CHAPTER 4

SKELETONIZATION ALGORITHMS FOR IMAGE REPRESENTATION

4.1. Introduction

4.1.1 Image Decomposition

Decomposition is a technique for separating a binary shape into a union of simple binary shapes. The decomposition is unique and invariant to translation, rotation, and scaling. The techniques used in the decomposition are based on mathematical morphology. The shape description produced can be used in object recognition and in binary image coding.

A morphological approach to skeleton decomposition is used to decompose complex shape into a simple component. The decomposition is invariant to translation, rotation and scaling. Thus the skeleton can be calculated entirely by the basic operations of mathematical morphology which makes the skeleton a morphological representation, enabling image analysis using morphological tools. The skeleton provides a decomposition of the original shape into features (discs) of different sizes which can be seen as components of different scales.

4.1.2 Literature Survey

(Kresch, R. and Malah, 1994) have presented new properties of the discrete morphological skeleton representation of binary images, along with a novel coding scheme for lossless binary image compression that was based on these properties. Following a short review of the theoretical background, two sets of new properties of the discrete morphological skeleton representation of binary images are proved. The first one leads to the conclusion that only the radii of skeleton points belonging to a subset of the ultimate erosions are needed for perfect reconstruction. This corresponds to a lossless sampling of the quench function. The second set of new properties was related to deterministic prediction of skeletal information in a progressive transmission scheme. Based on the new properties, a novel coding scheme for binary images was
presented. The proposed scheme was suitable for progressive transmission and fast implementation. Computer simulations, that were presented, also showed that the proposed coding scheme substantially improved the results obtained by previous skeleton-based coders, and performed better than classical coders, including run-length/Huffman, quad tree, and chain coders. For facsimile images, its performance can be placed between the modified read (MR) method (K=4) and modified modified read (MMR) method.

(Essam A. El-Kwae and Mansur R. Kabuka,2000) have introduced a binary shape representation called the Hilbert Morphological Skeleton Transform (HMST). This representation combined the Morphological Skeleton Transform (MST) with the clustering capabilities of the Hilbert transform. The HMST preserves the skeleton properties including information preservation, progressive visualization and compact representation. Then, an object recognition algorithm, the Hilbert Skeleton Matching Algorithm (HSMA), was introduced. This algorithm performs a single sweep over the HMSTs and renders the similarity between them as a distance measure. Testing the HSMA against the Skeleton Matching algorithm (SMA) and invariant moments revealed that the HSMA algorithm achieves slightly better object recognition rates while substantially reducing the complexity. In an experiment of 14,400 shape matches, the HSMA achieved a 90.36% recognition rate as opposed to 89.76% for the SMA and 89.49% for invariant moments. On the other hand, the HSMA improved the SMA processing more than 40%.

(P. A. Maragos and R. W. Schafer,1988) have presented the results of a study on the use of morphological set operations to represent and encode a discrete binary image by parts of its skeleton, a thinned version of the image containing complete information about its shape and size. Using morphological erosions and openings, a finite image can be uniquely decomposed into a finite number of skeleton subsets and then the image can be exactly reconstructed by dilating the skeleton subsets. The morphological skeleton is shown to unify many previous approaches to skeletonization, and some of its theoretical properties are investigated. Fast algorithms that reduce the original quadratic complexity to linear are developed for skeleton decomposition and reconstruction. Partial reconstructions of the image are quantified through the omission of subsets of skeleton points. The concepts of a globally and locally minimal skeleton
are introduced and fast algorithms are developed for obtaining minimal skeletons. For images containing blobs and large areas, the skeleton subsets are much thinner than the original image. Therefore, encoding of the skeleton information results in lower information rates than optimum block-Huffman or optimum run length-Huffman coding of the original image. The highest level of image compression was obtained by using Elias coding of the skeleton.

(V. Vijaya Kumar,2009) has presented the morphological skeleton method is a popular approach for morphological shape representation. This project proposes a new algorithm for skeletonization of 2D images, based on primitive concepts of morphology which is an extension of the morphological binary skeleton. The proposed project utilizes structuring elements of size \( n \times n \), where \( n=2, 3, 4...m \). A fine comparison is made on the effect of skeletonization based on the size of structuring element and their repetitiveness. One of the major problems of existing approaches is that at what iterative step the skeletonization process should stop. This problem is solved in this project by experimenting the existing and proposed method on various images with different structuring elements. One of the problems of multi resolution (scale-space) approach to skeleton construction is that they are sensitive to boundary noise. This project advocates this problem as well. The present method is experimented on various shapes, alphabets and digits. Good results are obtained and they are compared with the other existing techniques.

The Morphological Skeleton Representation and coding of binary images by (A. Maragos et al,1986) presented the results of a study on the use of morphological set operations to represent and encode a discrete binary image by parts of its skeleton, a thinned version of the image containing complete information about its shape and size. Using morphological erosions and openings, a finite image can be uniquely decomposed into a finite number of skeleton subsets and then the image can be exactly reconstructed by dilating the skeleton subsets. The morphological skeleton is shown to unify many previous approaches to skeletonization, and some of its theoretical properties are investigated. Fast algorithms that to reduce the original quadratic complexity to linear are developed for skeleton decomposition and reconstruction. Partial reconstructions of the image are quantified through the omission of subsets of skeleton points. The concepts of a globally and locally minimal skeleton are introduced and fast algorithms are developed for obtaining minimal skeletons.
The algorithm combines the advantages of the morphological skeleton transform (MST) (P.A. Maragos et al. 1988) and the morphological shape decomposition (MSD) (I. Pitas et al. 1992). The representative disks have simple and well-defined mathematical characterizations. The algorithm is simple and efficient to implement. The experimental results show that the number of representative disks used by this algorithm is significantly lower than that used by the MSD. The overlapping level between the representative disks is much lower than that of the MST. A simple procedure can be used to combine the representative disks into more meaningful shape components. These shape components seem to correspond better to the natural shape parts than those generated by the MSD. It is also possible to build a good approximation for a given shape using only a small number of major components.

The morphological shape decomposition (MSD) (S. Belongie et al., 2001) is another important morphological shape representation scheme, in which a given shape is represented as a union of certain disks contained in the shape. The overlapping among representative disks of different sizes is eliminated.

Another morphological shape representation algorithm that can be viewed as a compromise between the MST (S. Belongie et al., 2002) and the MSD was recently proposed. In this scheme, overlapping among representative disks of different sizes is allowed, but severe overlapping among such disks is avoided. We can call this algorithm overlapped morphological shape decomposition (OMSD) (C. Vasanthanayaki et al., 2005). The advantages of these basic algorithms include that they have simple and well-defined mathematical characterizations and they are easy and efficient to implement. There is a common problem shared by all three algorithms. In general, there is a lot of overlapping among representative disks of the same size. The MST is not typically considered as a shape decomposition algorithm because of the heavy overlapping among the representative disks. For the MSD and OMSD, there is a simple scheme for grouping representative disks into shape components. Each component is a maximal set of representative disks of the same size with connecting centers. In general, a component may contain many overlapping representative disks. Sometimes, a large number of such disks are
used to represent a simple shape component. At other times these disks form complicated structures.

Among the various approaches to shape representation and retrieval, the method based on morphological feature descriptors attracts more of our attention, because using the information from the region to describe a shape is the most straightforward idea. Using the shape primitives to represent the skeleton points has the advantage that when image is processed. Also, the processing procedures for solving the problem of invariant under scale, translation, and rotation are reasonably simple.

4.2. Existing Algorithms

4.2.1 Generalized Skeleton Transform (GST)

A common problem shared by several leading morphological Shape representation algorithms is that there is much overlapping among the representative disks of the same size. A shape component represented by a group of connected disk centers sometimes uses many heavily overlapping representative disks to represent a relatively simple shape part. A shape component may also contain a large number of representative disks that form a complicated structure. In this work, we introduce a generalized discrete morphological skeleton transform that uses eight structuring elements to generate skeleton subsets so that no two skeletal points from the same skeleton subset will be adjacent to each other. Each skeletal point will represent a shape part that is in general an octagon with four pairs of parallel opposing sides. The number of representative points needed to represent a given shape is significantly lower than that in the standard skeleton transform. A collection of shape components needed to build a structural representation is easily derived from the generalized skeleton transform. Each shape component covers a significant area of the given shape and severe overlapping is avoided. The given shape can also be accurately approximated using a small number of shape components. The mathematical skeleton transform is a leading morphological shape representation algorithm.
In MST a given shape is represented as a union of all maximal disks contained in the shape. In general there is much overlapping among the maximal disks. The morphological shape decomposition is another important morphological shape representation scheme in which a given shape is represented as a union of certain disks contained in the shape. The overlapping among representative disks of different size is eliminated. The main advantage of the generalized skeleton transform is that it leads to an efficient shape decomposition scheme. In this scheme, a given shape is decomposed into a collection of modestly overlapping octagonal shape components. One problem with this decomposition scheme is that the generalized skeleton transforms needs to be applied multiple times.

4.2.2 Octagon-Fitting Algorithm

In this section, an algorithm described that associates each image point with a special maximal shape part, or shape element. The size of the shape element is assigned to the point. In this algorithm, the shape elements are derived by repeatedly applying erosion operations using the eight structuring elements shown in Fig. in the following order: B0, B1, ..., B7, B0, B1, ..., B7, B0, B1, .... That is, these eight structuring elements will be applied in a cyclic sequence. A given shape image can be seen as a set of image points. The order in which we are applying the structural element determine the shape component generated. This algorithm can be viewed as a process of applying erosion operations to repeatedly divide a set of image points into two disjoint subsets. For a nonempty image that is not a set of isolated points, let

\[
Y_0(X) = X \ominus B_0
\]  

(4.1)

After first step of erosion we are going for the next step of erosion by next structuring element. While applying erosion iterations we are initializing an array of elements in order to store the maximal iteration count, the last structuring element used and the corresponding central pixel. We are applying erosions until we get the maximal octagon which can be occupied in the image. The corresponding iteration count, center pixel and last structuring element used are stored in the array.
For avoiding overlapping next step is to eliminate the part under the maximal octagon from next iteration. For this dilation operations are performed depending on the data in the array. Thus we are whitening the part under the maximal octagon and make it similar to background. Next series of erosions are applied for the remaining part of the image and we are finding the maximal octagon which can be occupied in the remaining part, store the corresponding data and we are eliminating that portion from next iteration. Thus we are finding all the maximal octagons which can be occupied in the image without overlapping.

For finding the overall skeleton structure of the image we are representing all the boundary points using dilation operations. For reconstructing an image back from the skeleton, on a white background we are placing a black pixel at the point corresponds to the center pixel of the first maximal octagon. On that point, applying dilation operations according to the maximal iteration count and the last structuring element used.

Thus we are reconstructing the first maximal octagon. Then we are placing a black pixel at point correspond to second center pixel, and again applying dilations according to maximal iteration count and the last structuring element used. Thus we are reconstructing the second maximal octagon. In a similar way by applying iterations using the data present in the array we are reconstructing the entire image.

There is a simple relationship between the shape of a shape element and the numbers of different structuring elements used. The number of $B_0$ or $B_4$ used to construct a shape element equals the size of the shape element’s two horizontal sides. The number of $B_2$ or $B_6$ used is same as the size of two vertical sides. The numbers of other two pairs of structuring elements used determine the sizes of two pairs of diagonal sides as shown in Fig 4.1.
In the generalized skeleton algorithm, only a small number of image points and the associated maximal octagons are identified to represent the original image. In the associated decomposition algorithm, however, it is too limiting to use only the skeleton points. An octagon represented by a non-skeleton point can often have neighboring octagons of the same size. The generalized skeleton algorithm needs to be applied again to combine them. In the new OFA, a maximal octagon is assigned to each image point.

4.2.3 Polygonal Components

The octagonal components are more efficient to implement. But the time required for the decomposition of these octagonal components are more. So we stop the procedure of using octagon within two iterations and continue the decomposition process using polygonal components. The polygonal components are decomposed using the basic shape primitives. Also a polygon can have at most eight different sides in eight predefined directions. Polygonal components of different shapes and sizes can be used in the description of the given shape. The main advantage of using polygonal components is that it can be used to compute elongated parts and small regions. We assign values for a 3x3 matrix $\Omega$ and it is used by the shape primitives.
The values assigned for a 3x3 matrix is given by \([1 \ 2 \ 4; \ 8 * 16; \ 32 \ 64 \ 128]\). These values are assigned in a form that the addition of any of these numbers may not produce the resultant that is used by the 3x3 image. For example, \(1+2=3\); the value 3 is not used in the 3x3 image and hence the next number 4 is used. Then the addition of four and one is five(4+1=5); So 5 is not used in the image. The same procedure is repeated for assigning all the values in the 3x3 image. The centering element is denoted by * and its value is one.

For the 13 basic shape primitives shown in Fig 4.2, the values are obtained by using the values in the 3x3 matrix. Value of the shape primitive 1 is 8 because it is present in the left side of the centering element. Value of the shape primitive 2 is 64 because it is present in the bottom of the centering element. Value of the shape primitive 3 is 4 because it is present to the right top side of the centering element. The Value of the shape primitive 4 is 1 because it is present in the left top side of the centering element. The Value of the shape primitive 5 is 10 (8+2) because it is present in the left and top side of the centering element and hence the same process is repeated for calculating all the values of the shape primitives. The values are given by:

\[
\begin{align*}
\text{Value of shape primitive P1} &= 8 \\
\text{Value of shape primitive P2} &= 64 \\
\text{Value of shape primitive P3} &= 4 \\
\text{Value of shape primitive P4} &= 1 \\
\text{Value of shape primitive P5} &= 10 \\
\text{Value of shape primitive P6} &= 72 \\
\text{Value of shape primitive P7} &= 80 \\
\text{Value of shape primitive P8} &= 18 \\
\text{Value of shape primitive P9} &= 26 \\
\text{Value of shape primitive P10} &= 88
\end{align*}
\]
Value of shape primitive P11 = 74
Value of shape primitive P12 = 82
Value of shape primitive P13 = 90

By using these values the polygonal components are decomposed. Here the repeated erosion operations are avoided. Hence the time consumption by using these polygonal components is less. Finally the single points in the image are found out and they are decomposed.

A new shape decomposition algorithm is used, that decomposes a two-dimensional (2-D) binary shape into a group of polygonal components. Polygonal component of the given image is first identified. The first component is determined incrementally by selecting a sequence of basic shape primitives.

Each component is a subset of the given image and the union of these components gives the original image. The shape information is extracted at different scale levels. The higher scale level information is used to guide the extraction of shape information at the low scale levels. The first shape primitive is determined based on the shape of the given image. The second primitive is determined based on the choice of the first one, and so on. This algorithm allows accurate approximations of binary shapes at low coding costs. We can use small number of polygons to achieve accurate approximations of the given shape.

4.3. Proposed Algorithm
4.3.1 Skeleton Representation

The skeleton points are derived by repeatedly applying erosion operation using eight structuring elements. The eight structuring elements applied in cyclic sequence. The proposed algorithm utilizes the number of skeleton points, in their co-ordinate positions, corresponding structuring elements and noise removal filter for reconstruction of the image. The process will be
repeated for the skeleton points obtained by the block diagram shown in Fig 4.3 to generate skeleton points.

The morphological skeleton theory has developed literature in topological and algebraic branches. From topological point of view, the skeleton of a shape can be seen as a thin caricature. From algebraic point of view, the skeleton is the result of the decomposition of a given set into simpler elements. The main algebraic properties of the skeleton representation are

1. The skeleton representation can be calculated by means of an algebraic closed-form formula.

2. The skeleton provides a decomposition of the original shape into the features of different sizes, which can be seen as components in different scales. The smallest maximal discs can often be considered as detail whereas the largest ones can often be considered as the main structure. This provides a hierarchical or pyramidal interpretation to the skeleton representation.

3. Simplified versions of the original shape are obtained by partial reconstruction from the skeleton representation.

The conventional skeleton representation has the following undesired characteristics.

1. It usually contains redundant points, that is, many skeleton points can be discarded and still the original shape can be fully reconstructed. The redundant points usually form long, often undesired, branches in the skeleton.

2. Unlike other binary image representations (e.g. chain code and quad tree), it is not a self-dual representation, because the skeleton of the complement of X is totally different from the skeleton of X.
Fig 4.3 Block diagram of the Skeleton Representation
4.3.2 Shape Reconstruction

Restoration attempts to recover an image that has been degraded by using a priori knowledge of the degradation phenomenon. It requires an efficient filtering procedure which restores the image from its noisy version. Restoration filter should be effective in eliminating the noise degradation. It should be able to restore various important aspects of the size- shape content and geometrical structure. Here in this project idempotent recursive soft morphological filters are introduced for the restoration of images from their noisy versions.

Algorithm to reconstruct the shape:

The following steps are used to reconstruct the shape from the skeleton points.

1. Read number of received skeleton points in N and initialize I=1 and initialize an array IM with zeros
2. Read the first skeletal point coordinate and the corresponding structuring element
3. Place 1 at the coordinate position and dilate with the corresponding structuring element.
4. If I<=N read the next skeletal coordinate and the corresponding structuring element.
5. Initialize array (X) with zeros and place 1 at the coordinate position and dilate with the corresponding structuring element.
6. Then add (X) to (IM) and I=I+1, then go to fourth point.
7. If the condition I<=N is false, print (IM), and get the reconstructed shape image.
4.3.3 Soft Morphological Filtering

4.3.3.1. Literature survey

A number of morphological shape representation and decomposition algorithms have been developed over the years. (J. Serra 1982) introduced a class of recursive transformations. These are widely used in signal and image processing applications such as sequential block labeling, predictive coding, and adaptive dithering. The main distinction of the recursive transformation is the pixel’s value of the transformed image depends upon the pixel’s values of both the input image and the transformed image itself. Due to this reason, some partial order has to be imposed on the underlying image domain so that the transformed image can be computed recursively according to this imposed partial order. In other words, a pixel’s value of the transformed image may not be processed until all the pixels preceding it have been processed.

Morphological shape decomposition is a very popular method for shape representation (Pitas 1990) but the main disadvantage of this method is the lack of robustness especially in impulsive noise.

Soft morphological filters (Koskinen 1991) that possess the desirable property of being less sensitive to additive noises and to small variations in the shape of the objects to be filtered. The structuring element in soft morphological filters is divided into two parts: one being the “hard center” and the other being the “Soft boundary.”

Soft morphological filters (Frank Y. Shih 1993) are used for smoothing signals with the advantage of being less sensitive to additive noises and to small variations in the shape of the objects to be transformed as compared to standard morphological filters. They also present the properties of soft morphological operation and the new definitions of binary soft morphological operations. It is shown that is soft morphological filtering an arbitrary signal is equivalent to decomposing the signal into binary signals, filtering each binary signal with a binary SOFT morphological filter, and then reversing the decomposition.
Shape representation scheme (Wang 1995) using recursive morphological operations. In this a given shape is decomposed into a union of certain disks contained in the shape. The overlapping between the representative disks is completely eliminated. The individual disks are defined recursively. The decomposition procedure is still simple. But the recursive morphological operations seem to be inherently serial and therefore not suitable for parallel implementations, also decomposition results depend on the order in which the image pixels are examined.

The recursive soft morphological (RSM) filters, (Shih 1995) their properties, cascade combinations and idempotent RSM filters are developed. In general, recursive structures usually provide better smoothing capabilities and take less computational time even though this is at the expense of increased detailed distortion.

A new class of recursive order-statistic soft morphological (ROSSM) filters and their important properties (S.C. Pei 1998) related to morphological filtering procedure. They also provided criteria for specific selection of parameters to achieve excellent performance in noise reduction and edge preservation.

A novel recursive algorithm for binary image area location (B.Gatos 2000) is developed. In this isothetic polygons with minimum number of vertices are used in order to achieve simplicity of description and efficiency of storage. These polygons are defined by a recursive formula, where the resulting areas are calculated from successive additions and subtractions of simple rectangular blocks.

A morphological shape decomposition algorithm (Jianning Xu 2001) decomposes a two-dimensional (2-D) binary shape into a collection of convex polygonal components. A single convex polygonal approximation for a given image is first identified. This first component is determined incrementally by selecting a sequence of basic shape primitives. These shape primitives are chosen based on shape information extracted from the given shape at different
scale levels. Additional shape components are identified recursively from the difference image between the given image and the first component. Simple operations were used to repair certain concavities caused by the set difference operation. The resulting hierarchical structure provides descriptions for the given shape at different detail levels. This algorithm allows accurate approximations for the given shapes at low coding costs.

A new class of morphological operations (F.Y Shih 2003) which allows one to select varying shapes and orientations of structuring elements. However, the sweep erosion and dilation do not satisfy the basic properties of mathematical morphology. In particular they are not increasing operators in general and the sweep dilation operator does not commute with the union.

A soft morphological filtering (Zhao Chun-hui 2006) method which is an important nonlinear filtering method especially in the field of digital image processing is widely applied. The operation of soft morphological filter can be divided into two basic problems that include morphological operation and structuring system (SS) selection. The rules for morphological operations are predefined so the filter's properties depend merely on the selection of SS. In this method the structuring system possesses the shape and structural characteristics of images; the compositions of hard center, soft boundary and repetition parameter are automatically adjusted. A soft morphological filter that provides good filtering result to images with complex noise can be realized.

A morphological shape decomposition algorithm (Jianning Xu 2007) uses a simple and well-defined process to generate an efficient set of representative disks. These representative disks can be seen as the shape components in a structural representation. The original shape can be reconstructed at different levels of accuracy. These representative disks do not seem to be as sensitive to small boundary changes as skeleton-based representations.

A new algorithm for skeletonization of 2D images (V.Vijaya Kumar 2008), based on primitive concepts of morphology which is an extension of the morphological binary skeleton.
One of the problems of multi resolution (scale-space) approach to skeleton construction is that they are sensitive to boundary noise. They advocate this problem as well.

### 4.3.3.2 Soft Morphological Filtering

After reconstruction of the input shape image, SMF will apply and get the noise free shape as output. In Soft Morphology the structuring element B is divided into two subsets: the hard structuring element A, \( A \subseteq B \) and the soft structuring element \( B \setminus A \), where ‘\( \setminus \)’ denotes the set difference. For input signal \( f \), the soft erosion and soft dilation with order \( r \) are defined as:

**Soft erosion**

\[
\varepsilon_{B,A,r}(f) = \min^{(r)}\{r \diamond f(a) : a \in A_m\} \\
\cup \{f(b) : b \in (B\setminus A)_m\},
\]  

(4.2)

**Soft dilation**

\[
\Delta_{B,A,r}(f) = \max^{(r)}\{r \diamond f(a) : a \in A_m\} \\
\cup \{f(b) : b \in (B\setminus A)_m\}
\]

(4.3)

Where \( \min^{(r)} \) or \( \max^{(r)} \) means taking the \( r^{\text{th}} \) element of input data set in which all elements are ranked in ascending or descending order, respectively; \( \{r \diamond f(a)\} \) denotes the repetition \( r \) times of signal \( f(a) \). That is, \( \{r \diamond f(a)\} = \{f(a), f(a), ..., f(a)\} \) (\( r \) times). \( A_m \) is the SE \( A \) shifted at the \( m^{\text{th}} \) position. If the order \( r \) is set to 1, equations (4.2) and (4.3) represent the erosion and dilation operators in standard morphology respectively. The result of soft morphological operators is less sensitive to small variations of the object being processed. Therefore, soft morphological operators are able to perform better in preserving signal’s detail than the standard morphological operators.
4.4 Results and Discussion

With an Example of butterfly image, the obtained Skeleton with repeated erosion operation is shown in Fig 4.4. By using octagonal shape components the image is first decomposed with the structuring elements. The image is then decomposed using polygonal components with the Shape primitives. The skeleton points are the locus of centers of the shape components. The image is reconstructed using Octagon and polygonal components. The same steps are shown in Fig 4.6 for fish image.

(i) Original image

(ii) Overall skeleton Structure

(iii) Octagon based decomposition

(iv) Polygon decomposition

Fig 4.4 Results of reconstructed butterfly images using octagonal and polygonal components (continued)
(v) Overall Skeleton

(vi) Reconstruction using octagon and polygon

Fig 4.4 Results of reconstructed butterfly images using octagonal and polygonal components
The original image of size 4 kilobytes is compressed by using Huffman coding into 854 bytes. This is shown in Fig 4.5 as a screen shot.
Fig 4.6 Results of reconstructed fish images using octagonal and polygonal components (continued)
The image is reconstructed using Octagon and polygonal components. The shape representation algorithm is very promising and yield more accurate result over various images. A closely related objective measure is MSE. RMSE, SNR (ms), SNR (rms), PSNR are the defacto standards used in the image processing community. It is so commonly used for three reasons 1) because some objective measure is needed; (2) because it is possible to relate MSE to theoretical issues related to rate/distortion curves and least-squares minimization in statistical theory more easily than with any other measures and (3) because PSNR is a logarithmic measure which correlates with the logarithmic response to image intensity. Generally speaking, as a rule of thumb, the higher PSNR will frequently correspond to better decompression noticeably. But the present study has tested this for reconstruction of the images. The error rate of proposed method
is less than the previous methods. The PSNR is high for the present method for all images. It indicates that it has high signal to noise ratio.

### 4.4.1 Error Calculations

The number of error functions as stated in the below equations has applied on all reconstruction images are calculated.

Let

\[ f(x,y) = \text{input shape image} \]
\[ g(x,y) = \text{reconstructed image} \]
\[ M \text{ and } N = \text{the sizes of input and reconstructed image} \]

### 4.4.2 Error functions

1. AEPP: Average error per pixel

\[
AEPP = \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} |f(x,y) - g(x,y)|
\]  \hspace{1cm} (4.4)

2. MSE: Mean square error

\[
MSE = \frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} (f(x,y) - g(x,y))^2
\]  \hspace{1cm} (4.5)

3. RMSE: Root mean square error

\[
RMSE = \sqrt{\frac{1}{MN} \sum_{i=0}^{M-1} \sum_{j=0}^{N-1} (f(x,y) - g(x,y))^2}
\]  \hspace{1cm} (4.6)

4. SNR (ms): Signal to noise ratio (mean square)

\[
\text{SNR}(\text{ms}) = \frac{\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} f(x,y)^2}{\sqrt{\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} (f(x,y) - g(x,y))^2}}
\]  \hspace{1cm} (4.7)
5. SNR (rms): Signal to noise ratio (root mean square)

\[
\text{SNR(rms)} = \sqrt{\frac{\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} g(x,y)^2}{\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} (g(x,y) - f(x,y))^2}}
\]  

6. PSNR: Peak signal to noise ratio

\[
\text{PSNR} = 10 \times \log_{10} \left( \frac{255^2}{\text{MSE}} \right)
\]

7. Error-Rate: Error-rate per pixel

\[
\text{Error Rate} = \frac{\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} |f(x,y) - g(x,y)|}{\sum_{i=0}^{M-1} \sum_{j=0}^{N-1} f(x,y)}
\]

Here we are introducing a decomposition algorithm of OFA. An octagon-fitting algorithm is one assigns a special maximal octagon to each image point in a given shape. These maximal octagons are derived using simple and well-defined operations and they represent meaningful and well-characterized shape parts of the original shape. The main advantage of the octagon fitting algorithm is, a given shape is decomposed into a collection of non-overlapping shape components and each shape component is represented by a single center point and the shape of a shape component is always primitive and explicitly specified using four integers. The drawback with this algorithm is that the time consumption is more. So we use OFA for two iterations and decompose the given shape using polygon. Using polygon we are able to decompose elongated parts and even small regions. Then the single points are decomposed and the original image is reconstructed in a better way.

Using Octagon and polygon we can compress more data and the time consumption is less. So the combined effect of Octagon and polygon yields accurate results. Again, this feature supports the development of shape matching algorithms. The error calculations are shown in Table 4.1.
Table 4.1 Error functions values for different images

<table>
<thead>
<tr>
<th>Images</th>
<th>AEPP</th>
<th>MSE</th>
<th>RMSE</th>
<th>SNR(ms)</th>
<th>SNR(rms)</th>
<th>PSNR</th>
<th>Error Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Digits</td>
<td>177.0618</td>
<td>221.0937</td>
<td>14.8692</td>
<td>1.0150</td>
<td>1.0075</td>
<td>24.68</td>
<td>1.072</td>
</tr>
<tr>
<td>Dog</td>
<td>191.5487</td>
<td>218.3915</td>
<td>14.7781</td>
<td>1.0084</td>
<td>1.0042</td>
<td>24.73</td>
<td>1.167</td>
</tr>
<tr>
<td>Fish</td>
<td>168.8377</td>
<td>197.9414</td>
<td>14.0692</td>
<td>1.0072</td>
<td>1.0036</td>
<td>25.16</td>
<td>1.031</td>
</tr>
<tr>
<td>Lamp</td>
<td>179.0923</td>
<td>192.5202</td>
<td>13.8752</td>
<td>1.0037</td>
<td>1.0019</td>
<td>25.28</td>
<td>1.968</td>
</tr>
<tr>
<td>Letters</td>
<td>187.4871</td>
<td>211.3743</td>
<td>14.5387</td>
<td>1.0129</td>
<td>1.0064</td>
<td>24.88</td>
<td>1.838</td>
</tr>
<tr>
<td>Telephone</td>
<td>178.3826</td>
<td>195.3853</td>
<td>13.9780</td>
<td>1.0068</td>
<td>1.0034</td>
<td>25.22</td>
<td>1.943</td>
</tr>
<tr>
<td>Butterfly</td>
<td>190.5409</td>
<td>214.8161</td>
<td>14.6566</td>
<td>1.0081</td>
<td>1.0041</td>
<td>24.81</td>
<td>1.123</td>
</tr>
<tr>
<td>Teapot</td>
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<td>195.8900</td>
<td>13.9961</td>
<td>1.0024</td>
<td>1.0012</td>
<td>25.21</td>
<td>8.953</td>
</tr>
<tr>
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<td>165.4293</td>
<td>255</td>
<td>15.9687</td>
<td>1</td>
<td>1</td>
<td>24.06</td>
<td>3.282</td>
</tr>
</tbody>
</table>

The shape based decomposition algorithm is very promising and yield more accurate result over various images. A study of values of PSNR and number of skeleton points required in the algorithms are given in the table below. Of these algorithms the more accurate result got from the combined effect of octagon and polygon. The following Table Analysis of images using combined effect of octagon & polygon decomposition algorithms. The performance of Octagon & Polygon and Octagon method on Butterfly and Fish Image are shown in Table 4.2.
Table 4.2 The performance of Octagon & Polygon and Octagon method on Butterfly and Fish Image

<table>
<thead>
<tr>
<th>Image</th>
<th>Method</th>
<th>Time Taken ms</th>
<th>MSE</th>
<th>PSNR</th>
<th>Number of skeleton points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Butterfly.bmp</td>
<td>Octagon &amp; Polygon</td>
<td>121.87</td>
<td>0.0215</td>
<td>58.82</td>
<td>235</td>
</tr>
<tr>
<td></td>
<td>Octagon</td>
<td>203.35</td>
<td>0.0244</td>
<td>58.26</td>
<td>214</td>
</tr>
<tr>
<td>Fish.bmp</td>
<td>Octagon &amp; Polygon</td>
<td>166.75</td>
<td>0.0263</td>
<td>57.94</td>
<td>250</td>
</tr>
<tr>
<td></td>
<td>Octagon</td>
<td>261</td>
<td>0.0305</td>
<td>57.29</td>
<td>243</td>
</tr>
</tbody>
</table>

The results of Proposed method in Skeletonization method for various images in MPEG-7 and Kimia data sets are shown in Fig. 4.7 to Fig.4.10.
Fig 4.7 Results of Proposed method in Skeletonization for MPEG-7 Dataset Apple Image
Fig 4.8 Results of Proposed method in Skeletonization for MPEG-7 Dataset Butterfly Image
Fig 4.9 Results of Proposed method in Skeletonization for Kimia Dataset Butterfly Image
Fig 4.10 Results of Proposed method in Skeletonization for Kimia Dataset Spanner Image