CHAPTER 3

INVENTORY TURNOVER RATIO MODELS IN A MULTI-DETERMINISTIC SITUATION
3. CHAPTER 3: INVENTORY TURNOVER RATIO MODELS IN A MULTI-DETERMINISTIC SITUATION

3.1. Introduction:

It is not possible to consider each item separately when there is some relationship between each other. It is multi deterministic situation where the turnover is maximized for a stock dependent consumption rate with respect to some limitations imposed on the amount invested on inventories. Here, the technique of lagragian multiplier is very helpful to determine the optimal turnover for various items.

When a restriction is imposed on the maximum amount to be invested on inventories, with respect to the maximum limit of the inventories, the excess of money that is spent on the idle resources is reduced. Due to this, the amount of inventory stocked is in turn, very less. A balance between a high turnover and a low turnover for various items is obtained through this approach.

Here, the turnover ratio has been considered under a multi-deterministic situation and there is a limit on the amount that can be incurred on inventories with respect to the maximum limit on an average inventory. The results obtained are justified with the help of a hypothetical example.

3.2. Model - I: Inventory Turnover Ratio Model for a Stock Dependent Consumption Rate in a Multi-Deterministic Situation under Fixed Setup Cost.

3.2.1. Introduction:-

In this chapter, the demand is not constant, but it is stock dependent. The inventory turnover ratio is maximized for a stock dependent consumption rate in a multi deterministic situation where a limit has been
imposed on the amount invested on inventories with respect to maximum limit of an average inventory.

This model was proposed by S.B.Srivastav (1978) and supported by Kanti Swaroop, Gupta and Man Mohan in 1994.

The results obtained are justified with the help of a hypothetical problem.

3.2.2. Notations:-

- \( Q_i \) = Lot Size of \( i^{th} \) item (In Units)
- \( D_i \) = Annual Demand of \( i^{th} \) item (In Units)
- \( C_{i1} \) = Holding Cost of \( i^{th} \) item (Rs./Units)
- \( C_{3i} \) = Setup Cost of \( i^{th} \) item (per production run or per order)
- \( K \) = Maximum limit on inventories (In Units)
- \( p_i \) = Purchase Cost of \( i^{th} \) item (Rs./Unit)

where \( i = 1, 2, 3..., n \)

3.2.3. Assumption:-

(i) Demand is stock dependent
(ii) Shortages are not permitted
(iii) Lead time is zero
(iv) Production or supply of commodity is instantaneous
(v) There is a limit imposed on inventories

3.2.4. Problem Formulation:-

As discussed earlier, the inventory turnover ratio for \( i^{th} \) item is defined as:

\[
I(Q_i) = \frac{D_i p_i}{T(Q_i)} \quad \cdots \quad 3.1
\]

Where \( T(Q_i) = \frac{Q C_{i1}}{2} + \frac{D_i C_{3i}}{Q_i} \)

where \( i = 1, 2, 3..., n \)
When the inventories consist of several items, the total cost is given by,

\[ TC(Q_i) = \sum_{i=1}^{n} T(Q_i) \]

\[ TC(Q_i) = \sum_{i=1}^{n} \left[ \frac{Q_i C_{ui}}{2} + \frac{D_i C_{3i}}{Q_i} \right] \quad \ldots \quad 3.2 \]

There is a limit on inventories; the objective is to minimize the inventory turnover ratio subject to the condition total \( \frac{1}{2} \sum_{i=1}^{n} Q_i \leq K \) where \( K \) is the maximum limit on inventories. Thus, the total cost function is redefined by the lagrangian function as:

\[ L = \sum_{i=1}^{n} \left[ \frac{C_{ui} Q_i}{2} + \frac{C_{3i} D_i}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \quad \ldots \quad 3.3 \]

Where \( \lambda \) is a lagrangian multiplier. Let us take the inventory turnover ratio for different stock dependent demand function as follows:

### 3.2.5. Case I: \( D = nQ \)

Where \( n \) = number of shipments and \( Q \) is a Contract quantity.

\[ I(Q_i) = \frac{D_i p_i}{\sum_{i=1}^{n} \left[ \frac{C_{ui} Q_i}{2} + \frac{C_{3i} D_i}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right]} \quad \ldots \quad 3.4 \]

Differentiate equation (3.4) with respect to \( Q \) and equate \( \varepsilon \) to zero, we have

\[ \frac{\partial I(Q_i)}{\partial Q_i} = -D_i p_i \left[ \frac{C_{ui} Q_i}{2} - \frac{C_{3i} D_i}{Q_i^2} + \lambda \left( \sum_{i=1}^{n} \left( \frac{C_{ui} Q_i}{2} + \frac{C_{3i} D_i}{Q_i} \right) \right) + \lambda \left( \sum_{i=1}^{n} Q_i - 2K \right) \right] ^2 = 0 \]

\[ \ldots \quad 3.5 \]

\[ \therefore \frac{C_{ui}}{2} - \frac{C_{3i} D_i}{Q_i^2} + \lambda = 0 \]

\[ \therefore Q_i = \sqrt{\frac{2D_i C_{3i}}{C_{ui} + 2\lambda}} \quad \quad i = 1, 2, 3, \ldots, n \quad \ldots \quad 3.6 \]
Which becomes optimal subject to the condition stated below:

\[
\frac{\partial r(Q_0)}{\partial \lambda} = -D_i P_i \left[ \sum_{i=1}^{n} \left( \frac{C_{iL}}{2} + \frac{C_{iS} D_i}{Q_i} \right) + \lambda \left( \sum_{i=1}^{n} Q_i - 2K \right) \right] \left[ 0 + \sum_{i=1}^{n} Q_i - 2K \right] = 0
\]

\[ \cdots \quad 3.7 \]

\[ \therefore \sum_{i=1}^{n} Q_i - 2K = 0 \]

\[ \therefore \sum_{i=1}^{n} Q_i = 2K \quad \cdots \quad 3.8 \]

Above results determines the optimal turnover when maximum limit on inventories is imposed.

3.2.6. Hypothetical Problem:-

Let us consider problem as follow:

<table>
<thead>
<tr>
<th>Item</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holding Cost (Rs.)</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Setup Cost (Rs.)</td>
<td>60</td>
<td>50</td>
<td>40</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>Cost per Unit (Rs.)</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>Yearly Demand (In Units)</td>
<td>7,500</td>
<td>10,000</td>
<td>13,500</td>
<td>17,000</td>
<td>20,500</td>
</tr>
</tbody>
</table>

(i) Maximum limit on amount of average inventories = Rs. 1500

(ii) Maximum limit on inventory K = 175 Units

By using the successive approximation, the value of lagragian multiplier \( \lambda \) is obtained as 88.20, such that \( \frac{1}{2} \sum_{i=1}^{n} Q_i \leq K \).

<table>
<thead>
<tr>
<th>Item</th>
<th>Demand (Units) ( D_i )</th>
<th>Holding Cost (Rs.) ( C_{iL} )</th>
<th>Setup Cost (Rs.) ( C_{iS} )</th>
<th>Purchase Cost (Rs.) ( P_i )</th>
<th>Optimal Turnover ( Q_{opt} )</th>
<th>Total Cost (Rs.) ( T(Q_i) )</th>
<th>ITOR ( I(Q_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7500</td>
<td>20</td>
<td>60</td>
<td>5</td>
<td>68</td>
<td>7324</td>
<td>5.12</td>
</tr>
<tr>
<td>2</td>
<td>10000</td>
<td>20</td>
<td>50</td>
<td>6</td>
<td>71</td>
<td>7721</td>
<td>7.77</td>
</tr>
<tr>
<td>3</td>
<td>13500</td>
<td>20</td>
<td>40</td>
<td>7</td>
<td>74</td>
<td>8024</td>
<td>11.78</td>
</tr>
<tr>
<td>4</td>
<td>17000</td>
<td>20</td>
<td>30</td>
<td>8</td>
<td>72</td>
<td>7797</td>
<td>17.44</td>
</tr>
<tr>
<td>5</td>
<td>20500</td>
<td>20</td>
<td>20</td>
<td>9</td>
<td>65</td>
<td>6991</td>
<td>26.39</td>
</tr>
</tbody>
</table>
3.2.7. Remark:-

From the hypothetical problem given above, it is clear that the cost of average inventory is less than the maximum limit imposed. Thus, the amount which is extra, can be spent on the resource utilization. Also, ITOR is in increasing order which denote that inventory management system is good. There are no excesses or shortages observed in inventories of the given items. Also this type of complete resource allotment is only possible through computer program which is mentioned in appendix II.

3.2.8. Case II: \( D = \alpha + \beta Q \)

Where \( \alpha \) and \( \beta \) are constants, \( Q \) is a Contract quantity.

The inventory turnover ratio is defined as:

\[
I(Q) = \frac{D_i P_i}{\sum_{i=1}^{n} \left( \frac{C_i Q_i}{2} + \frac{C_{Q_i} D_i}{Q_i} \right) + \lambda \left( \sum_{i=1}^{n} Q_i - 2K \right)}
\]

\[
: I(Q) = \frac{(\alpha + \beta Q) P_i}{\sum_{i=1}^{n} \left( \frac{C_i Q_i}{2} + \frac{\alpha C_i}{Q_i} + \beta C_i \right) + \lambda \left( \sum_{i=1}^{n} Q_i - 2K \right)}
\]

Differentiate equation (3.10) with respect to \( Q_i \) and equate it to zero, we have,

\[
\frac{\partial I(Q)}{\partial Q_i} = \beta P_i \left\{ \sum_{i=1}^{n} \left[ \frac{C_i Q_i}{2} + \frac{\alpha C_i}{Q_i} + \beta C_i \right] + \lambda \left( \sum_{i=1}^{n} Q_i - 2K \right) \right\}^{-1} - \beta P_i \left\{ \sum_{i=1}^{n} \left[ \frac{C_i Q_i}{2} + \frac{\alpha C_i}{Q_i} + \beta C_i \right] + \lambda \left( \sum_{i=1}^{n} Q_i - 2K \right) \right\}^{-2}
\]

\[
\left\{ \left[ \frac{C_i Q_i}{2} - \frac{\alpha C_i}{Q_i^2} \right] + \lambda \right\} = 0
\]

\[
: \beta P_i \left\{ \sum_{i=1}^{n} \left[ \frac{C_i Q_i}{2} + \frac{\alpha C_i}{Q_i} + \beta C_i \right] + \lambda \left( \sum_{i=1}^{n} Q_i - 2K \right) \right\}^{-1} -
\]

75
\[(\alpha p_i + \beta Q_i, p_i) \left\{ \left[ \frac{C_{i,1}}{2} - \frac{\alpha C_{3i}}{Q_i^2} \right] + \lambda \right\} = 0 \]

for \(i^{th}\) item, we have,

\[(2\beta^2 C_{3i} - \alpha C_{i,1} - 2\lambda \alpha - 4K\lambda \beta)Q_i^2 + 4\alpha \beta C_{3i}Q_i + 2\alpha^2 C_{3i} = 0 \quad \ldots. \quad 3.12\]

Which implies that,

\[Q_i = \frac{-4\alpha \beta C_{3i} \pm \sqrt{32\alpha^2 C_{3i} \lambda K \beta + 8\alpha^3 C_{3i} C_{i,1} + 16\alpha^3 \lambda C_{3i}}}{4\beta^2 C_{3i} - 8\lambda K \beta - 2\alpha C_{i,1} - 4\lambda \alpha} \quad \ldots. \quad 3.13\]

Where \(i = 1, 2, 3, \ldots, n\)

Which becomes optimal subject to the condition stated below:

\[\frac{\partial I(Q_i)}{\partial \lambda} = 0 - (\alpha p_i + \beta Q_i, p_i) \left\{ \sum_{i=1}^{n} \left[ \frac{C_{i,1}Q_i}{2} + \frac{\alpha C_{3i}}{Q_i} + \beta C_{3i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right\} \]

\[\left[ 0 + \sum_{i=1}^{n} Q_i - 2K \right] = 0 \quad \ldots. \quad 3.14\]

\[\therefore \sum_{i=1}^{n} Q_i - 2K = 0\]

\[\therefore \sum_{i=1}^{n} Q_i = 2K \quad \ldots. \quad 3.15\]

Above results determines the optimal turnover when maximum limit on inventories is imposed.

Above equations are solved by successive approximation with the help of trial & error method.

3.2.9. Hypothetical Problem:-

Let us take problem defined in (3.2.6)

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(\beta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>10</td>
</tr>
<tr>
<td>1500</td>
<td>20</td>
</tr>
<tr>
<td>2000</td>
<td>30</td>
</tr>
<tr>
<td>2500</td>
<td>40</td>
</tr>
<tr>
<td>3000</td>
<td>50</td>
</tr>
</tbody>
</table>

(i) Maximum limit on amount of average inventories = Rs. 1500
(ii) Maximum limit on inventory $K = 175$ Units

By using the successive approximation, the value of lagragian multiplier $\lambda$ is obtained as 7.97, such that $\frac{1}{2} \sum_{i=1}^{n} Q_i \leq K$.

<table>
<thead>
<tr>
<th>Item</th>
<th>Demand (Units) $D_i$</th>
<th>Holding Cost (Rs.) $C_{hi}$</th>
<th>Setup Cost (Rs.) $C_{si}$</th>
<th>Purchase Cost (Rs.) $P_i$</th>
<th>Optimal Turnover $Q_{opt}$</th>
<th>Total Cost (Rs.) $T(Q_i)$</th>
<th>ITOR $I(Q_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1567</td>
<td>20</td>
<td>60</td>
<td>5</td>
<td>57</td>
<td>2225</td>
<td>3.52</td>
</tr>
<tr>
<td>2</td>
<td>2951</td>
<td>20</td>
<td>50</td>
<td>6</td>
<td>73</td>
<td>2759</td>
<td>6.42</td>
</tr>
<tr>
<td>3</td>
<td>4431</td>
<td>20</td>
<td>40</td>
<td>7</td>
<td>81</td>
<td>2998</td>
<td>10.35</td>
</tr>
<tr>
<td>4</td>
<td>5604</td>
<td>20</td>
<td>30</td>
<td>8</td>
<td>78</td>
<td>2942</td>
<td>15.24</td>
</tr>
<tr>
<td>5</td>
<td>6105</td>
<td>20</td>
<td>20</td>
<td>9</td>
<td>62</td>
<td>2587</td>
<td>21.24</td>
</tr>
</tbody>
</table>

3.2.10. Remark:-

The above data indicates that with increases in the value of $\alpha$ and $\beta$, the size of inventory turnover also increases. There is also a reduction in total inventory cost. Therefore ITOR increases which minimizing the risk of stock outs. This proves that the firm handling a good inventory management even after the limit is imposed on amount of inventories. Here perfect allocation of resources is possible only through computer program which is mentioned in appendix II.

3.2.11. Case III: $D = \beta Q^x$

$0 < x < 1$, $x$ is a variable cost and $\beta$ is a constant and $Q$ is a contract quantity.

The Inventory turnover ratio is defined as

$$i(Q) = \frac{D_i P_i}{\sum_{i=1}^{n} \left[ \frac{C_{hi} Q_i}{2} + \frac{C_{si} D_i}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right]}$$

$$= \beta Q^x p \left[ \sum_{i=1}^{n} \left[ \frac{C_{hi} Q_i}{2} + \frac{C_{si} \beta Q^x_i}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right]$$

...... 3.16

...... 3.17
Necessary and sufficient conditions for optimising inventory turnover ratio are (i) \( \frac{\partial f(Q_i)}{\partial Q_i} = 0 \), (ii) \( \frac{\partial f(Q_i)}{\partial \lambda} = 0 \)

Differentiate equation (3.17) with respect to \( Q_i \), and equate it to zero, we get

\[
\frac{\partial f(Q_i)}{\partial Q_i} = x_i \beta Q_i^{x_i-1} p_i \left\{ \sum_{i=1}^{n} \left[ \frac{C_{hi}Q_i + C_{bi}Q_i^{x_i}}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right\}^{-1} \\
- \beta Q_i^{x_i-1} p_i \left\{ \sum_{i=1}^{n} \left[ \frac{C_{hi}Q_i + C_{bi}Q_i^{x_i}}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right\}^{-2} \\
\left\{ \frac{C_{hi}}{2} + (x_i - 1)C_{bi}Q_i^{x_i-2} + \lambda \right\} = 0 \\
\text{...... 3.18}
\]

For \( i \)th item we have,

\[
x_i Q_i^{-1} \left\{ \frac{C_{hi}Q_i + C_{bi}Q_i^{x_i}}{Q_i} \right\} + \lambda \left[ Q_i - 2K \right] - \left\{ \frac{C_{hi}}{2} + (x_i - 1)C_{bi}Q_i^{x_i-2} + \lambda \right\} = 0 \\
\therefore (x_i - 1)(C_{hi} + 2\lambda)Q_i^2 - 4K\lambda x_iQ_i + 2C_{bi}Q_i^{x_i} = 0 \\
\text{...... 3.19}
\]

Which implies that,

\[
Q_i = \sqrt{\frac{(1-x_i)(C_{hi} + 2\lambda)Q_i^2 + 4K\lambda x_iQ_i}{2C_{bi}}} \\
\text{...... 3.20}
\]

OR

\[
Q_i = \left[ \frac{(1-x_i)(C_{hi} + 2\lambda)Q_i^2 + 4K\lambda x_iQ_i}{2C_{bi}} \right]^{\frac{1}{x_i}} \\
\text{Where } i = 1, 2, 3, \ldots, n
\]

And

\[
\frac{\partial f(Q_i)}{\partial \lambda} = 0 - \beta Q_i^{x_i} p_i \left\{ \sum_{i=1}^{n} \left[ \frac{C_{hi}Q_i + C_{bi}Q_i^{x_i}}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right\}^{-2} \\
\left\{ 0 + \sum_{i=1}^{n} Q_i - 2K \right\} = 0 \\
\text{...... 3.21}
\]

\[
\therefore \sum_{i=1}^{n} Q_i - 2K = 0
\]
\[ \sum_{i=1}^{n} Q_i = 2K \] \[ \ldots \quad 3.22 \]

Above results determines the optimal turnover when maximum limit on inventories is imposed.

Above equations are solved by successive approximation with the help of trial and error method.

### 3.2.12. Hypothetical Problem:-

Let us take the example given in (3.2.6) in which values of \( x_i, \beta \) are as bellow:

<table>
<thead>
<tr>
<th>Item</th>
<th>( x_i )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td>55</td>
</tr>
<tr>
<td>3</td>
<td>0.7</td>
<td>125</td>
</tr>
</tbody>
</table>

(i) Maximum limit on amount of average inventories = Rs. 1500

(ii) Maximum limit on inventory \( K = 175 \) Units

By using the successive approximation, the value of lagragian multiplier \( \lambda \) is obtained as 2.34, such that \( \frac{1}{2} \sum_{i=1}^{n} Q_i \leq K \).

<table>
<thead>
<tr>
<th>Item</th>
<th>Demand (Units) ( D_i )</th>
<th>Holding Cost (Rs.) ( C_{hi} )</th>
<th>Setup Cost (Rs.) ( C_{si} )</th>
<th>Purchase Cost (Rs.) ( P_i )</th>
<th>Optimal Turnover ( Q_{opt} )</th>
<th>Total Cost (Rs.) ( T(Q_i) )</th>
<th>ITOR ( I(Q_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>732</td>
<td>20</td>
<td>60</td>
<td>5</td>
<td>55</td>
<td>1350</td>
<td>2.71</td>
</tr>
<tr>
<td>2</td>
<td>2962</td>
<td>20</td>
<td>50</td>
<td>6</td>
<td>146</td>
<td>2474</td>
<td>7.18</td>
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<td>3</td>
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<td>40</td>
<td>7</td>
<td>147</td>
<td>2591</td>
<td>11.12</td>
</tr>
</tbody>
</table>

### 3.2.13. Remark:-

The above result indicates that with increases in the value of \( x_i \) and \( \beta \) the size of inventory turnover also increases. Also, we observed increasing trend in ITOR. Therefore, this proves that the firm has good inventory management system. Here perfect allocation of resources is possible only through computer program which is mentioned in appendix II.
3.2.14. Case IV: \( D = \beta_1 Q - \beta_2 Q^2 \)

\( \beta_1, \beta_2 \) are constants and \( Q \) is contract quantity.

The Inventory turnover ratio is defined as:

\[
I(Q) = \frac{D_p}{\sum_{i=1}^{n} \left[ \frac{C_i Q_i}{2} + \frac{C_{2i} D_{2i}}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right]}
\]

\[
= [\beta_1 Q_i p_i - \beta_2 Q^2 p_i] \left\{ \sum_{i=1}^{n} \left[ \frac{C_i Q_i}{2} + \frac{C_{2i} (\beta_1 Q_i - \beta_2 Q_i^2)}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right\}^{-1}
\]

Differentiate equation (3.23) with respect to \( Q \) and equate it to zero, we get,

\[
\frac{\partial I(Q)}{\partial Q} = \left[ \beta_1 p_i - 2\beta_2 Q_i p_i \right] \left\{ \sum_{i=1}^{n} \left[ \frac{C_i Q_i}{2} + \frac{C_{2i} (\beta_1 Q_i - \beta_2 Q_i^2)}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right\}^{-1}
\]

\[- \left[ \beta_1 Q_i p_i - \beta_2 Q^2 p_i \right] \left\{ \sum_{i=1}^{n} \left[ \frac{C_i Q_i}{2} + \frac{C_{2i} (\beta_1 Q_i - \beta_2 Q_i^2)}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right\}^{-2}
\]

\[ \left\{ \frac{C_i}{2} - C_{2i} \beta_2 + \lambda \right\} = 0 \]

For \( i^{th} \) item, we have,

\[
(\beta_1 - 2\beta_2 Q_i) \left[ \frac{C_i Q_i}{2} + C_{2i} \beta_1 - C_{2i} \beta_2 Q_i + \lambda Q_i - 2\lambda K \right] -
\]

\[
(\beta_1 Q_i - \beta_2 Q_i^2) \left( \frac{C_i}{2} - C_{2i} \beta_2 + \lambda \right) = 0
\]

\[ \therefore \left( 2C_{2i} \beta_2^2 - C_{2i} \beta_2 - 2\lambda \beta_2 \right) Q_i^2 + 4\beta_2 (2\lambda K - \beta_1 C_{2i}) Q_i + \left( 2C_{2i} \beta_1^2 - 4\lambda \beta_1 K \right) = 0
\]

Which implies that,

\[
Q_i = \left\{ \frac{(4\beta_2 \lambda K - 2\beta_1 \beta_2 C_{2i}) \pm \left[ 16 \beta_2 \beta_1^2 \lambda K^2 + 4\beta_2 \lambda \beta_1^3 C_{2i} + 2\beta_1^2 \beta_2 C_{2i} C_{2i} - 8\beta_1 \beta_2 C_{2i} \lambda K - 8\beta_1 \beta_2 C_{2i} \lambda K - 4\lambda K \beta_2 \beta_1 C_{2i} \right]^{1/2}}{2 \beta_1^2 C_{2i} - 2\beta_2 \lambda - \beta_2 C_{2i}} \right\}^{1/2}
\]

\[ \therefore 3.26 \]
Where \( i = 1, 2, 3, \ldots, n \)

and

\[
\frac{\partial \ell(Q_i)}{\partial \lambda} = 0 - \left[ \beta_1 Q_i - \beta_2 Q_i^2 \right] \left[ \sum_{i=1}^{n} \left( \frac{C_i Q_i}{2} + \frac{C_k (\beta_1 Q_i - \beta_2 Q_i^2)}{Q_i} \right) + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right]^{-2} 
\]

\[
0 + \sum_{i=1}^{n} Q_i - 2K = 0
\]

\[
\therefore \sum_{i=1}^{n} Q_i = 2K
\]

Above results determines the optimal turnover when maximum limit on inventories is imposed.

Above equations are solved by successive approximation with the help of trial and error method.

### 3.2.15. Hypothetical Problem:-

Let us take the example given in (3.2.6) in which value of \( \beta_1 \) and \( \beta_2 \) are as below:

<table>
<thead>
<tr>
<th>Item</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>70</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>75</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(i) Maximum limit on amount of average inventories = Rs. 1500

(ii) Maximum limit on inventory \( K = 175 \) Units

By using the successive approximation, the value of lagragian multiplier \( \lambda \) is obtained as 2.34, such that \( \frac{1}{2} \sum_{i=1}^{n} Q_i \leq K \).
### Item Demand

<table>
<thead>
<tr>
<th>Item</th>
<th>Demand (Units) $D_i$</th>
<th>Holding Cost (Rs.) $C_{h_i}$</th>
<th>Setup Cost (Rs.) $C_{s_i}$</th>
<th>Purchase Cost (Rs.) $P_i$,</th>
<th>Optimal Turnover $Q_{opt}$</th>
<th>Total Cost (Rs.) $T(Q_i)$</th>
<th>ITOR $I(Q_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1125</td>
<td>20</td>
<td>60</td>
<td>5</td>
<td>36</td>
<td>2219</td>
<td>2.53</td>
</tr>
<tr>
<td>2</td>
<td>3641</td>
<td>20</td>
<td>50</td>
<td>6</td>
<td>79</td>
<td>3087</td>
<td>7.08</td>
</tr>
<tr>
<td>3</td>
<td>17200</td>
<td>20</td>
<td>40</td>
<td>7</td>
<td>234</td>
<td>5279</td>
<td>22.80</td>
</tr>
</tbody>
</table>

#### 3.2.16. Remark:-

Here we observed that with increases in the value of $\beta_1$ and decreases in the value of $\beta_2$ the optimum turnovers achieved with increasing trend in demand. Also, ITOR is in increasing order which indicates the good performance of the firm with reduction in total inventory cost. Here perfect allocation of resources is possible only through computer program which is mentioned in appendix II.

#### 3.3. Model - II: Inventory Turnover Ratio Model For A Stock Dependent Consumption Rate In A Multi-Deterministic Situation When Setup Cost Is Varies.

**3.3.1. Introduction:-**

In this chapter, the demand and setup/ordering cost is not constant, but it is stock dependent. The inventory turnover ratio is maximized for a stock dependent consumption rate in a multi deterministic situation where a limit has been imposed on the amount invested on inventories with respect to maximum limit of an average inventory.

This model was proposed by S.B.Srivastav (1978) and supported by Kanti Swaroop, Gupta and Man Mohan in 1994.

The results obtained are justified with the help of a hypothetical problem.
3.3.2. Notations:-

\[ Q_i = \text{Lot Size of } i^{th} \text{ item (In Units)} \]

\[ D_i = \text{Annual Demand of } i^{th} \text{ item (In Units)} \]

\[ C_{i1} = \text{Holding Cost of } i^{th} \text{ item (Rs./Units)} \]

\[ C_{3i} = C_{3i} + bQ_i = \text{Setup Cost of } i^{th} \text{ item (per production run or per order)} \]

\[ K = \text{Maximum limit on inventories (In Units)} \]

\[ p_i = \text{Purchase Cost of } i^{th} \text{ item (Rs./Unit)} \]

where \( i = 1, 2, 3..., n \)

3.3.3. Assumption:-

(i) Demand is stock dependent
(ii) Shortages are not permitted
(iii) Lead time is zero
(iv) Production or supply of commodity is instantaneous
(v) There is a limit imposed on inventories

3.3.4. Problem Formulation:-

As discussed earlier, the inventory turnover ratio for \( i^{th} \) item is defined as:

\[ I(Q_i) = \frac{D_i p_i}{T(Q_i)} \]  \quad \ldots \quad 3.29

Where

\[ T(Q_i) = \frac{Q_i C_{i1}}{2} + \frac{D_i (C_{3i} + bQ_i)}{Q_i} \]

where \( i = 1, 2, 3..., n \)

When the inventories consist of several items, the total cost is given be,

\[ TC(Q_i) = \sum_{i=1}^{n} T(Q_i) \]
\[ TC(Q_i) = \sum_{i=1}^{n} \left[ \frac{Q_iC_{3i}}{2} + \frac{D_i(C_{3i} + bQ_i)}{Q_i} \right] \] \hspace{1cm} 3.30

There is a limit on inventories; the objective is to minimize the inventory turnover ratio subject to the condition total \( \frac{1}{2} \sum_{i=1}^{n} Q_i \leq K \) where \( K \) is the maximum limit on inventories. Thus, the total cost function is redefined by the lagrangian function as:

\[ L = \sum_{i=1}^{n} \left[ \frac{C_{3i}Q_i}{2} + \frac{(C_{3i} + bQ_i)D_i}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \] \hspace{1cm} 3.31

Where \( \lambda \) is a lagrangian multiplier. Let us take the inventory turnover ratio for different stock dependent demand function as follows:

3.3.5. Case I: \( D = nQ, C_{3i} = C_{3i} + bQ_i \)

Where \( n \) is number of shipments and \( Q \) is a Contract quantity.

\[ I(Q_i) = \frac{D_iP_i}{\sum_{i=1}^{n} \left[ \frac{C_{3i}Q_i}{2} + \frac{(C_{3i} + bQ_i)D_i}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right]} \] \hspace{1cm} 3.32

Differentiate equation (3.32) with respect to \( Q_i \) and equate it to zero, we have

\[ \frac{dI(Q_i)}{dQ_i} = -D_iP_i \left[ \frac{C_{3i}}{2} - \frac{C_{3i}D_i}{Q_i^2} + \lambda \right] \]

\[ \left[ \sum_{i=1}^{n} \left( \frac{C_{3i}Q_i}{2} + \frac{(C_{3i} + bQ_i)D_i}{Q_i} \right) + \lambda \left( \sum_{i=1}^{n} Q_i - 2K \right) \right]^{-2} \]

\[ = 0 \] \hspace{1cm} 3.33

\[ \therefore \frac{C_{3i}}{2} - \frac{C_{3i}D_i}{Q_i^2} + \lambda = 0 \]

\[ \therefore Q_i = \sqrt{\frac{2D_iC_{3i}}{C_{3i} + 2\lambda}} \quad i = 1, 2, 3, \ldots, n \] \hspace{1cm} 3.34

Which becomes optimal subject to the condition stated below:
Above results determines the optimal turnover when maximum limit on inventories is imposed.

**3.3.6. Hypothetical Problem:-**

Let us consider problem as follow:

<table>
<thead>
<tr>
<th>Item</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Holding Cost (Rs.)</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Setup Cost (Rs.)</td>
<td>60</td>
<td>50</td>
<td>40</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>Cost per Unit (Rs.)</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
</tr>
<tr>
<td>Yearly Demand (In Units)</td>
<td>7,500</td>
<td>10,000</td>
<td>13,500</td>
<td>17,000</td>
<td>20,500</td>
</tr>
</tbody>
</table>

(i) Maximum limit on amount of average inventories = Rs. 1500
(ii) Maximum limit on inventory K = 175 Units

By using the successive approximation, the value of lagrangian multiplier \( \lambda \) is obtained as 88.20, such that \( \frac{1}{2} \sum Q_i \leq K \).

<table>
<thead>
<tr>
<th>Item</th>
<th>Demand (Units)</th>
<th>Holding Cost (Rs.)</th>
<th>Setup Cost (Rs.)</th>
<th>Purchase Cost (Rs.)</th>
<th>Optimal Turnover Qopt</th>
<th>Total Cost (Rs.)</th>
<th>ITOR I(Qopt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7500</td>
<td>20</td>
<td>61</td>
<td>5</td>
<td>68</td>
<td>7399</td>
<td>5.07</td>
</tr>
<tr>
<td>2</td>
<td>10000</td>
<td>20</td>
<td>61</td>
<td>5</td>
<td>71</td>
<td>7821</td>
<td>7.67</td>
</tr>
<tr>
<td>3</td>
<td>13500</td>
<td>20</td>
<td>51</td>
<td>6</td>
<td>74</td>
<td>8159</td>
<td>11.58</td>
</tr>
<tr>
<td>4</td>
<td>17000</td>
<td>20</td>
<td>31</td>
<td>8</td>
<td>72</td>
<td>7967</td>
<td>17.07</td>
</tr>
<tr>
<td>5</td>
<td>20500</td>
<td>20</td>
<td>21</td>
<td>9</td>
<td>65</td>
<td>7196</td>
<td>25.64</td>
</tr>
</tbody>
</table>
3.3.7. Remark:-

From the hypothetical problem given above, it is clear that the cost of average inventory is less than the maximum limit imposed. Thus, the amount which is extra can be spent on the resource utilization. Also, ITOR is in increasing order which denote that inventory management system is good. There are no excesses or shortages observed in inventories of the given items. Here perfect allocation of resources is possible only through computer program which is mentioned in appendix II.

3.3.8. Case II: \( D = \alpha + \beta Q, C_{3i} = C_{3i} + bQ_i \)

Where \( \alpha \) and \( \beta \) are constants, \( Q \) is a Contract quantity.

The inventory turnover ratio is defined as:

\[
I(Q_i) = \frac{D_i P_i}{\sum_{i=1}^{n} \left[ \frac{C_{1i} Q_i}{2} + \frac{(C_{3i} + bQ_i) D_i}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right]}
\]

\[
I(Q_i) = \frac{(\alpha + \beta Q_i) P_i}{\sum_{i=1}^{n} \left[ \frac{C_{1i} Q_i}{2} + \frac{\alpha(C_{3i} + bQ_i)}{Q_i} + \beta(C_{3i} + bQ_i) \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right]}
\]

\[
3.37
\]

Differentiate equation (3.38) with respect to \( Q_i \) and equate it to zero, we have,

\[
\frac{\partial I(Q_i)}{\partial Q_i} = \beta P_i \left\{ \sum_{i=1}^{n} \left[ \frac{C_{1i} Q_i}{2} + \frac{\alpha(C_{3i} + bQ_i)}{Q_i} + \beta(C_{3i} + bQ_i) \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right\}^{-1} - \\
(\alpha P_i + \beta Q_i P_i) \left\{ \sum_{i=1}^{n} \left[ \frac{C_{1i} Q_i}{2} + \frac{\alpha(C_{3i} + bQ_i)}{Q_i} + \beta(C_{3i} + bQ_i) \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right\}^{-2}
\]

\[
\left\{ \left[ \frac{C_{1i}}{2} - \frac{\alpha C_{3i}}{Q_i^2} + b\beta \right] + \lambda \right\} = 0
\]

\[
3.39
\]
\[ \begin{align*}
\therefore \beta p, \left\{ \sum_{i=1}^{n} \left[ \frac{C_i Q_i}{2} + \alpha C_{3i} + \beta C_{3i} + b\alpha + b\beta Q_i \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right\} = 0 \\
(\alpha p_i + \beta Q_i p_i) \left[ \frac{C_i}{2} - \frac{\alpha C_{3i}}{Q_i^2} + b\beta \right] + \lambda = 0
\end{align*} \]

for \( i \)th item, we have,
\[ (2\beta^2 C_{3i} - \alpha C_{3i} - 2\lambda\alpha - 4K\lambda\beta) Q_i^2 + 4\alpha\beta C_{3i} Q_i + 2\alpha^2 C_{3i} = 0 \quad \ldots \ldots \quad 3.40 \]

Which implies that,
\[ Q_i = \frac{-4\alpha\beta C_{3i} \pm \sqrt{32\alpha^2 C_{3i}^2 \lambda K\beta + 8\alpha^2 C_{3i}^2 C_{3i} + 16\alpha^2 \lambda C_{3i}^2}}{4\beta^2 C_{3i} - 8\lambda K\beta - 2\alpha C_{3i} - 4\lambda\alpha} \quad \ldots \ldots \quad 3.41 \]

Where \( i = 1, 2, 3, \ldots, n \)

Which becomes optimal subject to the condition stated below:
\[ \frac{\partial f(Q_i)}{\partial \lambda} = 0 - (\alpha p_i + \beta Q_i p_i) \left\{ \sum_{i=1}^{n} \left[ \frac{C_i Q_i}{2} + \frac{\alpha(C_{3i} + bQ_i)}{Q_i} + \beta(C_{3i} + bQ_i) \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right\}^2 + 0 + \sum_{i=1}^{n} Q_i - 2K = 0 \quad \ldots \ldots \quad 3.42 \]

\[ \therefore \sum_{i=1}^{n} Q_i - 2K = 0 \]

\[ \therefore \sum_{i=1}^{n} Q_i = 2K \quad \ldots \ldots \quad 3.43 \]

Above results determines the optimal turnover when maximum limit on inventories is imposed.

Above equations are solved by successive approximation with the help of trial & error method.
3.3.9. Hypothetical Problem:-

Let us take problem defined in (3.3.6)

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>10</td>
</tr>
<tr>
<td>1500</td>
<td>20</td>
</tr>
<tr>
<td>2000</td>
<td>30</td>
</tr>
<tr>
<td>2500</td>
<td>40</td>
</tr>
<tr>
<td>3000</td>
<td>50</td>
</tr>
</tbody>
</table>

(i) Maximum limit on amount of average inventories = Rs. 1500

(ii) Maximum limit on inventory $K = 175$ Units

By using the successive approximation, the value of lagragian multiplier $\lambda$ is obtained as 7.97, such that $\frac{1}{2} \sum_{i=1}^{n} Q_i \leq K$.

<table>
<thead>
<tr>
<th>Item</th>
<th>Demand (Units) $D_i$</th>
<th>Holding Cost (Rs.) $C_{ii}$</th>
<th>Setup Cost (Rs.) $C_{ii}$</th>
<th>Purchase Cost (Rs.) $P_i$</th>
<th>Optimal Turnover $Q_{opt}$</th>
<th>Total Cost (Rs.) $T(Q_i)$</th>
<th>ITOR $I(Q_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1567</td>
<td>20</td>
<td>61</td>
<td>5</td>
<td>57</td>
<td>2241</td>
<td>3.50</td>
</tr>
<tr>
<td>2</td>
<td>2951</td>
<td>20</td>
<td>51</td>
<td>6</td>
<td>73</td>
<td>2789</td>
<td>6.35</td>
</tr>
<tr>
<td>3</td>
<td>4431</td>
<td>20</td>
<td>41</td>
<td>7</td>
<td>81</td>
<td>3042</td>
<td>10.20</td>
</tr>
<tr>
<td>4</td>
<td>5604</td>
<td>20</td>
<td>31</td>
<td>8</td>
<td>78</td>
<td>2999</td>
<td>14.95</td>
</tr>
<tr>
<td>5</td>
<td>6105</td>
<td>20</td>
<td>21</td>
<td>9</td>
<td>62</td>
<td>2648</td>
<td>20.75</td>
</tr>
</tbody>
</table>

3.3.10. Remark:-

The above data indicates that with increases in the value of $\alpha$ and $\beta$ the size of inventory turnover also increases. There is also a reduction in total inventory cost. Therefore ITOR increases which minimizing the risk of stock outs. This proves that the firm handling a good inventory management even after the limit is imposed on amount of inventories. Here perfect allocation of resources is possible only through computer program which is mentioned in appendix II.
3.3.11. Case III: \( D = \beta Q^x, \ C_{3t} = C_{3t} Q^x + bQ_i \)

\( 0 < x < 1, \ x \) is a variable cost and \( \beta \) is a constant and \( Q \) is a contract quantity.

The Inventory turnover ratio is defined as

\[
I(Q) = \frac{D, P}{\sum_{i=1}^{n} \left[ C_{1i} Q_i + \frac{(C_{2i} + bQ_i) D_i}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right]} \ldots \ldots 3.44
\]

\[
= \beta Q_i^x P_i \left[ \sum_{i=1}^{n} \left[ \frac{C_{1i} Q_i}{2} + \frac{(C_{2i} + bQ_i) \beta Q_i^x}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right]^{-1} \ldots \ldots 3.45
\]

Necessary and sufficient conditions for optimising inventory turnover ratio are (i) \( \frac{\partial I(Q)}{\partial Q_i} = 0 \), (ii) \( \frac{\partial I(Q)}{\partial \lambda} = 0 \)

Differentiate equation (3.45) with respect to \( Q_i \) and equate it to zero, we get

\[
\frac{\partial I(Q_i)}{\partial Q_i} = x_i \beta Q_i^x - 1 P_i \left[ \sum_{i=1}^{n} \left[ \frac{C_{1i} Q_i}{2} + \frac{(C_{2i} + bQ_i) \beta Q_i^x}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right]^{-1}
\]

\[
- \beta Q_i^x P_i \left[ \sum_{i=1}^{n} \left[ \frac{C_{1i} Q_i}{2} + \frac{(C_{2i} + bQ_i) \beta Q_i^x}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right]^{-2}
\]

\[
\left\{ \frac{C_{1i}}{2} + (x_i - 1)C_{2i} \beta Q_i^x - 2 + x_i bQ_i^x + \lambda \right\} = 0 \ldots \ldots 3.46
\]

For \( i^{th} \) item we have,

\[
x_i Q_i^{-1} \left[ \sum_{i=1}^{n} \left[ \frac{C_{1i} Q_i}{2} + \frac{(C_{2i} + bQ_i) \beta Q_i^x}{Q_i} \right] + \lambda \left[ Q_i - 2K \right] \right] - \left\{ \frac{C_{1i}}{2} + (x_i - 1)C_{2i} \beta Q_i^x - 2 + x_i bQ_i^x + \lambda \right\} = 0
\]

\[
\therefore (x_i - 1)(C_{1i} + 2\lambda) Q_i^2 - 4K\lambda x_i Q_i + 2C_{3i} \beta Q_i^x = 0 \ldots \ldots 3.47
\]
Which implies that,

$$Q_i = \sqrt[2C_3, \beta]{(1-x_i)(C_{1i} + 2\lambda)Q_i + 4K\lambda x_i \cdot Q_i} \quad \ldots \quad 3.48$$

OR

$$Q_i = \left[\frac{(1-x_i)(C_{1i} + 2\lambda)Q_i + 4K\lambda x_i \cdot Q_i}{2C_3, \beta}\right]^{\frac{1}{2}}$$

Where \( i = 1, 2, 3, \ldots, n \)

And

$$\frac{\partial \mu(Q_i)}{\partial \lambda} = -\beta Q_i x_i p_i \left[ \sum_{i=1}^{n} \left( C_i Q_i + \frac{(C_{yi} + bQ_i)Q_i x_i}{Q_i} \right) + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right]^{-2}$$

$$0 + \sum_{i=1}^{n} Q_i - 2K = 0 \quad \ldots \quad 3.49$$

$$\therefore \sum_{i=1}^{n} Q_i - 2K = 0$$

$$\therefore \sum_{i=1}^{n} Q_i = 2K \quad \ldots \quad 3.50$$

Above results determines the optimal turnover when maximum limit on inventories is imposed.

Above equations are solved by successive approximation with the help of trial and error method.

**3.3.12. Hypothetical Problem:-**

Let us take the example given in (3.3.6) in which values of \( x_i, \beta \) are as bellow:

<table>
<thead>
<tr>
<th>Item</th>
<th>( x_i )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.9</td>
<td>20</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
<td>55</td>
</tr>
<tr>
<td>3</td>
<td>0.7</td>
<td>125</td>
</tr>
</tbody>
</table>

(i) Maximum limit on amount of average inventories = Rs. 1500

(ii) Maximum limit on inventory \( K = 175 \) Units
By using the successive approximation, the value of lagragian multiplier $\lambda$ is obtained as 2.34, such that \( \frac{1}{2} \sum_{i=1}^{n} Q_i \leq K \).

<table>
<thead>
<tr>
<th>Item</th>
<th>Demand (Units) $D_i$</th>
<th>Holding Cost (Rs.) $C_{1i}$</th>
<th>Setup Cost (Rs.) $C_{3i}$</th>
<th>Purchase Cost (Rs.) $P_i$</th>
<th>Optimal Turnover $Q_{opt}$</th>
<th>Total Cost (Rs.) $T(Q_i)$</th>
<th>ITOR I(Q_i)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>732</td>
<td>20</td>
<td>61</td>
<td>5</td>
<td>55</td>
<td>1357</td>
<td>2.69</td>
</tr>
<tr>
<td>2</td>
<td>2962</td>
<td>20</td>
<td>51</td>
<td>6</td>
<td>146</td>
<td>2503</td>
<td>7.10</td>
</tr>
<tr>
<td>3</td>
<td>4116</td>
<td>20</td>
<td>41</td>
<td>7</td>
<td>147</td>
<td>2632</td>
<td>10.95</td>
</tr>
</tbody>
</table>

3.3.13. Remark:-

The above result indicates that with increases in the value of $x_1$ and $\beta$ the size of inventory turnover also increases. Also, we observed increasing trend in ITOR. Therefore, this proves that the firm has good inventory management system. Here perfect allocation of resources is possible only through computer program which is mentioned in appendix II.

3.3.14. Case IV: $D = \beta_1 Q - \beta_2 Q^2$, $C_{3i}=C_{3i}+b_i Q_i$

$\beta_1, \beta_2$ are constants and $Q$ is contract quantity.

The Inventory turnover ratio is defined as:

\[
I(Q_i) = \frac{D_i P_i}{\sum_{i=1}^{n} \left[ \frac{C_{1i} Q_i}{2} + (C_{3i} + b_i Q_i) D_i \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right]^2} \\
= \left[ \beta_1 Q_i P_i - \beta_2 Q_i^2 P_i \right] \left[ \sum_{i=1}^{n} \left[ \frac{C_{1i} Q_i}{2} + \frac{(C_{3i} + b_i Q_i)(\beta_1 Q_i - \beta_2 Q_i^2)}{Q_i} \right] + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right] \right]^{-1}
\]

Differentiate equation (3.51) with respect to $Q_i$ and equate it to zero, we get,
\[ \frac{\partial t(Q_0)}{\partial Q_i} = [\beta_i Q_i - 2\beta_2 Q_i P_i] \left\{ \sum_{i=1}^{n} \left[ \frac{C_i Q_i}{2} + \frac{(C_y + b Q_i) \beta_i Q_i}{Q_i} \right] \right\} + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right]^{-1} \\
- \left\{ \beta_i Q_i P_i - \beta_2 Q_i^2 P_i \right\} \left\{ \sum_{i=1}^{n} \left[ \frac{C_i Q_i}{2} + \frac{(C_y + b Q_i) \beta_i Q_i}{Q_i} \right] \right\} + \lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right]^{-2} \\
\left\{ \frac{C_i}{2} - C_i \beta_2 + b \beta_1 - 2b \beta 2 Q_i + \lambda \right\} = 0 \\
\text{…… 3.52} \\
\]

For \( i^{th} \) item, we have,

\[ (\beta_i - 2\beta_2 Q_i) \left[ \frac{C_i Q_i}{2} + C_i \beta_1 + b \beta_1 Q_i - C_i \beta_2 Q_i - b \beta_2 Q_i^2 + \lambda Q_i - 2\lambda K \right] - \\
(\beta_i Q_i - \beta_2 Q_i^2) \left[ \frac{C_i}{2} + b \beta_1 - C_i \beta_2 - 2b \beta_2 Q_i + \lambda \right] = 0 \\
\therefore (2C_i \beta_2^3 - C_i \beta_2 - 2\lambda \beta_2)^2 + 4\beta_2 (2\lambda K - \beta_1 C_i) Q_i + (2C_i \beta_1^2 - 4\lambda \beta_1 K) = 0 \\
\text{…… 3.53} \\
\]

Which implies that,

\[ Q_i = \left\{ - (4\beta_2 \lambda K - 2\beta_1 \beta_2 C_i) \pm \sqrt{16\beta_2^2 \lambda^2 2^2 - 4\beta_2 \lambda^2 \beta_1 C_i + \\
2\beta_1^2 \beta_2^3 C_i C_3 - 8\beta_1 \beta_2^2 C_i \lambda K - 8\beta_2 \beta_1^2 \lambda^2 K - 4\lambda \beta_2 \beta_1 C_i \} \right\} / \\
\left\{ 2\beta_2^2 C_i - 2\lambda - \beta_2 C_i \right\} \\
\text{…… 3.54} \\
\]

Where \( i = 1, 2, 3, \ldots, n \)

and

\[ \frac{\partial t(Q_0)}{\partial \lambda} = 0 - [\beta_i Q_i P_i - \beta_2 Q_i^3 P_i] \left\{ \sum_{i=1}^{n} \left[ \frac{C_i Q_i}{2} + \frac{(C_y + b Q_i) \beta_i Q_i}{Q_i} \right] \right\} + \\
\lambda \left[ \sum_{i=1}^{n} Q_i - 2K \right]^{-2} \left[ 0 + \sum_{i=1}^{n} Q_i - 2K \right] = 0 \\
\text{…… 3.55} \\
\therefore \sum_{i=1}^{n} Q_i - 2K = 0 \\
\therefore \sum_{i=1}^{n} Q_i = 2K \\
\text{…… 3.56} \\
\]
Above results determines the optimal turnover when maximum limit on inventories is imposed.

Above equations are solved by successive approximation with the help of trial and error method.

3.3.15. Hypothetical Problem:-

Let us take the example given in (3.3.6) in which value of $\beta_1$ and $\beta_2$ are as below:

<table>
<thead>
<tr>
<th>Item</th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>0.3</td>
</tr>
<tr>
<td>2</td>
<td>70</td>
<td>0.2</td>
</tr>
<tr>
<td>3</td>
<td>75</td>
<td>0.1</td>
</tr>
</tbody>
</table>

(i) Maximum limit on amount of average inventories = Rs. 1500

(ii) Maximum limit on inventory $K = 175$ Units

By using the successive approximation, the value of lagragian multiplier $\lambda$ is obtained as 2.34, such that $\frac{1}{2} \sum Q_i \leq K$.

<table>
<thead>
<tr>
<th>Item</th>
<th>Demand (Units) $D_i$</th>
<th>Holding Cost (Rs.) $C_1$</th>
<th>Setup Cost (Rs.) $C_2$</th>
<th>Purchase Cost (Rs.) $P_i$</th>
<th>Optimal Turnover $Q_{opt}$</th>
<th>Total Cost (Rs.) $T(Q_i)$</th>
<th>ITOR $I(Q_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1125</td>
<td>20</td>
<td>60</td>
<td>5</td>
<td>36</td>
<td>2230</td>
<td>2.52</td>
</tr>
<tr>
<td>2</td>
<td>3641</td>
<td>20</td>
<td>51</td>
<td>6</td>
<td>79</td>
<td>3124</td>
<td>6.99</td>
</tr>
<tr>
<td>3</td>
<td>17200</td>
<td>20</td>
<td>42</td>
<td>7</td>
<td>234</td>
<td>5451</td>
<td>22.09</td>
</tr>
</tbody>
</table>

3.3.16. Remark:-

Here we observed that with increases in the value of $\beta_1$ and decreases in the value of $\beta_2$ the optimum turnovers achieved with increasing trend in demand. Also, ITOR is in increasing order which indicates the good performance of the firm with reduction in total inventory cost. Here perfect allocation of resources is possible only through computer program which is mentioned in appendix II.
3.4. Conclusion:-

The inventory turnover consists of many items; it is not possible to consider each one of them separately. Thus, by imposing the limit on the maximum amount that can be invested on inventories, the turnover ratio for the firm is maximized by minimizing the risk of stock outs.

In contrast to this, under the traditional inventory control there is not a single amount left, from the limit imposed on inventories. Thus, the costs incurred because of inventories are more as compared to that ITOR based system.

In this type of situation problem is efficiently solved by only with the help of computer program.