CHAPTER 9

INVENTORY TURNOVER RATIO MODELS UNDER INFLATIONARY SITUATION
9. CHAPTER 9: INVENTORY TURN OVER RATIO MODELS UNDER INFLATIONARY SITUATION.

9.1. Introduction:-

In this chapter, an attempt is made to determine the optimum lot size for replenishment, such that the supplier's inventory turn over ratio becomes the maximum. The different inventory costs consider under this situation. Main objective of this chapter is to find out the behaviour of ITOR under inflationary situations so firm handles the inventory efficiently.

Since its inception, the concept of an economic order quantity (EOQ) has become deeply rooted in the theory and practice of inventory management. The EOQ models that have appeared in the literature can be divided logically in two groups. One group of inventory models assumes that the objective of inventory management is to minimize average annual cost. The other group assumes that the appropriate objective is to minimize the discounted value of all future costs. Hadley compared optimal order quantities determined by minimizing annual cost and discounted costs for wide ranges of parameter values and concluded “there is only a negligible if any difference in the order quantities”. Hadley’s work is important because it justifies the use of less complicated models.

A generalization of Hadley’s work can be carried out by permitting a non-zero inflation rate. Thus to account for an inflationary situation that is actually existing, a constant inflation rate $i$ and nominal interest rate $r$ can be considered, so that the cost associated with the inventory model can be taken accordingly. On the basis of the above structure the annual cost model of John J. Kanet and James A. Miles can be considered.
9.1.1. **Inventory Cost Under Inflationary Situation:**

Let us consider annual cost model in which the objective is to minimize total annual inventory cost, denoted as $F(q)$, where $q$ is the quantity per order, specifically.

$$F(q) = \text{Purchase Cost} + \text{Order Cost} + \text{Storage Cost}$$

Let us define the following terms:

- $C_0$ = Cost per item at time Zero
- $C_3$ = Setup cost (or total fixed cost) per order as at time zero
- $i$ = inflation rate per year
- $R$ = quantity demanded per year
- $r$ = nominal annual interest rate
- $C_1$ = Storage cost per rupee of inventory per year exclusive of the opportunity cost for lost interest income.

If only one order is placed for the entire year, then all units will be purchased now at Rs. $C_0$ each. If two orders are placed for the entire year, half of the units will be purchased now at a cost of Rs. $C_0$ and the other half will be purchased in the middle of the year at an "inflated" Cost of Rs. $\left(1+\frac{i}{2}\right)C_0$. For the case of two orders, average per unit purchase cost is $\left(1+\frac{i}{4}\right)C_0$. As the number of orders are increased, average purchase cost approaches $\left(1+\frac{i}{2}\right)C_0$. In general, average purchase cost is $\left[1+i\left(1-\frac{q}{R}\right)/2\right]C_0$.

Similarly, the average setup cost for the year is $\left[1+i\left(1-\frac{q}{R}\right)/2\right]C_3$. The annual purchase cost is average per unit cost times quantity purchased. So that annual purchase cost is $\left[1+i\left(1-\frac{q}{R}\right)/2\right]C_0R$. Order cost for the year is simply average set up cost per order times number of order. Therefore order cost is
The holding cost is average rupee value of inventory times the holding cost per rupee per year. Therefore annual holding/storage cost is defined as

\[ 1+i \left(1 - \frac{q}{R}\right)/2 \]

\[ C_0 q \left( r + C_t \right)/2 \]

\[ F(q) = \left[1+i \left(1 - \frac{q}{R}\right)/2\right] C_0 q \left( r + C_t \right) / 2 \]

Thus the total inventory cost for the relevant model will be changed accordingly as shown in the above derivation.

In this chapter, Inventory turnover ratio model under inflationary situation, the above cost structure is taken in the relevant models and then the ITOR maximisation approach is considered.

9.2. **Model 1: Inventory Turnover Ratio Model for Inflationary Situation under EOQ Approach.**

9.2.1. **Introduction:**

This model relates to a warehouse inventory system from which the units are sold in the market depending upon their market requirements. The different inventories costs are subjected to inflation and accordingly the rate of inflation and the nominal rate of interest for actual investment would be more effective. The problem is to determine the optimum lot size for maximising the ITOR with respect to annual cost model of John J. Kanet and James A. Miles is considered.

9.2.2. **Assumptions and Notations:**

(i) Lots of size q are replenished for each replenishment

(ii) Demand rate is R units/year

(iii) Shortages are not allowed

(iv) Lead time is zero

(v) Inflation rate is i Rs./year
(vi) Nominal interest rate is \( r \) Rs/year \((r>i)\)

(vii) Initially the purchase cost per item is \( C_0 \) Rs/unit/year

(viii) Unit inventory holding cost is \( C_1 \) Rs/unit/year

(ix) The setup cost per order at time zero is \( S_0 \) Rs/order.

(x) Damaged and deteriorating of the items are not allowed

(xi) Advertisement cost is included in market price \( p \) per unit of the commodity.

9.2.3. Problem Formulation:

Purchase Cost

\[
\text{Purchase Cost} = \left(1 + \frac{i}{2}\left[1 - \frac{q}{R}\right]\right)C_0R
\]

Order Cost

\[
\text{Order Cost} = \left(1 + \frac{i}{2}\left[1 - \frac{q}{R}\right]\right)\frac{S_0R}{q}
\]

Storage Cost

\[
\text{Storage Cost} = \left(1 + \frac{i}{2}\left[1 - \frac{q}{R}\right]\right)\frac{C_0q(r+C_1)}{2}
\]

Total Inventory Cost = (Purchase Cost) + (Order Cost) + (Storage Cost)

\[
= \left(1 + \frac{i}{2}\left[1 - \frac{q}{R}\right]\right)C_0R + \left(1 + \frac{i}{2}\left[1 - \frac{q}{R}\right]\right)\frac{S_0R}{q} + \left(1 + \frac{i}{2}\left[1 - \frac{q}{R}\right]\right)\frac{C_0q(r+C_1)}{2}
\]

\[
= \left(1 + \frac{i}{2}\left[1 - \frac{q}{R}\right]\right)\left[C_0R + \frac{S_0R}{q} + \frac{C_0q(r+C_1)}{2}\right]
\]

ITOR is defined under this situation as,

\[
\therefore I(q) = \frac{pR}{\left\{1 + \frac{i}{2}\left[1 - \frac{q}{R}\right]\right\}\left[C_0R + \frac{S_0R}{q} + \frac{C_0q(r+C_1)}{2}\right]^{-1}}
\]

The necessary and sufficient conditions for maximisation of ITOR are

\[
(i) \frac{\partial I(q)}{\partial q} = 0 \quad (ii) \frac{\partial^2 I(q)}{\partial q^2} < 0
\]

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\[
\frac{\partial I(q)}{\partial q} = -pR\left\{\left[1 + \frac{i}{2}\left(1 - \frac{q}{R}\right)\right]\left[C_0 R + S_0 R + \frac{C_0 q(r + C_1)}{2}\right]\right\}^2 \\
\left\{0 + i\left(0 - \frac{1}{R}\right)\right\}\left[C_0 R + \frac{S_0 R}{q} + \frac{C_0 q(r + C_1)}{2}\right] + \left[1 + i\left(1 - \frac{q}{R}\right)\right]\left[0 - \frac{S_0 R}{q^2} + \frac{C_0 q(r + C_1)}{2}\right]
\]

\[
= -pR\left\{\left[1 + \frac{i}{2}\left(1 - \frac{q}{R}\right)\right]\left[C_0 R + \frac{S_0 R}{q} + \frac{C_0 q(r + C_1)}{2}\right]\right\}^2 \\
-i \frac{2R}{C_0 R + \frac{S_0 R}{q} + \frac{C_0 q(r + C_1)}{2}} \left\{\frac{i}{2R}C_0 R + \frac{S_0 R}{q} + \frac{C_0 q(r + C_1)}{2}\right\} + \left[1 + i\left(1 - \frac{q}{R}\right)\right]\left[-\frac{S_0 R}{q^2} + \frac{C_0 q(r + C_1)}{2}\right]
\]

and \(\frac{\partial^2 I(q)}{\partial q^2} < 0\)

\[
\frac{\partial I(q)}{\partial q} = 0 \text{ implies that}
\]

\[
-\frac{pR}{2R}\left[1 + \frac{i}{2}\left(1 - \frac{q}{R}\right)\right]\left[C_0 R + \frac{S_0 R}{q} + \frac{C_0 q(r + C_1)}{2}\right] + \left[1 + i\left(1 - \frac{q}{R}\right)\right]\left[-\frac{S_0 R}{q^2} + \frac{C_0 q(r + C_1)}{2}\right] = 0 \quad \text{..... 9.6}
\]

\[
\therefore \frac{i}{2R}\left[C_0 R + \frac{S_0 R}{q} + \frac{C_0 q(r + C_1)}{2}\right] + \left[1 + i\left(1 - \frac{q}{R}\right)\right]\left[-\frac{S_0 R}{q^2} + \frac{C_0 q(r + C_1)}{2}\right] = 0
\]

\[
\therefore \frac{i}{2R}\left[2C_0 Rq + 2S_0 R + C_0 q^2(r + C_1)\right] + \left[2R + i(R - q)\right]\left[-2S_0 R + C_0 q^2(r + C_1)\right] = 0
\]

\[
\therefore -iq\left[2C_0 Rq + 2S_0 R + C_0 q^2(r + C_1)\right] + \left[2R + i(R - q)\right]\left[-2S_0 R + C_0 q^2(r + C_1)\right] = 0
\]

\[
\therefore -iq\left[2S_0 R + C_0 q^2(r + C_1)\right] - 2q^2 C_0 R + \left[2R + iR\right]\left[-2S_0 R + C_0 q^2(r + C_1)\right] = 0
\]

\[
-iq\left[-2S_0 R + C_0 q^2(r + C_1)\right] = 0
\]
\[ -2iqS_0 R - iq^3 C_0 (r + C_1) - 2i^2 C_0 R - 4R^2 S_0 - 2iR^2 S_0 + 2RC_0 q^3 (r + C_1) + iRC_0 q^2 (r + C_1) + 2iqS_0 R - iC_0 q^3 (r + C_1) = 0 \]

\[ -2iq^3 C_0 (r + C_1) - q^2 RC_0 [2i - 2(r + C_1) - i(r + C_1)] - 2R^2 S_0 [2 + i] = 0 \]

\[ 2iC_0 (r + C_1) q^3 + RC_0 [2i - 2(r + C_1) - i(r + C_1)] q^2 + 2R^2 S_0 [2 + i] = 0 \]

\[ 2iC_0 (r + C_1) q^3 + RC_0 [2i - (r + C_1) (2 + i)] q^2 + 2R^2 S_0 [2 + i] = 0 \]

\[ iC_0 (r + C_1) q^3 + \frac{RC_0}{2} [2i - (r + C_1) (2 + i)] q^2 + R^2 S_0 [2 + i] = 0 \]

\[ iC_0 (r + C_1) q^3 + RC_0 \left[ i - \frac{(r + C_1) (2 + i)}{2} \right] q^2 + R^2 S_0 [2 + i] = 0 \]  

Value of \( q_{\text{opt}} \) is obtained by successive approximation method.

### 9.2.4. Hypothetical Problem:

To explain the problem formulation of model 1 of 9.2 let us take numerical example as bellow.

\[ C_0 = 1, r = 0.1 \text{ to } 0.12, C_1 = 0.05, S_0 = 0.1, i = 0.05 \text{ to } 0.10, \]

\[ R = 10000, p = \text{Rs. } 10. \]

Solution of the hypothetical problem is,

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<th>Demand (Units)</th>
<th>Optimal Total Inventory Turnover</th>
<th>Total Inventory Turnover</th>
<th>Inventory Cost Ratio</th>
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Rate of Interest = 0.11

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<th>R (Units)</th>
<th>Turnover</th>
<th>Cost Ratio</th>
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Rate of Interest = 0.12

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9.2.5. Remark:-

From the solution given above, we observe that (i) for fixed interest rate and varying inflation rate optimal turnover and total cost are increasing whereas ITOR decreasing resp. (ii) for fixed inflation rate and varying interest rate optimal turnover and ITOR decreasing whereas total cost increasing resp. Therefore management of the firm can be taken proper decision with respect to total cost and ITOR about inventories.

9.3. Model 2: Inventory Turnover Ratio Model under Inflationary Situations and Markup Variation

9.3.1. Introduction:-

Here, an attempt is made to determine the optimum lot size for replenishment, such that the supplier ITOR becomes the maximum. The model discussed here considers the quantity discount cases subjected to the
stipulated markup prices. Various cost of inventory is considered with respect to inflation. The cost of damaged goods and advertising are considered in total inventory control cost. Two particular cost functions for quantity discount are explained by hypothetical problem.

9.3.2. Assumptions and Notations:-

(i) In replenishment, lots of size q are replenished
(ii) Demand rate is R units/year and is a function of the unit selling price p Rs.
(iii) Shortages are not allowed
(iv) Lead time is zero
(v) Inflation rate is i Rs./year
(vi) Interest rate is r Rs/year; (r>1)
(vii) Initial purchase cost per item is C₀ Rs per unit per year
(viii) Inventory holding cost is C₁ Rs per unit per year
(ix) Setup cost per order at time zero is S₀ Rs per order.
(x) Advertisement cost αpR Rupees per year; α = Constant
(xi) Damaged cost is γC(q)R Rupees per year; γ = lot fraction defective and C(q) = Unit cost function.

9.3.3. Formulation of the Model:-

This problem relates with the warehouse inventory system from which the units are sold in the market depending upon their market requirements. The different inventory costs are subjected to inflation and accordingly the rate of inflation and the nominal rate of interest for actual investment would be more effective. The cost of damaged goods and advertising cost are to be considered with the routine inventory costs of EOQ. The problem is to determine the optimum lot size for maximising the ITOR.

Under the assumption given above, the model is derived as under.
Total Inventory Cost \( F(q) = \left\{1 + \frac{i}{2}\left[1 - \frac{q}{R}\right]\right\} \left[C_0R + \frac{S_0R}{q} + \frac{C_0q(r + C_1)}{2}\right] + \alpha pR + \gamma C(q)R \]

\[ \text{It is further assumed that market price per unit } p = \theta C(q) \text{ where } C(q) \text{ is the unit function and } \theta = \text{Markup parameter } (\theta > 1). \]

\[ \therefore \quad F(q) = \left\{1 + \frac{i}{2}\left[1 - \frac{q}{R}\right]\right\} \left[C_0R + \frac{S_0R}{q} + \frac{C_0q(r + C_1)}{2}\right] + \alpha pR + \gamma C(q)R \]

\[ \text{In the above total inventory cost, let us consider here a specific demand function given by Kotler as: } R = \frac{Kf}{p^n} \]

Where \( K = \text{Constant}; \ f = \text{Frequency of advertisement} \text{ and } n = \text{Elasticity of demand } \ (n > 0) \)

\[ \therefore \quad R = \frac{Kf}{\theta^n[C(q)]^n} \]

\[ \text{and for } n = 1, \therefore \quad R = \frac{Kf}{\theta[C(q)]} \]

\[ \therefore \quad F(q) = \left\{1 + \frac{i}{2}\left[1 - \frac{q\theta C(q)}{Kf}\right]\right\} \left[KfC_0 + \frac{KfS_0}{\theta C(q)} + \frac{C_0q(r + C_1)}{2}\right] + (\alpha \theta + \gamma)C(q) \frac{Kf}{\theta C(q)} \]

\[ \text{Now, Inventory turnover ratio is defined as: } \]

\[ I(q) = \frac{R[p - C(q)]}{F(q)} \]

\[ \text{9.3.4. Case-I: Linear Quantity Discount} \]

The unit cost function \( C(q) \) assume the linear quantity discount as

\[ C(q) = a - bq \]

\[ \therefore \quad p = \theta C(q) \]

\[ = \theta(a - bq) \]
\[
R[p - C(q)] = \frac{Kf}{\theta(a - bq)} \theta(a - bq) - (a - bq)\]
\[
= \frac{Kf}{\theta(a - bq)} (a - bq) [\theta - 1]\nonumber
= \frac{Kf (\theta - 1)}{\theta} \quad \ldots \quad 9.14
\]
\[
F(q) = \left( 1 + \frac{i}{2} \right) - \frac{iq \theta(a - bq)}{2Kf} \left[ \frac{C_0 Kf}{\theta(a - bq)} + \frac{S_0 Kf}{\theta(a - bq)q} + \frac{C_0 q (r + C_1)}{2} \right] + \frac{(\alpha \theta + \gamma) Kf}{\theta} \nonumber
\]
\[
= \left( 1 + \frac{i}{2} \right) \left[ \frac{C_0 Kf}{\theta(a - bq)} + \frac{S_0 Kf}{\theta(a - bq)q} + \frac{C_0 q (r + C_1)}{2} \right] - \nonumber
\]
\[
\frac{iq \theta(a - bq)}{2Kf} \left[ \frac{C_0 Kf}{\theta(a - bq)} + \frac{S_0 Kf}{\theta(a - bq)q} + \frac{C_0 q (r + C_1)}{2} \right] + \frac{(\alpha \theta + \gamma) Kf}{\theta} \nonumber
\]
\[
= \left[ 1 + \frac{i}{2} \right] C_0 (r + C_1) - i C_0 \left[ \frac{1}{2} \left( \frac{1}{\theta} \right) \frac{Kf S_0}{Kf} \right] - \nonumber
\]
\[
\left[ i C_0 \theta (r + C_1) \right] \left[ \frac{1}{4Kf} \right] (a q^2 - b q^2) + \left[ \frac{1 + i}{2} \right] \left( \frac{Kf C_0}{\theta} \right) \frac{1}{(a - bq)} - \nonumber
\]
\[
\frac{i S_0}{2} + \frac{(\alpha \theta + \gamma) Kf}{\theta} \quad \ldots \quad 9.15\nonumber
\]

Let us take \( A = \left[ 1 + \frac{i}{2} \right] C_0 (r + C_1) - i C_0 \right] / 2 \)
\[
B = \left[ \frac{1 + i}{2} \frac{Kf S_0}{\theta} \right] \]
\[ C = \left[ \frac{iC_0 \theta (r + C)}{4Kf} \right] \]

and
\[ D = \left( \frac{1 + i}{2} \right) \frac{KfC_0}{\theta} \]

Now from the result 9.15, we have,
\[ F(q) = Aq + B(aq - bq^2)^i - C(aq^2 - bq^3) + D(a - bq)^i - \frac{iS_0}{2} + \frac{(\alpha \theta + \gamma)Kf}{\theta} \]

From 9.14 and 9.16 ITOR is defined as:
\[ I(q) = \left[ \frac{Kf(\theta - 1)}{\theta} \right] F(q) \]

\[ I(q) = \left[ \frac{Kf(\theta - 1)}{\theta} \right] \left[ Aq + B(aq - bq^2)^i + D(a - bq)^i - \frac{iS_0}{2} + \frac{(\alpha \theta + \gamma)Kf}{\theta} \right] \]

\[ C(aq^2 - bq^3) \]

Necessary and sufficient condition to maximise ITOR are (i) \( \frac{\partial I(q)}{\partial q} = 0 \),

(ii) \( \frac{\partial^2 I(q)}{\partial q^2} < 0 \)

\[ \therefore \frac{\partial I(q)}{\partial q} = \left[ \frac{Kf(\theta - 1)}{\theta} \right] \frac{\partial}{\partial q} \left[ Aq + B(aq - bq^2)^i - C(aq^2 - bq^3) + D(a - bq)^i - \frac{iS_0}{2} \right. \]

\[ + \left. \left( \frac{2\theta + \gamma}{\theta} \right) Kf \right] \] \[ + \left. \left[ Aq + B(aq - bq^2)^i + D(a - bq)^i - \frac{iS_0}{2} + \left( \frac{2\theta + \gamma}{\theta} \right) Kf \right]^{-1} \]

\[ \frac{\partial}{\partial q} \left[ \frac{Kf(\theta - 1)}{\theta} \right] = \left[ \frac{Kf(\theta - 1)}{\theta} \right] \left[ Aq + B(aq - bq^2)^i - C(aq^2 - bq^3) + D(a - bq)^i - \frac{iS_0}{2} + \left( \frac{2\theta + \gamma}{\theta} \right) Kf \right]^{-2} \]

\[ A - B(aq - bq^2)^2(a - 2bq) - C(2aq - 3bq^2) - D(a - bq)^2(-b) - 0 + 0 \]

Therefore, from result (9.18), we have,
\[ A - \frac{B}{q^2(a-bq)}(a-2bq) - C(2aq-3bq^2) + \frac{bD}{(a-bq)^2} = 0 \]
\[ Aq^2(a-bq)^3 - B(a-2bq) - C(2aq-3bq^2)q^2(a-bq)^2 + bDq^2 = 0 \]
\[ Aq^2(a^2-2abq+b^2q^2) - aA + 2bBq - \]
\[ C(2aq^3-3bq^4)(a^2-2abq+b^2q^2) + bDq^2 = 0 \]
\[ a^2Aq^2 - 2abAq^3 + b^2Aq^4 - Ba + 2bBq - \]
\[ C[2a^3q^3 - 4a^2bq^4 + 2ab^2q^5 - 3a^2bq^4 + 6ab^2q^5 - 3b^3q^6] + bDq^2 = 0 \]
\[ 3b^3Cq^6 - 8ab^2Cq^5 + (7a^2bC + b^2A)q^4 - (2abA + 2a^3C)q^3 \]
\[ + [a^2A + bD]q^2 + 2bBq - Ba = 0 \]

Put the value of A, B, C and D we have,

\[ \left[ \frac{3b^3iC_0\theta(r + C_1)}{4Kf} \right]q^6 - \left[ \frac{8ab^2iC_0\theta(r + C_1)}{4Kf} \right]q^5 + \]
\[ \left\{ \frac{b^2}{2} \left[ \left( 1 + \frac{i}{2} \right) C_0(r + C_1) - iC_0 \right] + \left[ \frac{7a^2biC_0(r + C_1)}{4Kf} \right] \right\}q^4 - \]
\[ \left\{ \frac{2ab}{2} \left[ \left( 1 + \frac{i}{2} \right) C_0(r + C_1) - iC_0 \right] + \left[ \frac{2a^2iC_0\theta(r + C_1)}{4Kf} \right] \right\}q^3 + \]
\[ \left\{ \frac{a}{2} \left[ \left( 1 + \frac{i}{2} \right) C_0(r + C_1) - iC_0 \right] + \left[ \frac{\left( 1 + \frac{i}{2} \right) KfC_0}{\theta} \right] \right\}q^2 - \]
\[ \left\{ -2b \left( 1 + \frac{i}{2} \right) KfS_0 \right\}q + \left\{ -a \left( 1 + \frac{i}{2} \right) KfS_0 \right\} = 0 \]

Let us take,

\[ \beta_0 = \left[ \frac{3b^3iC_0\theta(r + C_1)}{4Kf} \right] \]
\[ \beta_1 = \left[ \frac{8ab^2iC_0\theta(r + C_1)}{4Kf} \right] \]

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\[ \beta_2 = \frac{b^2}{2} \left[ \left(1 + \frac{i}{2}\right) c_0 (r + c_1) - i c_0 \right] + \frac{7a^3 b c_0 (r + c_1)}{4Kf} \]

\[ \beta_3 = ab \left[ \left(1 + \frac{i}{2}\right) c_0 (r + c_1) - i c_0 \right] + \frac{a^3 c_0 \theta (r + c_1)}{2Kf} \]

\[ \beta_4 = \frac{a^2}{2} \left[ \left(1 + \frac{i}{2}\right) c_0 (r + c_1) - i c_0 \right] + \left(1 + \frac{i}{2}\right) K f c_0 \]

\[ \beta_5 = - \frac{2b (1 + \frac{i}{2}) K f S_0}{\theta} \]

and \( \beta_6 = - \frac{a (1 + \frac{i}{2}) K f S_0}{\theta} \)

\[ \therefore \beta_6 q^6 - \beta_5 q^5 + \beta_4 q^4 + \beta_3 q^3 - \beta_2 q^2 - \beta_1 q + \beta_0 = 0 \]

\[ \therefore \sum_{j=0}^{6} (-1)^j \beta_j q^{6-j} = 0 \]

\( q_{\text{opt}} \) is obtained by solving the result 9.20 by successive approximation method.

9.3.5. Hypothetical Problem:-

To explain the case-I of model 2 of 9.3 let us take numerical example as below.

\( C_0 = 1, r = 0.1, C_1 = 0.05, S_0 = 0.1, i = 0.05, K = 10, f = 50, a = 50, \)

\( b = 0.0001, \alpha = 0.1, \gamma = 0.2 \) and mark up parameter \( \theta = 2, 3, 4, 5. \)

Solution of the hypothetical problem is,

<table>
<thead>
<tr>
<th>Mark up</th>
<th>Optimal Total Inventory</th>
<th>Parameter Turnover Inventory Turnover</th>
<th>theta (Units)</th>
<th>Cost</th>
<th>Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qopt</td>
<td>C(Qopt)</td>
<td>I(Qopt)</td>
<td>Rs.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.00</td>
<td>69</td>
<td>106.92</td>
<td>2.3381</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.00</td>
<td>46</td>
<td>87.95</td>
<td>3.7900</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.00</td>
<td>34</td>
<td>78.46</td>
<td>4.7792</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.00</td>
<td>28</td>
<td>72.77</td>
<td>5.4966</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Chapter 9 : Model 2 . Case - I

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9.3.6. Remark:-

From the optimal solution given above and situation stated in the model may be conclude that with the increases in the value of mark up parameter $\theta$ the optimal value of ITOR also increases and the optimum lot size decreases. Advertising, rate of inflation and damaged goods are highly sensitive with respect to total cost and such that ITOR.

9.3.7. Case-li: Hyperbolic Quantity Discount

Let us consider the hyperbolic quantity discount as:

$C(q) = b + \frac{d}{q}$

Where, $b$ and $d$ are constants.

From 9.11 and 9.13, inventory turnover ratio is defined as:

\[
I(q) = \frac{\left[ \frac{Kf}{\theta} (\theta - 1) \right]}{F(q)} \]

\[
= \left[ \frac{Kf}{\theta} (\theta - 1) \right] \frac{1}{\left[ 1 + \frac{i}{2} \left( 1 + \frac{\theta C(q)}{Kf} \right) \right]} \left[ \frac{Kf C_0}{\theta C(q)} + \frac{Kf S_0}{2} + \frac{C_0 q (r + C)}{2} \right] + \frac{(\alpha \theta + \gamma) Kf}{\theta}
\]

\[
= \left[ \frac{Kf}{\theta} (\theta - 1) \right] \frac{1}{\left[ 1 + \frac{i}{2} \left( 1 + \frac{\theta (bq + d)}{q Kf} \right) \right]} \left[ \frac{Kf C_0 q}{\theta (bq + d)} + \frac{Kf S_0 q}{\theta (bq + d)} + \frac{C_0 q (r + C)}{2} \right] + \frac{(\alpha \theta + \gamma) Kf}{\theta}
\]

Necessary and sufficient condition to maximise ITOR are (i) $\frac{\partial I(q)}{\partial q} = 0$

(ii) $\frac{\partial^2 I(q)}{\partial q^2} < 0$

\[
\frac{\partial I(q)}{\partial q} = \left[ \frac{Kf}{\theta} (\theta - 1) \right] \frac{\partial}{\partial q} \left[ 1 + \frac{i}{2} \left( 1 + \frac{\theta (bq + d)}{Kf} \right) \right] \left[ \frac{Kf C_0 q}{\theta (bq + d)} + \frac{Kf S_0 q}{\theta (bq + d)} + \frac{C_0 q (r + C)}{2} \right] + \frac{(\alpha \theta + \gamma) Kf}{\theta}
\]

\[
\frac{\partial^2 I(q)}{\partial q^2} < 0
\]

...... 9.22
\[ 
\left[ \frac{Kf(\theta - 1)}{\theta} \right] \left[ \frac{1 + \frac{i}{2} \left( 1 - \frac{\theta(bq + d)}{Kf} \right) KfC_0 q}{\theta(bq + d)} + \frac{KfS_0}{\theta(bq + d)} + \frac{C_0 q(r + C_1)}{2} \right] + \frac{\theta(bq + d)}{C_0 q(r + C_1)} \right]^{-2} \cdot \\
\frac{\partial}{\partial q} \left[ \left[ 1 + \frac{i}{2} \left( \frac{1 - \theta(bq + d)}{Kf} \right) KfC_0 q \right] \frac{KfS_0}{\theta(bq + d)} + \frac{C_0 q(r + C_1)}{2} \right] = 0 \\
\frac{\partial^2 I(q)}{\partial q^2} < 0 \text{ Satisfied such that } \frac{\partial I(q)}{\partial q} = 0 \\
\therefore \frac{\partial I(q)}{\partial q} = \frac{\partial}{\partial q} \left[ \left[ 1 + \frac{i}{2} \left( \frac{1 - \theta(bq + d)}{Kf} \right) KfC_0 q \right] \frac{KfS_0}{\theta(bq + d)} + \frac{C_0 q(r + C_1)}{2} \right] = 0 \\
\text{From 9.23} \\
g(q) = \left[ 1 + \frac{i}{2} \left( \frac{1 - \theta(bq + d)}{Kf} \right) KfC_0 q \right] \frac{KfS_0}{\theta(bq + d)} + \frac{C_0 q(r + C_1)}{2} \\
= \left[ \left( 1 + \frac{i}{2} \right) \frac{KfC_0}{\theta} \left( \frac{q}{bq + d} \right) \right] \frac{KfS_0}{\theta(bq + d)} + \frac{C_0 q(r + C_1)}{2} \\
= \left[ \left( 1 + \frac{i}{2} \right) \frac{KfC_0 q}{\theta(bq + d)} \right] \left[ \frac{1}{bq + d} \right] \left[ \frac{1 + \frac{i}{2} C_0 q(r + C_1)}{2} \right] + \frac{i \theta(bq + d)}{2 Kf} \times \frac{KfC_0 q}{\theta(bq + d)} + \frac{i \theta(bq + d)}{2 Kf} \times \frac{KfS_0}{\theta(bq + d)} + \frac{i \theta(bq + d)}{2 Kf} \times \frac{C_0 q(r + C_1)}{2} \\
= \left[ \left( 1 + \frac{i}{2} \right) \frac{C_0 q}{\theta} \left( \frac{q}{bq + d} \right) \right] \left[ \frac{1}{bq + d} \right] \left[ \frac{1 + \frac{i}{2} S_0 Kf}{\theta} \right] + \frac{1 + \frac{i}{2} C_0 q(r + C_1)}{2} \\
= \left[ \frac{i C_0 q}{2} \right] \left[ \frac{i S_0 q(r + C_1)}{4 Kf} \right] (bq + d) q \\
\]
\[
\left[ 1 + \frac{1}{2} \right] C_0 \frac{r + C_1 - iC_0}{2} + \left[ \frac{1 + i}{2} \right] C_0 Kf \frac{q}{bq + d} + \\
\left[ \frac{1 + i}{2} \right] S_0 Kf \frac{1}{bq + d} - iS_0 \frac{1}{2} - \left[ \frac{iC_0 \theta (r + C_1) Kf}{4Kf} \right] (bq^2 + dq)
\]

From 9.24, Let us take,

\[
A = \left[ 1 + \frac{i}{2} \right] C_0 (r + C_1 - iC_0) \sqrt{2}
\]

\[
B = \left[ \frac{1 + i}{2} \right] C_0 Kf \frac{q}{bq + d}
\]

\[
C = \left[ \frac{1 + i}{2} \right] S_0 Kf \frac{1}{bq + d}
\]

and

\[
D = \left[ \frac{iC_0 \theta (r + C_1) Kf}{4Kf} \right]
\]

therefore,

\[
g(q) = \left[ Aq + B \frac{q}{bq + d} + C \frac{1}{bq + d} - iS_0 \frac{1}{2} - D(bq^2 + dq) \right] \quad \ldots \ldots \quad 9.25
\]

From 9.25, we have,

\[
\frac{\partial I(q)}{\partial q} = \frac{\partial}{\partial q} \left[ Aq + B \frac{q}{bq + d} + C \frac{1}{bq + d} - iS_0 \frac{1}{2} - D(bq^2 + dq) \right] = 0
\]

\[
= A + B \frac{(bq + d) \times 1 - q(b)}{(bq + d)^2} + C \frac{(bq + d) \times 0 - 1(b)}{(bq + d)^2} - 0 - D(2bq - d) = 0
\]

\[
\therefore A(bq + d)^2 + Bd - Cb - D(2bq + d)(bq + d)^2 = 0
\]

\[
\therefore A(b^2q^2 + 2bq + d^2) + Bd - Cb - (2bDq + dD)(b^2q^2 + 2bq + d^2) = 0
\]
\[-b^2 dDq^2 - 2bd^2 Dq - d^3 D = 0\]

\[\therefore -2b^3 Dq^3 + b^2 Aq^2 - 5b^2 dDq^2 + 2bdAq - 4bd^2 Dq + d^2 A + Bd - Cb - d^3 D = 0\]

\[\therefore 2b^3 Dq^3 - [b^2 A - 5b^2 dD]q^2 + [4bd^2 D - 2bdA]q - d^2 A - Bd + Cb + d^3 D = 0\]

\[\ldots \quad 9.26\]

Put the value of A, B, C and D in 9.26, we have,

\[\therefore \frac{2b^3 iC^0 \theta (r + C_1)}{4Kf} q^3 - \left\{ \frac{b^2}{2} \left[ \left( 1 + \frac{i}{2} \right) C_0 (r + C_1) - iC_0 \right] - \frac{5b^2 diC^0 \theta (r + C_1)}{4Kf} \right\} q^2 +\]

\[\left\{ \frac{4bd^2 iC_0 \theta (r + C_1)}{4Kf} - \frac{2bd}{2} \left[ \left( 1 + \frac{i}{2} \right) C_0 (r + C_1) - iC_0 \right] \right\} q - \]

\[\frac{d^2}{2} \left[ \left( 1 + \frac{i}{2} \right) C_0 (r + C_1) - iC_0 \right] - \frac{d \left( 1 + \frac{i}{2} \right) C_0 Kf}{\theta} +\]

\[\frac{b \left( 1 + \frac{i}{2} \right) S_o Kf}{\theta} + \frac{d^3 iC^0 \theta (r + C_1)}{4Kf} = 0\]

\[\therefore \frac{b^3 iC_0 \theta (r + C_1)}{2Kf} q^3 - \left\{ \frac{b^2}{2} \left[ \left( 1 + \frac{i}{2} \right) C_0 (r + C_1) - iC_0 \right] - \frac{5b^2 diC_0 \theta (r + C_1)}{4Kf} \right\} q^2 +\]

\[\left\{ \frac{bd^2 iC_0 \theta (r + C_1)}{Kf} - bd \left[ \left( 1 + \frac{i}{2} \right) C_0 (r + C_1) - iC_0 \right] \right\} q -\]

\[\frac{d^2}{2} \left[ \left( 1 + \frac{i}{2} \right) C_0 (r + C_1) - iC_0 \right] - \frac{d^3 iC_0 \theta (r + C_1)}{4Kf} + \left( \frac{1 + \frac{i}{2}}{2} \right) \frac{Kf}{\theta} \left( dC_0 - bS_o \right) = 0\]

\[\ldots \quad 9.27\]

Where

\[\beta_0 = \frac{b^3 iC_0 \theta (r + C_1)}{2Kf}\]

\[\beta_1 = \frac{b^2}{2} \left[ \left( 1 + \frac{i}{2} \right) C_0 (r + C_1) - iC_0 \right] - \frac{5b^2 diC_0 \theta (r + C_1)}{4Kf}\]
\[ \beta_2 = \left\{ \frac{bd^2 i C_0 \theta (r + C_1)}{Kf} - bd \left[ \left( 1 + \frac{i}{2} \right) C_0 (r + C_1) - i C_0 \right] \right\} \]

and

\[ \beta_3 = \left\{ \frac{d^2}{2} \left[ \left( 1 + \frac{i}{2} \right) C_0 (r + C_1) - i C_0 \right] - \frac{d^3 i C_0 \theta (r + C_1)}{4Kf} + \left( 1 + \frac{i}{2} \right) \frac{Kf}{\theta} \left( d C_0 - b S_0 \right) \right\} \]

\[ \therefore \beta_0 q^3 - \beta_1 q^2 + \beta_2 q - \beta_3 = 0 \]

......... 9.28

9.3.8. Hypothetical Problem:-

To explain the case-II of model 2 of 9.3 let us take numerical example as bellow.

\[ C_0 = 1, r = 0.1, C_1 = 0.05, S_0 = 0.1, i = 0.05, K = 50, f = 50, b = 10, \]

\[ d = 50, \alpha = 0.1, \gamma = 0.2 \] and mark up parameter \( \theta = 2, 3, 4, 5, 6. \)

Solution of the hypothetical problem is,

<table>
<thead>
<tr>
<th>Mark up Parameter ( \theta )</th>
<th>Optimal Total Inventory</th>
</tr>
</thead>
<tbody>
<tr>
<td>theta (Units)</td>
<td>Turnover</td>
</tr>
<tr>
<td>Qopt</td>
<td>C(Qopt)</td>
</tr>
<tr>
<td>Rs.</td>
<td></td>
</tr>
<tr>
<td>2.00</td>
<td>1734</td>
</tr>
<tr>
<td>3.00</td>
<td>1157</td>
</tr>
<tr>
<td>4.00</td>
<td>869</td>
</tr>
<tr>
<td>5.00</td>
<td>696</td>
</tr>
<tr>
<td>6.00</td>
<td>581</td>
</tr>
</tbody>
</table>

9.3.9. Remark:-

In hyperbolic quantity discount case, above results justifies that with the increases in the value of mark up parameter \( \theta \) the optimal value of ITOR also increases and the optimum lot size decreases. Advertising, rate of inflation and damaged goods are highly sensitive with respect to total cost and such that ITOR.
9.4. **Model 3: Inventory Turnover Ratio Model under Inflationary Situations and Markup Variation for Finite Production Rate.**

9.4.1. **Introduction:**

The model to be discussed here refers the inventory management problem of a production firm. The produced units are stored in the godown and the units are supplied as per their actual demand. The price is to be fixed by considering a proper mark-up and a quantity discounting policy. The cost of purchasing, ordering and storing are subject to inflation. The replenishment of the goods is to be considered. Also, cost of advertisement and damaged goods are considered.

9.4.2. **Assumptions and Notations:**

(i) Lots of size $q$ is replenished for every replenishment.
(ii) Demand rate is $R$ units/year
(iii) Production rate is $\lambda$ Units per year is finite and $\lambda > R$
(iv) Shortages are not allowed
(v) Inflation rate is $i$ Rs./year
(vi) Interest rate is $r$ Rs/year where $r$ is very small and $r > i$
(vii) Initial purchase cost of an item is $C_0$ Rs per unit per year
(viii) Inventory holding cost of an item is $C_1$ Rs per unit per year
(ix) Setup cost per order is $S_0$ Rs at time Zero.
(x) Advertisement cost $apR$ Rs. per year. Where $\alpha$ is a Constant, $0<\alpha<1$.
(xi) Cost due to damage goods is given by $\gamma C(q)R$ Rs. per year. Where $\gamma$ is the lot fraction defective and $C(q)$ represents the cost of $q$ units replenished.
9.4.3. **Problem Formulation:**

In this model, purchase cost, order cost, advertising cost and damaged goods cost remains same but storage cost is defined as:

\[
\text{Storage cost} = \left[1 + \frac{t}{2} \left(1 - \frac{q}{R}\right)\right] C_0 q \left(r + C_1\right) \left(1 - \frac{R}{\lambda}\right) \quad \ldots \ldots \quad 9.29
\]

Total inventory cost = Purchase Cost + Order Cost + Storage cost + Advertising Cost + Damaged goods cost

\[
F(q) = \left[1 + \frac{t}{2} \left(1 - \frac{q}{R}\right)\right] C_0 R + \frac{S_0 R}{q} + C_0 \left(r + C_1\right) \left(1 - \frac{R}{\lambda}\right) q
\]

\[+ \alpha p R + \gamma C(q) R \quad \ldots \ldots \quad 9.30
\]

Further assumed that market price per unit of the commodity is given by:

\[
p = \theta C(q) \quad \ldots \ldots \quad 9.31
\]

where \(\theta = \text{Mark-up parameter; } \theta > 1\).

Let us consider a specific demand function given by Kotler as under:

\[
R = K f / p^n \quad \ldots \ldots \quad 9.32
\]

Where

- \(K = \text{Constant; } K > 0\)
- \(f = \text{frequency of advertisement}\)
- \(n = \text{Elasticity of demand; } n > 0\)

For \(n=1\), we have,

\[
R = K f / p = \frac{K f}{\partial C(q)} \quad \ldots \ldots \quad 9.33
\]

From result 9.32 and 9.33,

\[
F(q) = \left[1 + \frac{t}{2} \left(1 - \frac{\partial C(q) q}{K f}\right)\right] \left(\frac{K f C_0}{\partial C(q)} + \frac{K f S_0}{\partial C(q) q^2} + \frac{C_0 \left(r + C_1\right) \left(1 - \frac{K f}{\partial C(q) \lambda}\right) q}{2}\right)
\]

\[+ \frac{\alpha \partial C(q) K f}{\partial C(q)} + \frac{\gamma C(q) K f}{\partial C(q)}
\]

\[= \left[1 + \frac{t}{2} \left(1 - \frac{\partial C(q) q}{K f}\right)\right] \left(\frac{K f C_0}{\partial C(q)} + \frac{K f S_0}{\partial C(q) q^2} + \frac{C_0 \left(r + C_1\right) \left(1 - \frac{K f}{\partial C(q) \lambda}\right) q}{2}\right)
\]
Therefore from 9.33 and 9.34 Inventory turnover ratio is defined as:

\[ I(q) = \frac{R[p - C(q)]}{F(q)} \]

\[ = \frac{KfC(q)(\theta - 1)}{\theta C(q)} / F(q) \]

\[ = \left( \frac{\theta - 1}{\theta} \right) Kf / F(q) \]  \[ \ldots \quad 9.35 \]

9.4.4. Case-I: Linear Quantity Discount

The unit cost function C(q) assume the linear quantity discount as

\[ C(q) = a - bq \]  \[ \ldots \quad 9.36 \]

Where a and b are positive constants

From result 9.35 and 9.36, Inventory turnover ratio is given by

\[ I(q) = \left( \frac{\theta - 1}{\theta} \right) Kf / \left[ \left\{ 1 + \frac{i}{2} \left( 1 - \frac{\theta(a - bq)q}{Kf} \right) \right\} + \left( \frac{\alpha\theta + \gamma)Kf}{\theta} \right] \]

\[ \left[ \frac{KfC_0}{\theta(a - bq)} + \frac{KfS_0}{\theta(a - bq)q} + \frac{C_0(r + C_1)}{2} \left( 1 - \frac{Kf}{\theta\lambda(a - bq)} \right) q + \frac{(\alpha\theta + \gamma)Kf}{\theta} \right] \]  \[ \ldots \quad 9.37 \]

Necessary and sufficient condition to maximisation of ITOR are (i)

\[ \frac{\partial I(q)}{\partial q} = 0 \quad \text{and} \quad \frac{\partial^2 I(q)}{\partial q^2} < 0 \]

\[ \therefore \frac{\partial I(q)}{\partial q} = \left( \frac{\theta - 1}{\theta} \right) Kf \frac{\partial}{\partial q} \left[ \left\{ 1 + \frac{i}{2} \left( 1 - \frac{\theta(a - bq)q}{Kf} \right) \right\} + \left( \frac{\alpha\theta + \gamma)Kf}{\theta} \right] \]

\[ \left[ \frac{KfC_0}{\theta(a - bq)} + \frac{KfS_0}{\theta(a - bq)q} + \frac{C_0(r + C_1)}{2} \left( 1 - \frac{Kf}{\theta\lambda(a - bq)} \right) q + \frac{(\alpha\theta + \gamma)Kf}{\theta} \right]^{-1} \]

\[ \therefore \frac{\partial I(q)}{\partial q} = \left( \frac{\theta - 1}{\theta} \right) Kf \left[ \left\{ 1 + \frac{i}{2} \left( 1 - \frac{\theta(a - bq)q}{Kf} \right) \right\} \right]^{-1} \]

\[ \left[ \frac{KfC_0}{\theta(a - bq)} + \frac{KfS_0}{\theta(a - bq)q} + \frac{C_0(r + C_1)}{2} q - \frac{C_0(r + C_1)Kf}{2\theta\lambda(a - bq)q} + \frac{(\alpha\theta + \gamma)Kf}{\theta} \right]^{-2} . \]
\[
\frac{\partial}{\partial q}\left[1 + i\left(1 - \frac{\theta(a-bq)}{Kf}\right)\right].
\]

\[
\left[\frac{KfC_0}{\theta(a-bq)} + \frac{KfS_0}{\theta(a-bq)} + \frac{C_0(r+C_1)q - C_0(r+C_1)Kfq}{2 + \frac{2\theta\lambda(a-bq)}{2\theta\lambda(a-bq)}}\right] + 0
\]

Also \(\frac{\partial^2 I(q)}{\partial q^2} = \frac{\partial}{\partial q}\left(\frac{\partial I(q)}{\partial q}\right) < 0\), implies that \(\frac{\partial I(q)}{\partial q} = 0\).

\[
\frac{\partial I(q)}{\partial q} = \frac{\partial}{\partial q}\left[1 + i\left(1 - \frac{\theta(a-bq)}{Kf}\right)\right].
\]

\[
\left[\frac{KfC_0}{\theta(a-bq)} + \frac{KfS_0}{\theta(a-bq)} + \frac{C_0(r+C_1)q - C_0(r+C_1)Kfq}{2 + \frac{2\theta\lambda(a-bq)}{2\theta\lambda(a-bq)}}\right] = 0 \quad \text{...... 9.38}
\]

Now from 9.38

\[
\left[1 + i\left(1 - \frac{\theta(a-bq)}{Kf}\right)\right]
\]

\[
\left[\frac{KfC_0}{\theta(a-bq)} + \frac{KfS_0}{\theta(a-bq)} + \frac{C_0(r+C_1)q - C_0(r+C_1)Kfq}{2 + \frac{2\theta\lambda(a-bq)}{2\theta\lambda(a-bq)}}\right]
\]

\[
\left[1 + i\left(1 - \frac{\theta(a-bq)}{Kf}\right)\right]
\]

\[
\frac{\theta(a-bq)q}{Kf} \times \frac{KfC_0}{\theta(a-bq)} + \frac{\theta(a-bq)q}{Kf} \times \frac{KfS_0}{\theta(a-bq)} + \frac{\theta(a-bq)q}{Kf} \times \frac{C_0(r+C_1)q - C_0(r+C_1)Kfq}{2 + \frac{2\theta\lambda(a-bq)}{2\theta\lambda(a-bq)}}\right]
\]

\[
= \left[1 + i\left(1 - \frac{\theta(a-bq)}{Kf}\right)\right]
\]

\[
\left[\frac{1}{\theta}\left(\frac{C_0(a-bq)(r+C_1)q^2}{2Kf} - \frac{C_0(r+C_1)q^2}{2\lambda}\right)\right]
\]

\[
\left[1 + \frac{i}{2}\left(\frac{KfC_0}{\theta(a-bq)} + \frac{KfS_0}{\theta(a-bq)} + \frac{C_0(r+C_1)q - C_0(r+C_1)Kfq}{2 + \frac{2\theta\lambda(a-bq)}{2\theta\lambda(a-bq)}}\right)\right]
\]

\[
\left[1 + \frac{i}{2}\left(\frac{KfC_0}{\theta} + \frac{KfS_0}{\theta(a-bq)} + \frac{1}{q(a-bq)}\right)\right]
\]

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\[
\begin{align*}
&\left[\frac{1+i}{2} C_0 (r+C_1) \right] q - \left[\frac{1+i}{2} C_0 (r+C_1) Kf \right] \frac{q}{(a-bq)} - \\
&\frac{iqC_0}{2} - \frac{i\theta C_0 (a-bq) (r+C_1) q^2}{4Kf} + \frac{iC_0 (r+C_1) q^2}{4\lambda} \\
&= \left[\left(1+i\right) C_0 (r+C_1) - iC_0 \right] \frac{q}{2} + \left[\frac{1+i}{2} Kf S_0 \right] \frac{1}{aq-bq^2} - \\
&\frac{iC_0 (r+C_1) \theta}{4Kf} (aq^2-bq^3) + \left[\frac{1+i}{2} Kf C_0 \right] \frac{1}{(a-bq)} \\
&- \left[\frac{1+i}{2} C_0 (r+C_1) Kf \right] \frac{q}{2 \theta \lambda} + \frac{iC_0 (r+C_1) q^2}{4\lambda} - \frac{iS_0}{2} \quad \ldots \quad 9.39 \\
\end{align*}
\]

Let

\[
A = \left[\left(1+i\right) C_0 (r+C_1) - iC_0 \right] / 2 \\
B = \left[\frac{1+i}{2} Kf S_0 \right] \\
C = \left[\frac{iC_0 (r+C_1) \theta}{4Kf} \right] \\
D = \left[\frac{1+i}{2} Kf C_0 \right] \\
E = \left[\frac{1+i}{2} C_0 (r+C_1) Kf \right] / 2 \theta \lambda
\]
\[ F = \frac{iC_0(r+C_1)}{4\lambda} \]

From 9.39, we get,

\[
Aq + \frac{B}{aq-bq^2} - C(aq^2 - bq^3) + \frac{D}{aq-bq} - E\left(\frac{q}{a-bq}\right) + Fq^2 - \frac{iS_0}{2} \quad \ldots \quad 9.40
\]

\[
\frac{\partial I(q)}{\partial q} = A + \frac{B(a-2bq)}{q^2(a-bq)^2}(-1) - C(2aq - 3bq^2) + \frac{Db}{(a-bq)^2} -
\]

\[
\frac{Eq}{(a-bq)^2} + 2Fq - 0 = 0
\]

\[
A - \frac{B(a-2bq)}{q^2(a-bq)^2} - C(2aq - 3bq^2) + \frac{Db}{(a-bq)^2} - \frac{Ea}{(a-bq)^2} + 2Fq = 0
\]

\[
\ldots \quad 9.41
\]

\[
|Aq^2(a-bq)^2 - B(a-2bq) - C(2aq - 3bq^2)q^2(a-bq)^2 + Dbq^2 - Eaq^2 + 2Fq^3(a^2 - 2abq + b^2q^2)| = 0
\]

\[
|Aq^2(a^2 - 2abq + b^2q^2) - Ba + 2bBq - C(2aq^3 - 3bq^4)q^2(a-2abq + b^2q^2) + + Dbq^2 - Eaq^2 + 2Fq^3(a^2 - 2abq + b^2q^2)| = 0
\]

\[
\ldots \quad 9.42
\]

Put the value of \( A, B, C, D, E \) and \( F \) in 9.42, we have,

\[
\frac{3b^3C_0(r+C_1)}{4Kf}q^6 - 2b^2\left[4aiC_0(r+C_1)q\frac{4b}{4Kf} - iC_0(r+C_1)\right]q^5
\]

\[
+ \left\{\frac{b^2}{2}\left[1 + \frac{1}{2}\right]C_0(r+C_1) - iC_0\right\} + \frac{7a^2bC_0(r+C_1)q}{4Kf} - \frac{4abC_0(r+C_1)}{4\lambda}\right\}q^4
\]

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\[-\frac{2ab}{2}\left[(1 + \frac{i}{2})C_0(r + C_1) - iC_0\right] + \frac{2a^2iC_0(r + C_1)\theta}{4Kf} - \frac{2a^2iC_0(r + C_1)}{4\lambda}\right]q^3
\]
\[+ \left[\frac{a^2}{2}\left[(1 + \frac{i}{2})C_0(r + C_1) - iC_0\right] + \frac{b\left(1 + \frac{i}{2}\right)KfC_0}{\theta} - a\left(1 + \frac{i}{2}\right)C_0(r + C_1)Kf\right]q^2
\]
\[+ \frac{2b\left(1 + \frac{i}{2}\right)KfS_0}{\theta} - \frac{a\left(1 + \frac{i}{2}\right)KfS_0}{\theta} = 0
\]
\[\therefore \frac{3b^3iC_0(r + C_1)\theta}{4Kf} - b^2iC_0(r + C_1)\left[\frac{2a\theta}{Kf} - \frac{1}{2\lambda}\right]q^5
\]
\[+ \left[\frac{b^2}{2}\left[(1 + \frac{i}{2})C_0(r + C_1) - iC_0\right] + abiC_0(r + C_1)\left[\frac{7a\theta}{4Kf} - \frac{1}{\lambda}\right]\right]q^4
\]
\[+ \left[- ab\left[(1 + \frac{i}{2})C_0(r + C_1) - iC_0\right] + \frac{a^2iC_0(r + C_1)}{2}\left[\frac{a\theta}{Kf} - \frac{1}{\lambda}\right]\right]q^3
\]
\[+ \left[\frac{a}{2}\left(1 + \frac{i}{2}\right)C_0(r + C_1)\left[a - \frac{Kf}{2\lambda}\right] - \frac{iC_0a^2}{2} + \frac{b\left(1 + \frac{i}{2}\right)KfC_0}{\theta}\right]q^2
\]
\[+ \frac{2b\left(1 + \frac{i}{2}\right)KfS_0}{\theta} - \frac{a\left(1 + \frac{i}{2}\right)KfS_0}{\theta} = 0 \quad \ldots \quad 9.43
\]

Let us take,
\[\beta_0 = \frac{3b^3iC_0(r + C_1)\theta}{4Kf}\]
\[\beta_1 = b^3iC_0(r + C_1)\left[\frac{2a\theta}{Kf} - \frac{1}{2\lambda}\right]\]
\[\beta_2 = \left\{\frac{b^2}{2}\left[(1 + \frac{i}{2})C_0(r + C_1) - iC_0\right] + abiC_0(r + C_1)\left[\frac{7a\theta}{4Kf} - \frac{1}{\lambda}\right]\right\}\]
\[\beta_3 = \left\{ab\left[(1 + \frac{i}{2})C_0(r + C_1) - iC_0\right] + \frac{a^2iC_0(r + C_1)}{2}\left[\frac{a\theta}{Kf} - \frac{1}{\lambda}\right]\right\}\]
\[ \beta_4 = \frac{a}{2} \left( 1 + \frac{i}{2} \right) C_0 (r + C_1) \left[ a - \frac{Kf}{\theta \lambda} \right] - \frac{i C_0 a^2}{2} + \frac{b \left( 1 + \frac{i}{2} \right) K f C_0}{\theta} \]

\[ \beta_5 = \frac{2 b \left( 1 + \frac{i}{2} \right) K f S_0}{\theta} \]

and \[ \beta_6 = -\frac{a \left( 1 + \frac{i}{2} \right) K f S_0}{\theta} \]

\[ \therefore \beta_6 q^6 - \beta_5 q^5 + \beta_4 q^4 - \beta_3 q^3 + \beta_2 q^2 - \beta_1 q + \beta_0 = 0 \]

\[ \therefore \sum_{j=0}^{6} (-1)^j \beta_j q^{6-j} = 0 \]

Which is solved by the successive approximation method.

### 9.4.5. Hypothetical Problem:

To explain the case-I of model 3 of 9.4 let us take numerical example as bellow.

- \( C_0 = 1, r = 0.1, C_1 = 0.05, S_0 = 0.1, i = 0.05, K = 10, f = 50, a = 50, \)
- \( b = 0.0001, \alpha = 0.1, \gamma = 0.2, \lambda = 1000 \) and mark up parameter
- \( \theta = 2, 3, 4, 5. \)

Solution of the hypothetical problem is,

| Mark up Parameter theta (Units) | Optimal Total Inventory Cost Ratio Qopt C(Qopt) I(Qopt) Rs. |
|-------------------------------|---------------------------------|----------------|---|-------------------|----------------|
| 2.00                          | 69                              | 106.89         | 2.3388 |
| 3.00                          | 46                              | 87.94          | 3.7906 |
| 4.00                          | 34                              | 78.46          | 4.7797 |
| 5.00                          | 27                              | 72.77          | 5.4970 |

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9.4.6. Remark:

From the optimal solution given above and situation stated in the model for the finite production rate, it may be conclude that with the increases in the value of mark up parameter \( \theta \) the optimal value of ITOR also increases and the optimum lot size decreases. Advertising, rate of inflation and damaged goods are highly sensitive with respect to total cost and such that ITOR.

9.4.7. Case-II: Hyperbolic Quantity Discount

The unit cost function \( C(q) \) assume the hyperbolic quantity discount as

\[
C(q) = b + \frac{d}{q}
\]

.... 9.45

Where \( b \) and \( d \) are positive constants.

From result 9.34, 9.35 and 9.45, Inventory turnover ratio is given by

\[
I(q) = \left( \frac{\theta - 1}{\theta} \right) Kf \left[ \frac{1 + i}{2} \left( 1 - \frac{\theta (bq + d)q}{q Kf} \right) \right].
\]

\[
= \left[ \frac{KfC_0q}{\theta (bq + d)} + \frac{Kfs_0q}{\theta (bq + d)} + \frac{C_0 (r + C_1)}{2} - \frac{C_0 (r + C_1) Kf q^2}{2 \theta \lambda (bq + d)} \right] + \frac{(\alpha \theta + \gamma) Kf}{\theta}
\]

Necessary and sufficient condition to maximise the ITOR are (i)

\[
\frac{\partial I(q)}{\partial q} = 0, \quad (\text{ii}) \quad \frac{\partial^2 I(q)}{\partial q^2} < 0
\]

\[
\therefore \frac{\partial I(q)}{\partial q} = \left( \frac{\theta - 1}{\theta} \right) Kf \left[ \frac{1 + i}{2} \left( 1 - \frac{\theta (bq + d)q}{q Kf} \right) \right].
\]

\[
= \left[ \frac{KfC_0q}{\theta (bq + d)} + \frac{Kfs_0q}{\theta (bq + d)} + \frac{C_0 (r + C_1)}{2} - \frac{C_0 (r + C_1) Kf q^2}{2 \theta \lambda (bq + d)} \right] + \frac{(\alpha \theta + \gamma) Kf}{\theta}
\]

\[
= \left( \frac{\theta - 1}{\theta} \right) Kf \left[ \frac{1 + i}{2} \left( 1 - \frac{\theta (bq + d)q}{q Kf} \right) \right].
\]

\[
\therefore \frac{\partial^2 I(q)}{\partial q^2} < 0
\]

\[
\left[ \frac{KfC_0q}{\theta (bq + d)} + \frac{Kfs_0q}{\theta (bq + d)} + \frac{C_0 (r + C_1)}{2} - \frac{C_0 (r + C_1) Kf q^2}{2 \theta \lambda (bq + d)} \right] + \frac{(\alpha \theta + \gamma) Kf}{\theta}
\]

\[
= \left( \frac{\theta - 1}{\theta} \right) Kf \left[ \frac{1 + i}{2} \left( 1 - \frac{\theta (bq + d)q}{q Kf} \right) \right].
\]

\[
\therefore \frac{\partial^2 I(q)}{\partial q^2} < 0
\]
\[
\frac{\partial}{\partial q} \left[ 1 + i \left( 1 - \frac{\theta(bq + d)}{Kf} \right) \right].
\]

\[
\left[ \frac{KfC_0q}{\theta(bq + d)} + \frac{KfS_0}{\theta(bq + d)} + \frac{C_0(r + C_1)q}{2} - \frac{C_0(r + C_1)Kfq^2}{2 \theta \lambda(bq + d)} \right] + 0 \quad \ldots \quad 9.46
\]

Also \( \frac{\partial^2 I(q)}{\partial q^2} < 0 \) satisfies \( \frac{\partial I(q)}{\partial q} = 0. \)

From 9.46, we get,

\[
\frac{\partial I(q)}{\partial q} = \frac{\partial}{\partial q} \left[ 1 + i \left( 1 - \frac{\theta(bq + d)}{Kf} \right) \right].
\]

\[
\left[ \frac{KfC_0q}{\theta(bq + d)} + \frac{KfS_0}{\theta(bq + d)} + \frac{C_0(r + C_1)q}{2} - \frac{C_0(r + C_1)Kfq^2}{2 \theta \lambda(bq + d)} \right] = 0 \quad \ldots \quad 9.47
\]

Where,

\[
\left[ 1 + \frac{i}{2} \left( 1 - \frac{\theta(bq + d)q}{Kf} \right) \right] =
\]

\[
\left[ \frac{KfC_0q}{\theta(bq + d)} + \frac{KfS_0}{\theta(bq + d)} + \frac{C_0(r + C_1)q}{2} - \frac{C_0(r + C_1)Kfq^2}{2 \theta \lambda(bq + d)} \right]
\]

\[
= \left( 1 + \frac{i}{2} \right) \frac{KfC_0}{\theta} \left( \frac{q}{bq + d} \right) + \left( 1 + \frac{i}{2} \right) \frac{KfS_0}{\theta} \left( \frac{1}{bq + d} \right) +
\]

\[
\left( 1 + \frac{i}{2} \right) \frac{C_0(r + C_1)q}{2} - \frac{\left( 1 + \frac{i}{2} \right) \frac{C_0(r + C_1)Kf}{2 \theta \lambda} \left( \frac{q^2}{bq + d} \right)} -
\]

\[
\frac{i \theta(bq + d)}{2Kf} \times \frac{KfC_0q}{\theta(bq + d)} - \frac{i \theta(bq + d)}{2Kf} \times \frac{KfS_0}{\theta(bq + d)} -
\]

\[
\frac{i \theta(bq + d)}{2Kf} \times \frac{C_0(r + C_1)q}{2} + \frac{i \theta(bq + d)}{2Kf} \times \frac{C_0(r + C_1)Kf}{2 \theta \lambda(bq + d)}
\]

\[
= \left( 1 + \frac{i}{2} \right) \frac{KfC_0}{\theta} \left( \frac{q}{bq + d} \right) + \left( 1 + \frac{i}{2} \right) \frac{KfS_0}{\theta} \left( \frac{1}{bq + d} \right) +
\]

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\[
\left(1 + \frac{i}{2}\right)C_0(r + C_1)q - \left(1 + \frac{i}{2}\right)C_0(r + C_1)Kf \frac{q^2}{2\theta \lambda} \left(\frac{bq + d}{(bq + d)^2}\right)
\]

\[
- \frac{iC_0q}{2} - \frac{i\theta C_0(r + C_1)}{4Kf} \left(\frac{bq + d}{1}\right)q + \frac{iC_0(r + C_1)}{4\lambda} q^2
\]

\[
= \left[\left(1 + \frac{i}{2}\right)C_0(r + C_1) - iC_0\right]q + \left[\left(1 + \frac{i}{2}\right)Kf C_0\right] \frac{q}{(bq + d)}
\]

\[
= \left[\left(1 + \frac{i}{2}\right)Kf S_0\right] \left(\frac{1}{bq + d}\right) - \left[\left(1 + \frac{i}{2}\right)C_0(r + C_1)Kf\right] \frac{q^2}{2\theta \lambda} \left(\frac{bq + d}{(bq + d)^2}\right)
\]

\[
- \frac{iS_0}{2} - \left[\frac{i\theta C_0(r + C_1)}{4Kf}\right] (bq^2 + dq) + \frac{iC_0(r + C_1)}{4\lambda} q^2
\]

Let

\[
A = \left[\left(1 + \frac{i}{2}\right)C_0(r + C_1) - iC_0\right] \sqrt{2}
\]

\[
B = \left[\left(1 + \frac{i}{2}\right)Kf C_0\right] \frac{1}{\theta}
\]

\[
C = \left[\left(1 + \frac{i}{2}\right)Kf S_0\right] \frac{1}{\theta}
\]

\[
D = \left[\left(1 + \frac{i}{2}\right)C_0(r + C_1)Kf\right] \frac{1}{2\theta \lambda}
\]

\[
E = \left[\frac{i\theta C_0(r + C_1)}{4Kf}\right]
\]

\[
F = \frac{iC_0(r + C_1)}{4\lambda}
\]

From 9.48, we get,
\[
\begin{align*}
Aq + B & \left(\frac{q}{bq + d}\right) + \frac{C}{bq + d} - D \left(\frac{q^2}{bq + d}\right) - \frac{i\omega_0}{2} - E\left(\frac{q}{bq + d}\right) - Fq^2 \\
\frac{\partial I(q)}{\partial q} & = \frac{\partial}{\partial q} \left[ Aq + B \left(\frac{q}{bq + d}\right) + \frac{C}{bq + d} - D \left(\frac{q^2}{bq + d}\right) - \frac{i\omega_0}{2} - E\left(\frac{q}{bq + d}\right) - Fq^2 \right] \\
\end{align*}
\]

\[E(\frac{q}{bq + d}) + Fq^2 = 0\]

\[A + B \left(\frac{d}{(bq + d)^2}\right) - \frac{Cb}{(bq + d)^3} - D \left[\frac{bq^2 + 2qd}{(bq + d)^3}\right] - E(2bq + d) + 2Fq = 0\]

\[A(bq + d)^2 + Bd - Cd - D(bq^2 + 2qd) - E(2bq + d)(bq + d)^2 + 2Fq(bq + d)^2 = 0\]

\[(2b^2F - 2b^3E)\mathcal{H}^3 + \left(b^2A - bD - 5b^3dE + 4bdF\right)\mathcal{H}^3 + (2bdA - 2dD - 4bd^2E + 2d^2F)\mathcal{H} + Ad^2 + Bd - Cb - Ed^3 = 0 \quad \cdots \quad 9.49\]

Put value of A, B, C, D, E and F in 9.49, we get,

\[\left\{2b^2 \frac{iC_0(r + C_1)}{4\lambda} - 2b^3 \frac{i\theta C_0(r + C_1)}{4Kf}\right\} q^3 +\]

\[\left\{\frac{5b^2d i\theta C_0(r + C_1)}{4Kf} + 4bd iC_0(r + C_1)\right\} q^2 +\]

\[\left\{\frac{4bd^2 i\theta C_0(r + C_1)}{4Kf} + 2d^2 iC_0(r + C_1)\right\} q +\]

\[\left\{\left[\left(\frac{1 + i}{2}\right)C_0(r + C_1) - iC_0\right] \frac{d^2}{2} + \frac{1 + i}{2} \frac{KfC_0}{\theta} - \frac{1 + i}{2} \frac{Kfs_0}{\theta} - \frac{i\theta C_0(r + C_1)}{4Kf} \right\} = 0\]

\[\therefore \left\{\frac{b^2}{2} iC_0(r + C_1) \left[\frac{1}{2\lambda} - \frac{b\theta}{Kf}\right]\right\} q^3 +\]

\[\left\{\frac{b^2}{2} \left[\left(1 + \frac{i}{2}\right)C_0(r + C_1) - iC_0\right] \frac{1}{\theta\lambda} - \frac{b\theta}{Kf}\right] + bdiC_0(r + C_1) \left[\frac{1}{\lambda} - \frac{5b\theta}{4Kf}\right]\right\} q^2 +\]
\[
\begin{align*}
\left(d\left(1+\frac{i}{2}\right)c_0(r+C_1)\left[b-\frac{1}{\theta\lambda}\right]-bdic_0+d^2ic_0(r+C_1)\left[\frac{1}{2\lambda}\frac{b\theta}{Kf}\right]\right)q^2 + \\
\left(d^2c_0(r+C_1)\left[\left(1+\frac{i}{2}\right)\frac{id\theta}{2Kf}\right]-\frac{d^2ic_0}{2}+\frac{1+i}{2}\frac{Kf}{\theta}(dc_0-bs_0)-\frac{d^3ic_0(r+C_1)}{4Kf}\right) = 0
\end{align*}
\]

Where,
\[
\beta_0 = \frac{b^2}{2}ic_0(r+C_1)\left[\frac{1}{2\lambda}\frac{b\theta}{Kf}\right]
\]
\[
\beta_1 = \frac{b}{2}\left(1+\frac{i}{2}\right)c_0(r+C_1)\left[b-\frac{1}{\theta\lambda}\right]ic_0\frac{b^2}{2}+bdic_0(r+C_1)\left[\frac{1}{\lambda}\frac{5b\theta}{4Kf}\right]
\]
\[
\beta_2 = d\left(1+\frac{i}{2}\right)c_0(r+C_1)\left[b-\frac{1}{\theta\lambda}\right]-bdic_0+d^2ic_0(r+C_1)\left[\frac{1}{2\lambda}\frac{b\theta}{Kf}\right]
\]

and
\[
\beta_3 = \frac{d^2}{2}c_0(r+C_1)\left[\left(1+\frac{i}{2}\right)\frac{id\theta}{2Kf}\right]-\frac{d^2ic_0}{2}+\frac{1+i}{2}\frac{Kf}{\theta}(dc_0-bs_0)-\frac{d^3ic_0(r+C_1)}{4Kf}
\]

\[
\therefore \beta_0q^3 + \beta_1q^2 + \beta_2q + \beta_3 = 0
\]

\[
\therefore \sum_{j=0}^{3} \beta_j q^{3-j} = 0
\]

Which is solved by the successive approximation.

9.4.8. Hypothetical Problem:-

To explain the case-II of model 3 of 9.4 let us take numerical example as below.

\[
\begin{align*}
C_0 = 1, r = 0.1, C_1 = 0.05, S_0 = 0.1, i = 0.05. K = 50, f = 50, b = 10, \\
d = 50, \alpha = 0.1, \gamma = 0.2, \lambda = 1000 \text{ and mark up parameter} \\
\theta = 2, 3, 4, 5, 6.
\end{align*}
\]

Solution of the hypothetical problem is,
Chapter 9 : Model 3 : Case - II

<table>
<thead>
<tr>
<th>Mark up Parameter ( \theta ) (Units)</th>
<th>Optimal Total Inventory Turnover ( Q_{opt} ), ( C(Q_{opt}) ), ( I(Q_{opt}) ) Rs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.00</td>
<td>1801</td>
</tr>
<tr>
<td>3.00</td>
<td>1160</td>
</tr>
<tr>
<td>4.00</td>
<td>849</td>
</tr>
<tr>
<td>5.00</td>
<td>666</td>
</tr>
<tr>
<td>6.00</td>
<td>546</td>
</tr>
</tbody>
</table>

9.4.9. Remark:-

For finite production rate and hyperbolic quantity discount, results obtained above justifies that with the increases in the value of mark up parameter \( \theta \) the optimal value of ITOR also increases and the optimum lot size decreases. Advertising, rate of inflation and damaged goods are highly sensitive with respect to total cost and such that ITOR.

9.5. Conclusion:-

From the optimal solutions given above for the hypothetical problem stated for various situations, it may be concluded that with the increases in the mark up parameter \( \theta \), the optimum value of ITOR also increases and the optimum lot size decreases it is clear that ITOR plays very important roll about inventory management of the firm under situation stated in the model.

We also conclude that,

(i) Advertising plays very important roll to bust up demand and hence raises the ITOR of the supplier.

(ii) Rate of inflation and rate of interest are highly sensitive to take care of the total inventory cost.

(iii) Lot size can be controlled in such a manner so that the damaged goods problem can be settled when ever needed.