, the reason being that the fluxes ratios at detector due to standard oscillation for both (i.e., 0.78 : 0.61 : 0.61 and 0.22 : 0.39 : 0.39 respectively) are already quite close to the 1 : 1 : 1 that decoherence would result in. Effective probe for decoherence are high energy neutrinos from neutron decay, for instance, which gives a flux ratio of 1 : 0 : 0 at source [116], and not neutrinos from pion decay. The results for $\bar{\nu}$ and $\nu$ with a particular choice of the decoherence parameter is shown in Fig. 5.9. For our calculation, we have chosen the parameter $\delta \propto E^2$
which is expected within the context of string theories\footnote{The choice of $\delta \propto E^2$ also violates Lorentz symmetry which introduces weaker secondary effects not taken into account here.}. Upper limits on such a parameter are got from the Super-Kamiokande as $\sim 10^{-10}$ GeV.