CHAPTER III

REVIEW ON SPATIAL CORRELATION APPROACHES

3.1 Introduction

In this chapter, we discuss the performance of pure entropy, entropy with spatial correlation, fuzzy based entropy approaches are discussed. Gray level histogram based methods does not consider the spatial correlation between pixels in an image. This is simple to implement and does not consider the physical location of pixel and its interaction with neighboring pixels. When different images with identical histograms will result in the same threshold value, spatial methods are employed to avoid this kind of a problem. In the two dimensional entropy based segmentation procedure the gray value and its local average gray value of the pixels and their probability distribution are considered, resulting in betterment than its earlier methods. In the literature many researchers worked and made much progress in this direction. In this section, a brief review is conducted on some of the popular spatial approaches and presents the proposed work in connection with the particular area.

3.2 Two-dimensional Entropy versus Pure Entropy Methods

In this section we have discussed different entropy methods and its combination with other techniques such as spatial correlation, and fuzzy probability.

3.2.1 Shannon’s Entropy Algorithm

The first of Shannon’s logarithmic information measures is considered as entropy or Shannon entropy. It is denoted by $\phi(X)$. Think of $X$ as a collection of labels, each denoted by a lower-case letter $x$, for a set of $N$ messages. E.g., $X = \{1, 2, 3, 4, 5\}$, $x = 2$
or \( x = 3 \), etc. Let \( p(x) \) be the probability for message \( x \), therefore \( p(X) = 1 \). More generally, \( X \) is a random variable, a numerical function on some sample space. Each \( x \) may correspond to several different points in the sample space, and \( p(X) \) is the sum of the probabilities associated with these points.

Then define \( \phi(x) = -\sum x p(x) \log p(x) \)

For a gray scale or intensity image in which the number of labels are \( L \) equal to 256. The entropy of a discrete source is often obtained from the probability distribution, where \( p = \{ p_i \} \) is the probability of finding the system in each probable state \( i \). Therefore \( 0 \leq p_i \leq 1 \) and \( \sum_{i=0}^{L-1} p_i = 1 \), where \( L \) is the total number of possible states. The Shannon entropy may be described as \( \phi(t) = -\sum_{i=0}^{L-1} p_i \ln (p_i) \). This can be written as

\[
\phi(t) = H_O(t) + H_B(t) \tag{3.1}
\]

Where

\[
H_O(t) = -\sum_{i=0}^{t} p_i \ln (p_i) \tag{3.2}
\]

\[
H_B(t) = -\sum_{i=t+1}^{L-1} p_i \ln (p_i) \tag{3.3}
\]

Equation (5.1) is the entropic criterion function, \( H_O(t) \) and \( H_B(t) \) are entropies associated with object and background respectively by Equations (5.2) and (5.3), and we obtain optimal threshold \( t^* \) by maximizing \( \phi(t) \) from Equation (5.4).

\[
t^* = \operatorname{Arg} \max \{\phi(t)\} \tag{3.4}
\]
3.2.2 One-dimensional Entropic method of Kapur

In this method the Object and background classes are considered as two different sources. When the sum of the two class entropies is a maximum the image is said to be optimally thresholded. Thus using the definitions of the Object and background entropies,

\[
H_O(T) = - \sum_{g=T+1}^{G} \frac{p(g)}{P(T)} \log \frac{p(g)}{P(T)}
\]  

(3.5)

\[
H_B(T) = - \sum_{g=0}^{T} \frac{p(g)}{P(T)} \log \frac{p(g)}{P(T)}
\]  

(3.6)

and one has the optimal threshold from the following equation.

\[
T_{opt} = \arg \max \left[ H_O(T) + H_B(T) \right]
\]  

(3.7)

3.2.3 Two-Dimensional Histogram based Entropy By Abutaleb

Kapur et al. raised the concern that “what happens if two different pictures have the same gray level histogram and thus same histogram? Will it be suitable for both?” They also suggested that a second-order statistics or some local properties with the entropic concept of thresholding might give a better solution to this problem. Abutaleb and A. D. Brink extended the entropy based thresholding approach of Kapure et al. using two-dimensional entropy.

For selection of threshold Abutaleb made an attempt to utilize spatial information, that is the basis of two-dimensional entropy approach. These two-dimensional entropies were derived from a two-dimensional histogram or scatterplot, which is obtained by using gray value and its immediate neighborhood average gray value of the pixel. This is
the way Abutaleb extended one-dimensional entropy approach using scatterplot. The separation between the objects and background was obtained by locating the maximum of a two-dimensional entropy criterion function as follows:

\[
\psi(s, t) = H(A) + H(B)
\]  

(3.8)

Where

\( \psi(s, t) \) is an entropy based function,

\( H(A) \) is the entropy of the foreground and

\( H(B) \) is the entropy of the background.

Since \( n^2 \) bins are developed in this approach, the time complexity is more. Some results are shown in the figure below.

![Figure 3.1](image)

Figure 3.1 (a) Original image of coins (b) Thresholded image of Coins (c) Original image of Lena and (d) thresholded image of Lena due to Abutaleb.
3.2.4 Two-dimensional Entropy by A. D. Brink

When only a gray level histogram is used a lot of useful information contained in the image will be lost. If the loss can be overcome, it could result in much better threshold value for segmentation. Brink also employed the scatterplot which is constructed using gray value and its immediate neighborhood average gray value of the pixel. Brink technique is popularly known as ‘maximin technique’. This is an extension of the Abutaleb’s method. Instead of maximizing the sum of two entropies of background and foreground the smaller of the two entropies is maximized. This is the magic of Brink’s technique which reduces the time complexity over Abutaleb approach. According to Brink the normalized probabilities of background and objects, respectively, are:

\[ P_0(t,s) = \sum_{j=0}^{s} \sum_{i=0}^{t} P_{ij} \]

\[ P_1(t,s) = \sum_{j=s+1}^{n-1} \sum_{i=r+1}^{n-1} P_{ij} \]

but Abutaleb considers the second one that is object probability \( P_1(t,s) \) as

\[ P_1(t,s) = 1 - P_0(t,s) \]

In this approach the process estimates a \((T,S)\) as threshold vector that maximizes the smaller value of \( H_0(t,s) \) and \( H_1(t,s) \) as

\[ H(T,S) = \max (\min \{ H_0(t,s), H_1(t,s) \}) \quad (3.9) \]

Finally, this threshold vector is used for segmentation of a given image. Some results are shown below.
Figure 3.2 (a) Original and (b) Thresholded image of Lena due to Brink.

3.2.5 Entropic Thresholding based on GLSC Histogram by Yang Xiao

In this method, Spatial correlation is included through GLSC histogram for obtaining threshold. Gray level spatial correlation (GLSC) histogram differentiates two images in which the frequencies of intensities are exactly same, but differ in their placements, where two dimensional histograms fail. Therefore different images with identical gray level histogram will result in the same threshold value. To overcome the problem GLSC histogram is constructed by considering the similarity in neighborhood pixels with some adaptive threshold value as similarity measure ($\zeta$) as always 4 by Yang Xiao et al. [8 - 9].

**Novel similarity measure ($\zeta$)**

References [8] and [9] considered only local properties of the image and stated similarity measure $\zeta$ as constant at 4 producing reasonable results. Reference[10] has considered global properties into account along with local properties in computation of similarity measure $\zeta$ as, for every $N \times N$ map, with the help of Otsu’s discrimination analysis to measure image global and local characteristic $C_g$ and $C_l$ respectively. That is,
\[ \zeta = 2|C_g - C_l|. \]

**Figure 3.3 GLSC Histogram (with constant \( \zeta = 4 \)) of Cameraman image.**

**Computation of GLSC histogram**

Let \( f(x, y) \) be the gray level intensity of image at \((x, y)\). \( F = \{ f(x, y) | x \in [1, Q], y \in [1, R] \} \) of size \( Q \times R \). The gray level set \( \{0, 1, 2, ..., 255\} \) is considered as \( G \). The image GLSC histogram is computed by taking only image local properties into account as follows. Let \( g(x, y) \) be the similarity count corresponding to pixel of image \( f(x, y) \) in \( N \times N \) neighborhood, where \( N \) is any positive odd number in range \([3, \min(Q/2, R/2)]\).

\[
g(x+1,y+1) = \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} \mathbb{1}(|f(x+1,y+1) - f(x+i,y+j)| \leq \zeta)
\]  

(3.10)

Where,

\[
\mathbb{1}(|f(x+1,y+1) - f(x+i,y+j)|) = \\
\begin{cases} 
1 & \text{if } |f(x+1,y+1) - f(x+i,y+j)| \leq \zeta \\
0 & \text{otherwise}
\end{cases}
\]
GLSC histogram is constructed as shown in fig. 7 with the correlated probability at different gray level intensities from equations (15) and (16) as follows.

\[ h(k, m) = P(f(x, y) = k \text{ and } g(x, y) = m) \]  

(3.11)

Where, \( P \) is the gray level correlation probability computed for all pixels with intensity \( k \in G \) with correlation \( m \in \{1, 2, \ldots, N \times N\} \) and histogram is normalized.

**Non-linear Weight function.**

![Weight plot](image)

Figure 3.4 Weight plot

Where, \( \text{Weight}(m, N) \) is a nonlinear function as shown in fig. 3.4, applied to GLSC histogram elements for entropic calculation derived as.

\[ \text{Weight}(m, N) = \frac{1 + e^{-9m}}{1 - e^{-9m}} \]  

(3.12)

Where, \( N \) is any positive odd number in range \([3, \min(Q/2, R/2)]\) and \( m \in \{1, 2, \ldots, N \times N\} \), Fig. 3.4 shows \( \text{Weight}(m, N) \) a plot when \( N=3 \). Probability function \( p(k, m) \) is calculated as

\[ p(k, m) = \frac{\text{no. of pixels with gray value } k \text{ and } m \text{ correlation in } N \times N \text{ map}}{\text{total no. of pixels in the image}} \]
Where,

\[ \sum_{k=0}^{255} \sum_{m=1}^{NXN} p(k, m) = 1 \]

**Probability computations**

The two disjoint sets corresponding to object and background \( G_O, G_B \) are partitioned from set \( G \) corresponding to entire image with \( t \). The probability distribution associated with object and background is given by probability function as shown below:

\[ \hat{p}(i, j) = \{ p(i, j) / (255*N^2) \mid \forall \ i \in [0, 255], \ j \in [1, N \times N] \} \quad (3.13) \]

\[
\begin{bmatrix}
\hat{p}(0,1) / P_O(t) & \cdots & \hat{p}(0, N \times N) / P_O(t) \\
\hat{p}(1,1) / P_O(t) & \cdots & \hat{p}(1, N \times N) / P_O(t) \\
\vdots & \ddots & \vdots \\
\hat{p}(t,1) / P_O(t) & \cdots & \hat{p}(t, N \times N) / P_O(t)
\end{bmatrix}
\]

\[ \hat{p}(t+1, 1) / P_B(t) \quad \hat{p}(t+1, N \times N) / P_B(t) \quad \hat{p}(t+2, 1) / P_B(t) \quad \cdots \quad \hat{p}(255, N \times N) / P_B(t) \]

\[ \hat{p}(t,1) / P_B(t) \quad \hat{p}(t, N \times N) / P_B(t) \quad \hat{p}(t+1, 1) / P_B(t) \quad \cdots \quad \hat{p}(255, N \times N) / P_B(t) \]

Where, \( P_O(t) = \sum_{k=0}^{t} \sum_{m=1}^{NXN} \hat{p}(k, m) \quad (3.16) \)

and \( P_B(t) = 1 - P_A(t) \quad (3.17) \)
Kapur et al. criterion function is used to estimate the threshold on GLSC histogram instead of 2D gray level histogram with a defined weight for the taken map \( N=3 \).

\[
\phi(t, N) = H_O(t, n) + H_B(t, n) \tag{3.18}
\]

where, \( \phi(t, N) \) is the entropic criterion function, \( H_O(t, n) \) and \( H_B(t, n) \) are entropies associated with object and background respectively and optimal threshold \( t^* \) can be obtained by maximizing \( \phi(t) \).

\[
t^* = \text{Arg } \max \{ \phi(t, N) \} \tag{3.19}
\]

From equation (5) \( H_O, H_B \) are entropies associated with object and background distribution

\[
H_O(t, N) = \sum_{k=0}^{t} \sum_{m=1}^{N \times N} \frac{p(k, m)}{p_O(t)} \times \ln \left( \frac{p(k, m)}{p_O(t)} \right) \times \text{Weight}(m, N) \tag{3.20}
\]

And

\[
H_B(t, N) = \sum_{k=t+1}^{255} \sum_{m=1}^{N \times N} \frac{p(k, m)}{p_B(t)} \times \ln \left( \frac{p(k, m)}{p_B(t)} \right) \times \text{Weight}(m, N) \tag{3.21}
\]

In one-dimensional histogram the two peaks represents object and background, in GLSC histogram the same are represented in 2-dimmensional, which gives the information about spatial correlation. But it fails to identify the ambiguous region.

### 3.3 probability partitions based Entropic thresholding using GLSC histogram and Type-II Fuzzy - PROPOSED METHOD
The main problem with fuzzy set of type I, regardless of which shape and what algorithm is applied, that the assignment of a membership degree to an element or a pixel is not certain. Membership functions are usually defined by the expert and are based on his intuition/knowledge. The fact that different fuzzy techniques differ mainly in the way that they define the membership function is for the most part due to this dilemma. To find a more robust solution, type II fuzzy sets should be introduced.

Methodology

Now Equations (3.15) and (3.16) can be rewritten as

\[ P_O(t) = \sum_{k=0}^{255} \sum_{m=1}^{N} \hat{p}(k,m) \ast \mu(k) \]  \hspace{1cm} (3.22)

and \[ P_B(t) = 1 - P_O(t) \]  \hspace{1cm} (3.23)

Equations (3.6) and (3.7) are now changed to As with Entropy maximization

\[ H_O(t, N) = - \sum_{k=t+1}^{255} \sum_{m=1}^{N} \frac{\hat{p}(k,m) \mu(k)}{P_O(t)} \ast \ln \left( \frac{\hat{p}(k,m) \mu(k)}{P_O(t)} \right) \ast \text{Weight}(m,N) \]  \hspace{1cm} (3.24)

\[ H_B(t, N) = - \sum_{k=t+1}^{255} \sum_{m=1}^{N} \frac{p(k,m) \ast \mu_B(k)}{P_B(t)} \ast \ln \left( \frac{p(k,m) \ast \mu_B(k)}{P_B(t)} \right) \ast \text{Weight}(m,N) \]
Figure 3.6 (a) Original image of Lena (b) Thresholded image of Lena. (c) Original image of Trees (d) Thresholded image of Trees.

Weight \((m, N)\) is computed by Equation (3.11).

\[
\phi(t, N) = H_O(t, n) + H_B(t, n) \tag{3.25}
\]

Where, \(\phi(t, N)\) is the entropic criterion function, \(H_O(t, n)\) and \(H_B(t, n)\) are entropies associated with object and background respectively. The optimal threshold \(t^*\) is obtained by maximizing \(\phi(t)\) and \(t^* = \text{Arg max}\{ \phi(t, N)\}\). \(\tag{3.26}\)

The thresholded results are shown in the next page for the appreciation of our work.

3.4 Chapter Summary

In this chapter various spatial correlation based image thresholding techniques were studied. At the end of the chapter, probability partition based algorithm has been outlined.