CHAPTER 5

INVENTORY TURNOVER RATIO MODELS WITH SHORTAGE COSTS
5. CHAPTER 5: INVENTORY TURNOVER RATIO MODELS WITH SHORTAGE COSTS

5.1. Introduction:-

In this chapter ITOR Models are discussed with shortages costs. If shortages occur then firm suffers with direct and indirect losses. In direct losses, demand of the customer is met in the beginning of new production run otherwise the customer moves to some other firm to fulfill his requirements. This chapter deals with those cases of shortages where left over inventory turnover are supplied in such a way that risk of not fulfilling a customer requirements is minimized irrespective of maximum total cost. Here, the total cost on inventory turnover is maximum with increases in the size of inventory turnover due to instantaneous production. To handle such situation the given period is divided in to two equal parts. During first interval as soon as the desired units of a certain commodity arrive in inventory turnover the back turnovers of previous period are entertained according to the demand of a customer. In second interval when production does not take place then chance of shortages might occur. So our efforts is to minimize such risk and allow a customer not to go back to other firm but to provide him inventory turnover produced during first interval according to his genuine demand. The total inventory turnover cost also includes shortages costs.

This Inventory Turnover Ratio Model was suggested by S.B.Srivastava(1978) and Supported by Kanti Swaroop, Manmohan and Gupta (1994)
5.2. **Model – I: Inventory Turnover Ratio Model with Shortage Costs For Fixed Setup Cost.**

5.2.1. **Introduction:-**

In this section, we consider shortage cost along with holding cost and setup cost. Shortage cost indicates that how much cost of inventory turnover has caused the firm in loss. Here, replenishing of inventory turnover is instantaneous, i.e. for a given fixed period. Some cases are also proved and justified with the help of hypothetical examples. The EOQ approach is also been considered to formulate an inventory turnover ratio model.

5.2.2. **Notations:-**

\[
\begin{align*}
C_1 &= \text{Inventory holding Cost (Rs/Unit)} \\
C_2 &= \text{Shortage cost Rs. per unit} \\
C_3 &= \text{Inventory setup cost (Rs/Unit)} \\
p &= \text{Manufacturing cost per unit} \\
Q &= \text{A contract quantity produced at time period } t \\
t_1 &= \text{The first time interval during which the desired inventory turnover are drawn.} \\
t_2 &= \text{The second time interval during which the obtained inventory turnovers are accumulated but not fulfilled.} \\
Q_1 &= \text{The amount which goes into the inventory turnover} \\
Q_2 &= \text{The amount which is immediately taken to satisfy unfilled demand.}
\end{align*}
\]

5.2.3. **Assumption:-**

(i) Annual demand is known and constant
(ii) Lead time is zero
(iii) Chances of shortages may be occur
(iv) Rate of Replenishment is instantaneous
5.2.4. Problem Formulation:

Inventory holding cost during $t_1$ time interval:

$$\frac{Q_1 C_1 t_1}{2} = \frac{Q_1 C_1}{2} \times \frac{Q_1}{t_1} = \frac{Q_1^2 C_1}{2D}$$

Shortage cost during $t_2$ time interval

$$\frac{Q_2 C_2 t_2}{2} = \frac{Q_2 C_2}{2} \times \frac{Q_2}{t_2}$$

$$= \frac{Q_2^2 C_2}{2D}$$

but $Q = Q_1 + Q_2$

$$= \frac{(Q - Q_1)^2 C_2}{2D}$$

∴ Total cost for $t$ time interval

$$= \frac{Q_1^2 C_1}{2D} + \frac{(Q - Q_1)^2 C_2}{2D} + C_3$$

∴ Average inventory cost is defined as:

$$= \frac{1}{t} \left[ \frac{Q_1^2 C_1}{2D} + \frac{(Q - Q_1)^2 C_2}{2D} + C_3 \right]$$

but $Q = Dt$  \( \therefore t = \frac{Q}{D} \)

∴ Average inventory cost is defined as:

$$= \frac{D}{Q} \left[ \frac{Q_1^2 C_1}{2D} + \frac{(Q - Q_1)^2 C_2}{2D} + C_3 \right]$$

$$= \frac{Q_1^2 C_1}{2Q} + \frac{(Q - Q_1)^2 C_2}{2Q} + \frac{C_3 D}{Q}$$

∴ The inventor turnover ratio under this situation is defined as:

$$I(Q) = \frac{DP}{\left[ \frac{Q_1^2 C_1}{2Q} + \frac{(Q - Q_1)^2 C_2}{2Q} + \frac{C_3 D}{Q} \right]}$$  \( \ldots \ldots \quad 5.1 \)
5.2.5. Case I: \( D = nQ \)

\[
I(Q) = \frac{DP}{Q \left( \frac{Q_2 C_1}{2Q} + \frac{(Q - Q_1)^2 C_2}{2Q} + \frac{C_3 Q}{Q} \right)}
\]

...... 5.2

Differentiate equation (5.2) with respect to \( Q \) and equate it to zero, we get

\[
\frac{dI(Q)}{dQ} = -DP \left[ \frac{Q_2 C_1}{2Q} + \frac{(Q - Q_1)^2 C_2}{2Q} + \frac{C_3 Q}{Q} \right]^{2} \left[ -\frac{Q_2 C_1}{2Q^2} + \left( 1 - \frac{Q_1^2}{Q^2} \right) \frac{C_2}{2} - \frac{C_3 D}{Q^2} \right] = 0
\]

...... 5.3

\[
\therefore -DP \left[ \frac{Q_2 C_1}{2Q} + \frac{C_2}{2} - \frac{C_3 Q}{2Q^2} - \frac{C_3 D}{Q^2} \right] = 0
\]

\[
\therefore Q_2 C_1 + C_2 Q^2 - C_3 Q_1^2 - 2C_3 D = 0
\]

\[
\therefore C_2 Q^2 = (C_1 + C_2) Q_1^2 + 2C_3 D
\]

\[
\therefore Q = \sqrt{\frac{(C_1 + C_2) Q_1^2 + 2C_3 D}{C_2}}
\]

...... 5.4

Equation (5.4) is solved by successive approximation with the help of computer program under certain constraints it is observed that the inventory turnover ratio is negative if the second derivative \( \frac{d^2 I(Q)}{dQ^2} < 0 \).

\[
\frac{d^2 I(Q)}{dQ^2} = \sqrt{\left[ \frac{-DP C_2 Q_1^2}{Q^3} - \frac{Dp C_3 Q_1^2}{Q^3} - \frac{2D^2 C_3 P}{Q^3} \right] \left[ \frac{C_1 Q_1^2}{2Q} + \frac{C_2 Q}{2} - \frac{C_2 Q_1 + C_3 Q_2}{2Q} + \frac{C_3 D}{Q} \right]}
\]

\[
- \left[ \frac{2}{2Q^3} \left( \frac{Dp C_2 Q_1^2}{2} - \frac{Dp C_3 Q_1^2}{2Q^2} + \frac{D^3 C_3 P}{Q} \right) \right] \left[ \frac{-C_1 Q_1^2}{2Q^2} + \frac{C_2}{2} - \frac{C_2 Q_1 + C_3 Q_2}{2Q^2} - \frac{C_3 D}{Q^2} \right]
\]

\[
\left[ \frac{C_2 Q_1^2}{2Q} + \frac{C_2 Q}{2} - \frac{C_2 Q_1 + C_3 Q_2}{2Q} + \frac{C_3 D}{Q} \right] < 0
\]
5.2.6. Hypothetical Problem:-

<table>
<thead>
<tr>
<th>No.</th>
<th>Holding Cost (per Unit)</th>
<th>Shortages Cost (per Unit)</th>
<th>Setup Cost (per order)</th>
<th>Purchase Cost (per Unit)</th>
<th>Demand</th>
<th>Maximum Inventory Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20 Rs.</td>
<td>10 Rs.</td>
<td>Rs. 60</td>
<td>Rs. 5</td>
<td>10,000</td>
<td>1,000</td>
</tr>
<tr>
<td>2</td>
<td>20 Rs.</td>
<td>11 Rs.</td>
<td>Rs. 50</td>
<td>Rs. 6</td>
<td>12,000</td>
<td>1,000</td>
</tr>
<tr>
<td>3</td>
<td>20 Rs.</td>
<td>12 Rs.</td>
<td>Rs. 40</td>
<td>Rs. 7</td>
<td>15,000</td>
<td>1,000</td>
</tr>
</tbody>
</table>

Output of the hypothetical problem is,

| Item Demand | Holding Short Order | Purchase | Maximum | Optimal | Total Inventory Turnover | Inventory Turnover | Demand Cost | Shortage Cost | Setup Cost | Purchase Cost | Maximum Inventory Level | Inventory Turnover |
|-------------|---------------------|----------|---------|---------|--------------------------|--------------------|-------------|--------------|-------------|-------------|------------------------|---------------------|-------------|
| D           | C1                  | C2       | C3      | p       | Q1                       | Qopt               | C(Qopt)     | C(Qopt)      | C(Qopt)     | C(Qopt)     | Q1                     | C(Qopt)             |             |
| 1 10000     | 20.00               | 10.00    | 60.00   | 5       | 1000                     | 1766               | 7664        | 6.52         |             |             |                        |                     |             |
| 2 12000     | 20.00               | 11.00    | 50.00   | 6       | 1000                     | 1711               | 7820        | 9.21         |             |             |                        |                     |             |
| 3 15000     | 20.00               | 12.00    | 40.00   | 7       | 1000                     | 1663               | 7960        | 13.19        |             |             |                        |                     |             |

5.2.7. Remark:-

It is clear from the above result, optimal turnover and ITOR is depending on demand. The size of the turnover decreases, total inventory cost increases as shortage cost increases. Fluctuation observed in ITOR with respect to above reasons. Thus, with this approach firm must be considering optimum turnover regarding the minimum total inventory cost.

5.2.8. Case II: \( D = \alpha + \beta Q \)

Where \( \alpha \) and \( \beta \) are constants, \( Q \) is a Contract quantity.

The Inventory Turnover Ratio is defined as:

\[
I(Q) = \frac{(\alpha P + \beta Q P)}{\left[ \frac{C_1 Q^2}{2Q} + \frac{C_2 Q}{2} - C_3 Q, + \frac{C_3 Q^2}{2Q} + \frac{C_3 \alpha}{Q} + \beta C_3 \right]} \quad \cdots \quad 5.5
\]
Differentiate equation (5.5) with respect to $Q$ and equate it to zero, we get

$$\frac{\partial I(Q)}{\partial Q} = \left\{ \beta p \left[ \frac{C_1 Q_1^2}{2Q} + \frac{C_2 Q_2^2}{2} - C_2 Q_1 + \frac{C_2 Q_2^2}{2Q} + \frac{C_3}{Q} + \beta C_3 \right] - (\alpha p + \beta Q p) \right\}$$

$$\left[ -\frac{C_1 Q_1^2}{2Q^2} + \frac{C_2 Q_2^2}{2} - C_2 Q_1 + \frac{C_2 Q_2^2}{2Q} + \frac{C_3}{Q} + \beta C_3 \right]^2 = 0$$

$$\therefore \left( 2\beta^2 C_3 - 2\beta C_2 Q_1 - \alpha C_2 \right) Q_1^2 + \left( 2\beta (C_1 + C_2) Q_2^2 + 4\alpha \beta C_3 \right) Q + \left( \alpha (C_1 + C_2) Q_1^3 + 2\alpha^2 C_3 \right) = 0 \quad \ldots \quad 5.6$$

This implies that,

$$Q = \left\{ \frac{2\beta Q_1^3 (C_1 + C_2) + 4\beta C_3 \alpha \pm \sqrt{4\beta^2 Q_1^2 (C_1 + C_2)^2 Q_1^2 (C_1 + C_2) + 4 \alpha C_3 \alpha} \right\}$$

$$+ 4\alpha Q_1^2 C_2 (C_1 + C_2) (2\beta Q_1 + \alpha) + 8\alpha \beta C_3 (2\alpha C_2 - \beta Q_1 (C_1 + C_2))$$

$$+ 8\alpha C_2 C_3 \right\} \left[ 4\beta^2 C_3 - 4\beta C_2 Q_1 - 2\alpha C_2 \right] \quad \ldots \quad 5.7$$

The equation (5.7) gives optimum turnover which is solved by successive approximation with help of computer program. Under certain constraints it is observed that the turnover ratio is negative if $\frac{\partial^2 I(Q)}{\partial Q^2} > 0$.

$$\frac{\partial^3 I(Q)}{\partial Q^3} = \left[ \beta p Q_1^2 (C_1 + C_2) - \frac{2\alpha p Q_1^2}{2Q^3} (C_1 + C_2) - \frac{\beta p Q_1^2}{2Q^3} (C_1 + C_2) - \frac{2\alpha \beta p C_3}{Q^2} + \frac{2\alpha^2 p C_3}{Q^3} \right]$$

$$\left[ \frac{C_1 Q_1^2}{2Q} + \frac{C_2 Q_2^2}{2} - C_2 Q_1 + \frac{C_2 Q_2^2}{2Q} + \frac{C_3}{Q} + \beta C_3 \right]^2$$

$$\left[ \frac{\beta p Q_1^2}{2Q} (C_1 + C_2) - \frac{2\alpha p Q_1^2}{2Q^2} (C_1 + C_2) - \frac{\beta p Q_1^2}{2Q} (C_1 + C_2) - \beta p C_2 Q_1 + \frac{2\alpha \beta p C_3}{Q^2} + \frac{\beta^2 p C_3}{2} + \frac{\alpha^2 p C_3}{Q^2} \left[ -\frac{C_1 Q_1^2}{2Q^2} + \frac{C_2}{2} - \frac{C_2 Q_2^2}{2Q^2} - \frac{C_3}{Q} \right] \right]$$

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5.2.9. Hypothetical Problem:

Let us take a problem discussed in 5.2.6 where

\[
\left[ \frac{C_1 Q^2}{2Q} + \frac{C_2 Q}{2} - \frac{C_2 Q}{Q} + \frac{C_3 Q^2}{2Q} + \frac{C_3 \alpha}{Q} + \beta C_3 \right]^3 < 0
\]

<table>
<thead>
<tr>
<th>No</th>
<th>(\alpha)</th>
<th>(\beta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>250</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
<td>7</td>
</tr>
</tbody>
</table>

Output of the hypothetical problem is,

<table>
<thead>
<tr>
<th>Item</th>
<th>Demand (Units)</th>
<th>Holding Cost</th>
<th>Short Order Cost</th>
<th>Purchasing Cost</th>
<th>Maximum Level</th>
<th>Optimal Cost</th>
<th>Total Inventory Cost</th>
<th>Inventory Turnover Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15463</td>
<td>20.00</td>
<td>10.00</td>
<td>60.00</td>
<td>5</td>
<td>1000</td>
<td>3053</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>17407</td>
<td>20.00</td>
<td>11.00</td>
<td>50.00</td>
<td>6</td>
<td>1000</td>
<td>2859</td>
<td>1.000</td>
</tr>
<tr>
<td>3</td>
<td>19165</td>
<td>20.00</td>
<td>12.00</td>
<td>40.00</td>
<td>7</td>
<td>1000</td>
<td>2695</td>
<td>1.000</td>
</tr>
</tbody>
</table>

5.2.10. Remark:

Here, consumption rate depend on \(\alpha\), \(\beta\) and shortage cost. Total inventory cost with respect to demand is in decreasing order which is beneficial for the firm under this approach.

5.2.11. Case III: \(D = \beta Q^x\)

Where \(0 < x < 1\); \(x\) is a variable \(\beta\) is a constant and \(Q\) is a Contract quantity. The Inventory turnover ratio is defined as:

\[
I(Q) = \beta p Q^x \left[ \frac{C_1 Q^2}{2Q} + \frac{C_2 Q}{2} - \frac{C_3 Q}{Q} + \frac{C_3 Q^2}{2Q} + \beta C_3 Q^{-1} \right]^{-1}
\]

Differentiate equation (5.8) with respect to \(Q\) and equate it to zero, we get

\[
\frac{\partial I(Q)}{\partial Q} = x \beta p Q^{x-1} \left[ \frac{C_1 Q^2}{2Q} + \frac{C_2 Q}{2} - \frac{C_3 Q}{Q} + \frac{C_3 Q^2}{2Q} + \beta C_3 Q^{-1} \right]^{-1}
\]
\[ \beta p Q \left[ \frac{-C_2 Q_1^2}{2Q^2} + \frac{C_2}{2} - \frac{C_2 Q_1^2}{2Q^2} + (x-1)\beta C_3 Q^{-2} \right] \]
\[ \left[ \frac{C_2 Q_1^2}{2Q} + \frac{C_2 Q_1^2}{2Q} - C_3 Q_1 + \frac{C_2 Q_1^2}{2Q} + \beta C_3 Q^{-1} \right] ^2 = 0 \]

\[ \therefore 2 \beta C_3 Q^* + (x-1)C_3 Q^2 - 2xC_3 Q_1 Q + (x+1)(C_1 + C_2) Q_1^2 = 0 \quad \ldots \phantom{.} 5.9 \]

This implies that,
\[ Q_{\text{opt}} = \left[ \frac{(1-x)C_3 Q^2 + 2xC_3 Q_1 Q - (x+1)(C_1 + C_2) Q_1^2}{2\beta C_3} \right]^{\nu_t} \quad \ldots \phantom{.} 5.10 \]

The equation (5.10) indicates that \( Q_{\text{opt}} \) is optimum turnover which is solved by successive approximation with the help of computer program. Under certain constraints it is observed that the turnover is negative if second order derivative \( \frac{\partial^2 I(Q)}{\partial Q^2} > 0 \)

\[ \frac{\partial^2 I(Q)}{\partial Q^2} = \left[ \frac{x(x-1)\beta p C_2 Q^{-1}}{2} \right] + \left[ \frac{(x-2)(C_1 + C_2)\beta p Q_1^2 Q^{-3}}{2} \right] + \left[ \frac{x(x+1)(x-2)\beta p Q_1^2 Q^{-3}}{2} \right] - \left[ x(x-1)\beta p C_3 Q^{-2} \right] + \left[ \frac{(2x-2)\beta^2 p C_3 Q^{21-3}}{2} \right] \left( \left[ \frac{C_2 Q_1^2}{2Q} + \frac{C_2 Q_1^2}{2Q} - C_3 Q_1 + \frac{C_2 Q_1^2}{2Q} + \beta C_3 Q^{-1} \right] ^2 \right) \]
\[ \times 2 \left[ \left[ \frac{(x-1)\beta p C_3 Q^* + \beta p Q_1^2 Q^{-2}(C_1 - C_2) + \frac{x(x+1)\beta p Q_1^2 Q^{-2}}{2} - \frac{x\beta p C_2 Q_1 Q^{-1}}{2} }{2} \right] + \beta^2 p C_3 Q^{21-2} \left[ \left[ \frac{-C_1 Q_1^2}{2Q^2} + \frac{C_2}{2} - \frac{C_2 Q_1^2}{2Q^2} + (x-1)\beta C_3 Q^{-2} \right] \right] \right] \]
\[ \left[ \frac{C_2 Q_1^2}{2Q} + \frac{C_2 Q_1^2}{2Q} - C_3 Q_1 + \frac{C_2 Q_1^2}{2Q} + \beta C_3 Q^{-1} \right] ^3 < 0 \]

5.2.12. Hypothetical Problem:-

Let us take a problem discussed in 5.2.6 where:-

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Output of the hypothetical problem is,

<table>
<thead>
<tr>
<th>Item Demand (Units)</th>
<th>Holding Cost C1</th>
<th>Short Order Cost C2</th>
<th>Order Level Q</th>
<th>Purchasing Cost C3</th>
<th>Maximum Inventory (Units) Qopt</th>
<th>Optimal Cost C(Qopt)</th>
<th>Total Inventory Cost I(Qopt)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11414</td>
<td>20.00</td>
<td>10.00</td>
<td>60.00</td>
<td>5</td>
<td>1000</td>
<td>2805</td>
</tr>
<tr>
<td>2</td>
<td>10349</td>
<td>20.00</td>
<td>11.00</td>
<td>50.00</td>
<td>6</td>
<td>1000</td>
<td>2468</td>
</tr>
<tr>
<td>3</td>
<td>11026</td>
<td>20.00</td>
<td>12.00</td>
<td>40.00</td>
<td>7</td>
<td>1000</td>
<td>2227</td>
</tr>
</tbody>
</table>

5.2.13. Remark:-

In this case, demand, optimal turnover and total inventory cost are too high and ITOR is small which indicates unexpected market situation. Therefore financial manager of the firm must be taken appropriate steps with respect to the inventories.

5.2.14. Case IV: \( D = \beta_1 Q - \beta_2 Q^2 \)

\( \beta_1, \beta_2 \) are constants and \( Q \) is contract quantity.

The Inventory turnover ratio is defined as:

\[
I(Q) = \left[ \beta_1 Q - \beta_2 Q^2 \right] \left[ \frac{C_1 Q^2}{2Q} + \frac{C_2 Q}{2} - C_3 Q + \frac{C_3 Q^2}{2Q} + \beta_1 C_3 + \beta_2 C_3 Q \right]^{-1}
\]

...... 5.11

Differentiate equation (5.11) with respect to \( Q \) and equate it to zero, we get

\[
\frac{dT(Q)}{dQ} = \left[ \beta_1 Q - 2\beta_2 Q^2 \left[ \frac{C_1 Q^2}{2Q} + \frac{C_2 Q}{2} - C_3 Q + \frac{C_3 Q^2}{2Q} + \beta_1 C_3 + \beta_2 C_3 Q \right] - \left[ \beta_1 Q - \beta_2 Q^2 \left[ \frac{-C_1 Q^2}{2Q^2} + \frac{C_2}{2} - \frac{C_3 Q^2}{2Q^2} + \beta_3 C_3 \right] \right] / \right]
\]
\[
\left[ \frac{c_1q^3}{2q} + \frac{c_2q}{2} - c_2q_1 + \frac{c_2q_1^2}{2q} + \beta_1c_3 + \beta_2c_3q_1 \right]^2 = 0 \quad \text{(5.12)}
\]

\[
\therefore \beta_1(c_2 + 2\beta_2c_3)q^3 - 4\beta_2(c_2q_1 - \beta_1c_3)q^2 + \\
\left(3\beta_2q^2(c_1 + c_2) + 2\beta_1(c_2q_1 + \beta_1c_3)q - 2\beta_1(c_1 + c_2)q^2 \right) = 0
\]

This implies that,
\[
q_{opt} = \{-[2\beta_2c_2q_1 - 2\beta_1c_3] \pm \sqrt{\beta_2^2q^4(c_1^2 - 3c_1c_2) + 6\beta_2^3c_3q^2} \}
\]

\[
(c_1 + c_2) + 2\beta_1\beta_2c_2c_3(c_1 - 2\beta_2q_1 - \frac{4\beta_1\beta_2^2c_3q^2}{q}(c_1 + c_2)
\]

\[
+ \frac{2\beta_1\beta_2q^2}{q}(c_1c_2 + c_2^2) - 2\beta_1\beta_2c_2^2q_1 \}
\end{array}
\right] \frac{\sqrt{\beta_2^2c_3 - \beta_2^2c_2}}{2} \quad \text{(5.13)}
\]

The equation (5.13) indicates that \(q_{opt}\) is the optimum turnover and is obtained by successive approximation with the help of computer program. Under certain constraints it is observed that the turnover ratio is negative if second derivative is positive.

\[
\frac{\partial^2 I(Q)}{\partial Q^2} = \left[ -\frac{\beta_1pC_1q^2}{Q^2} - \beta_2pC_2Q + 2\beta_2^2pC_3Q - \frac{\beta_1pC_2q^2}{Q^2} + 2\beta_2pC_2Q - 2\beta_1C_3 \right]
\]

\[
\frac{2\beta_1\beta_2pC_3}{Q} \left[ \frac{c_1q^2}{2q} + \frac{c_2q}{2} - c_2q_1 + \frac{c_2q_1^2}{2q} + \beta_1c_3 + \beta_2c_3Q \right]^2 \}
\]

\[
\left[ \frac{\beta_1pq^2}{Q}(c_1 - c_2) - \frac{3\beta_2pq^2}{2}(c_1 - c_2) + \beta_2pQ^2 \left[ \beta_2c_3 - \frac{c_2}{2} \right] + \\
pQ_1(2\beta_2c_2q - \beta_1c_3) + \beta_1pC_3(\beta_1 - 2\beta_2q) \right] \left[ \frac{c_1q^2}{2q^2} + \frac{c_2}{2} - \frac{c_2q^3}{2q^3} - \beta_2c_3 \right]
\]

\[
\left[ \frac{c_2q^2}{2q} + \frac{c_2q}{2} - c_2q_1 + \frac{c_2q_1^2}{2q} + \beta_2c_1q_1 \right] < 0
\]

5.2.15. Hypothetical Problem:

Let us take a problem discussed in 5.2.6 where:

<table>
<thead>
<tr>
<th>No.</th>
<th>(\beta_1)</th>
<th>(\beta_2)</th>
</tr>
</thead>
</table>

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Output of the hypothetical problem is,

<table>
<thead>
<tr>
<th>Item</th>
<th>Demand (Units)</th>
<th>Holding Cost C1</th>
<th>Short Order Cost C2</th>
<th>Purchasing Cost C3</th>
<th>Minimum Order Quantity Q1</th>
<th>Maximum Inventory Level Qopt</th>
<th>Optimal Inventory Level Qopt</th>
<th>Total Inventory Turnover (Units)</th>
<th>Cost C(Qopt)</th>
<th>Inventory Turnover Rate (Ratio)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2232</td>
<td>20.00</td>
<td>10.00</td>
<td>60.00</td>
<td>5</td>
<td>1000</td>
<td>91</td>
<td>2383</td>
<td>4.68</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>3667</td>
<td>20.00</td>
<td>11.00</td>
<td>50.00</td>
<td>6</td>
<td>1000</td>
<td>129</td>
<td>2712</td>
<td>8.11</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>7767</td>
<td>20.00</td>
<td>12.00</td>
<td>40.00</td>
<td>7</td>
<td>1000</td>
<td>210</td>
<td>3580</td>
<td>15.19</td>
<td></td>
</tr>
</tbody>
</table>

5.2.16. Remarks:-

Here, consumption rate $\beta_1$ and $\beta_2$. Therefore optimal turnover, total inventory cost and ITOR increases with respect to $\beta_1$ and $\beta_2$. Thus financial manager of the firm find out optimum order quantity with respect to the situation stated above.

5.3. Model – II: Inventory Turnover Ratio Model with Shortage Costs for Varying Setup Cost.

5.3.1. Introduction:-

In this model, we consider shortage cost along with holding cost and setup cost is not fixed but varies with respect to optimal turnover. Shortage cost indicates that how much cost of inventory turnover has cause the firm in to loss. Here, replenishing of inventory turnover is instantaneous. i.e. for a given fixed period. Some results are also proved and justified with the help of hypothetical examples. The EOQ approach is also been considered to formulate an inventory turnover ratio model.
5.3.2. Notations:-

\[ \begin{align*}
C_1 &= \text{Inventory holding Cost (Rs/Unit)} \\
C_2 &= \text{Shortage cost Rs. per unit} \\
C_3 &= \text{Inventory setup cost (Rs/Unit)} \\
p &= \text{Manufacturing cost per unit} \\
Q &= \text{A contract quantity produced at time period } t \\
t_1 &= \text{The first time interval during which the desired inventory turnover are drawn.} \\
t_2 &= \text{The second time interval during which the obtained inventory turnovers are accumulated but not fulfilled.} \\
Q_1 &= \text{The amount which goes into the inventory turnover} \\
Q_2 &= \text{The amount which is immediately taken to satisfy unfilled demand.}
\end{align*} \]

5.3.3. Assumption:-

(i) Annual demand is known and constant
(ii) Lead time is zero
(iii) Chances of shortages may be occur
(iv) Rate of Replenishment is instantaneous

5.3.4. Problem Formulation:-

\[ \begin{align*}
\text{Average inventory cost is defined as:} \\
T(Q) &= \frac{D}{Q} \left[ \frac{Q_1^2 C_1}{2D} + \frac{(Q - Q_1)^2 C_2}{2D} + C_3 \right] \\
&= \frac{Q_1^2 C_1}{2Q} + \frac{(Q - Q_1)^2 C_2}{2Q} + \frac{C_3 D}{Q}
\end{align*} \]

\[ \begin{align*}
\text{The inventor turnover ratio under this situation is defined as:} \\
I(Q) &= \frac{DP}{\left[ \frac{Q_1^2 C_1}{2Q} + \frac{(Q - Q_1)^2 C_2}{2Q} + \frac{C_3 D}{Q} \right]} \\
&= \frac{DP}{\left[ \frac{Q_1^2 C_1}{2Q} + \frac{(Q - Q_1)^2 C_2}{2Q} + \frac{C_3 D}{Q} \right]} \\
&= 5.14
\end{align*} \]
5.3.5. Case I: \( D = nQ, C_3 = C_3 + bQ \)

\[
I(Q) = \frac{DP}{Q_1^2C_1 + \left(\frac{Q - Q_1}{2Q}\right)C_2 + \frac{(C_3 + bQ)D}{Q}}
\]

Differentiate equation (5.15) with respect to \( Q \) and equate it to zero, we get

\[
\frac{\partial I(Q)}{\partial Q} = -Dp \left[ \frac{Q_1^2C_1 + \left(\frac{Q - Q_1}{2Q}\right)C_2 + \frac{(C_3 + bQ)D}{Q}}{2Q} \right]^2
\]

\[
\left[ -\frac{Q_1^2C_1}{2Q^2} + \left(1 - \frac{Q_1^2}{Q^2}\right)\frac{C_2}{2} - \frac{C_1D}{Q^2} \right] = 0
\]

\[
\frac{\partial^2 I(Q)}{\partial Q^2} < 0
\]

\[
\therefore -Dp \left[ -\frac{Q_1^2C_1}{2Q} + \frac{C_2}{2} - \frac{Q_2Q_1^2}{2Q^2} - \frac{C_1D}{Q^2} \right] = 0
\]

\[
\therefore Q_1^2C_1 + C_2Q_2^2 - C_2Q_1^2 - 2C_3D = 0
\]

\[
\therefore Q_2^2\left(C_1 + C_3\right) + C_2Q_2^2 - 2C_3D = 0
\]

\[
\therefore C_2Q_2^2 = (C_1 + C_3)Q_1^2 + 2C_3D
\]

\[
\therefore Q_{opt} = \frac{\sqrt{(C_1 + C_2)Q_1^2 + 2C_3D}}{C_2}
\]

Equation (5.17) is solved by successive approximation with the help of computer program under certain constraints it is observed that the inventory turnover ratio is negative if the second derivative \( \frac{\partial^2 I(Q)}{\partial Q^2} > 0 \).

5.3.6. Hypothetical Problem:-

<table>
<thead>
<tr>
<th>No</th>
<th>Holding Cost (per Unit)</th>
<th>Shortages Cost (per Unit)</th>
<th>Setup Cost (per order)</th>
<th>Purchase Cost (per Unit)</th>
<th>Demand D</th>
<th>Maximum Inventory Level Q_1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20 Rs.</td>
<td>10 Rs.</td>
<td>Rs. 60</td>
<td>Rs. 5</td>
<td>10,000</td>
<td>1,000</td>
</tr>
<tr>
<td>2</td>
<td>20 Rs.</td>
<td>11 Rs.</td>
<td>Rs. 50</td>
<td>Rs. 6</td>
<td>12,000</td>
<td>1,000</td>
</tr>
</tbody>
</table>
Solution of the hypothetical problem is,

<table>
<thead>
<tr>
<th>Item</th>
<th>Demand</th>
<th>Holding Cost</th>
<th>Short Order Cost</th>
<th>Purchasing Cost</th>
<th>Maximum Inventory Level</th>
<th>Optimal Inventory Level</th>
<th>Total Inventory Cost</th>
<th>Inventory Turnover</th>
<th>Inventory Turnover Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10000</td>
<td>20.00</td>
<td>10.00</td>
<td>77.66</td>
<td>5</td>
<td>1000</td>
<td>1766</td>
<td>764</td>
<td>6.44</td>
</tr>
<tr>
<td>2</td>
<td>12000</td>
<td>20.00</td>
<td>11.00</td>
<td>67.11</td>
<td>6</td>
<td>1000</td>
<td>1711</td>
<td>7940</td>
<td>9.07</td>
</tr>
<tr>
<td>3</td>
<td>15000</td>
<td>20.00</td>
<td>12.00</td>
<td>56.63</td>
<td>7</td>
<td>1000</td>
<td>1663</td>
<td>8110</td>
<td>12.95</td>
</tr>
</tbody>
</table>

5.3.7. Remark:

It is clear from the above result, optimal turnover and ITOR is depend on demand and setup cost. The size of the turnover decreases, total inventory cost increases as shortage cost increases. Fluctuation observed in ITOR with respect to above reasons. Thus, with this approach firm must be considering optimum turnover regarding the minimum total inventory cost. Also we observe marginal effect on ITOR with respect to setup cost.

5.3.8. Case II: $D = \alpha + \beta Q$, $C_3 = C_3 + bQ$

Where $\alpha$ and $\beta$ are constants, $Q$ is a Contract quantity.

The Inventory Turnover Ratio is defined as:

$$I(Q) = \frac{(\alpha p + \beta Qp)}{\left[\frac{c_1 Q^2}{2Q} + \frac{c_2 Q^2}{2Q} - c_3 Q + \left(\frac{c_3 + bQ)\alpha}{Q} + \beta(c_3 + bQ)\right]\right]}$$

5.18

Differentiate equation (5.18) with respect to $Q$ and equate it to zero, we get

$$\frac{dI(Q)}{dQ} = \beta p \left[\frac{c_1 Q^2}{2Q} + \frac{c_2 Q^2}{2Q} - c_3 Q + \left(\frac{c_3 + bQ)\alpha}{Q} + \beta(c_3 + bQ)\right]\right]$$

$$= \left[\frac{c_1 Q^2}{2Q} + \frac{c_2 Q^2}{2Q} - c_3 Q + \left(\frac{c_3 + bQ)\alpha}{Q} + \beta(c_3 + bQ)\right]\right] = 0$$
\[ \begin{aligned}
&\{(2\beta^2 C_3 - 2\beta C_2 Q_1 - \alpha C_2)Q_2^2 + (2\beta(C_1 + C_2)Q_1^2 + 4\alpha\beta C_3)Q + \\
&(\alpha(C_1 + C_2)Q_1^2 + 2\alpha^2 C_3) = 0 \quad \ldots \quad 5.19
\end{aligned} \]

and
\[ \frac{\partial^2 I(Q)}{\partial Q^2} < 0 \]

This implies that,
\[ Q_{\text{opt}} = \left\{ \frac{2\beta Q_1^2 (C_1 + C_2) + 4\beta C_3 \alpha}{4\beta^2 Q_1^2 (C_1 + C_2)Q_1^2 (C_1 + C_2) + 4C_3 \alpha} \right\} \]
\[ + 4 \alpha Q_1^2 C_2 (C_1 + C_2) (2\beta Q_1 + \alpha) + 8 \alpha \beta C_3 Q_1 (2\alpha C_2 - \beta Q_1 (C_1 + C_2)) \]
\[ + 8 \alpha^2 C_2 C_3 \right\}^{1/2} [4\beta^2 C_3 - 4\beta C_2 Q_1 - 2\alpha C_2] \quad \ldots \quad 5.20 \]

The equation (5.20) gives optimum turnover, which is solved by successive approximation with help of computer program. Under certain constraints it is observed that the turnover ratio is negative if the \( \frac{\partial^2 I(Q)}{\partial Q^2} > 0 \).

**5.3.9. Hypothetical Problem:-**

Let us take a problem discussed in 5.3.6 where

<table>
<thead>
<tr>
<th>No</th>
<th>( \alpha )</th>
<th>( \beta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>200</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>250</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>300</td>
<td>7</td>
</tr>
</tbody>
</table>

Output of the hypothetical problem is,

<table>
<thead>
<tr>
<th>Item</th>
<th>Demand (Units)</th>
<th>Cost (( C_1 ))</th>
<th>Short Order Cost (( C_2 ))</th>
<th>Order Aged Cost (( C_3 ))</th>
<th>Purch (( p ))</th>
<th>Maximum Inventory (( Q_1 ))</th>
<th>Optimal Inventory (( Q_{\text{opt}} ))</th>
<th>Total Inventory Turnover</th>
<th>Inventory Turnover Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15463</td>
<td>20.00</td>
<td>10.00</td>
<td>90.53</td>
<td>5</td>
<td>1000</td>
<td>3053</td>
<td>10635</td>
<td>7.27</td>
</tr>
<tr>
<td>2</td>
<td>17407</td>
<td>20.00</td>
<td>11.00</td>
<td>78.59</td>
<td>6</td>
<td>1000</td>
<td>2859</td>
<td>10626</td>
<td>9.83</td>
</tr>
<tr>
<td>3</td>
<td>19165</td>
<td>20.00</td>
<td>12.00</td>
<td>66.95</td>
<td>7</td>
<td>1000</td>
<td>2695</td>
<td>10583</td>
<td>12.68</td>
</tr>
</tbody>
</table>
5.3.10. Remark:-

Here, consumption rate depend on $a$, $b$ and shortage cost. Total inventory cost with respect to demand is in decreasing order which is beneficial for the firm under this approach. Also we observe marginal effect on ITOR with respect to setup cost.

5.3.11. Case III: $D = \beta Q^x$, $C_3 = C_3 + bQ$

Where $0 < x < 1$; $x$ is a variable $\beta$ is a constant and $Q$ is a Contract quantity.

The Inventory turnover ratio is defined as:

$$I(Q) = \beta pQ \left[ \frac{C_1Q^2}{2Q} + \frac{C_2Q}{2} - C_2Q + \frac{C_2Q^2}{2Q} + \beta(C_3 + bQ)Q^{-1} \right]^{-1}$$

…….. 5.21

Differentiate equation (5.21) with respect to $Q$ and equate it to zero, we get

$$\frac{dI(Q)}{dQ} = x\beta pQ^{-1} \left[ \frac{C_1Q^2}{2Q} + \frac{C_2Q}{2} - C_2Q + \frac{C_2Q^2}{2Q} + \beta(C_3 + bQ)Q^{-1} \right]^{-2}$$

$$\beta pQ^{-1} \left[ -\frac{C_1Q}{2Q^2} + \frac{C_2}{2} - \frac{C_2Q^2}{2Q^2} + (x-1)\beta C_3Q^{-x} + bxQ^{-1} \right]$$

$$\left[ \frac{C_1Q^2}{2Q} + \frac{C_2Q}{2} - C_2Q + \frac{C_2Q^2}{2Q} + \beta(C_3 + bQ)Q^{-1} \right]^2 = 0$$

$\therefore 2\beta C_3Q^x + (x-1)C_2Q^2 - 2xC_2Q_1Q + (x+1)(C_1 + C_3)Q_1^2 = 0$…….. 5.22

and $\frac{d^2I(Q)}{dQ^2} < 0$

This implies that,

$$Q_{opt} = \left[ \frac{(1-x)C_2Q^2 + 2xC_2Q_1Q - (x+1)(C_1 + C_3)Q_1^2}{2\beta C_3} \right]^{1/4}$$

…….. 5.23
The equation (5.23) indicates that $Q_{\text{opt}}$ is optimum turnover which is solved by successive approximation with the help of computer program. Under certain constraints it is observed that the turnover is negative if second order derivative $\frac{d^2 I(Q)}{dQ^2} > 0$

5.3.12. Hypothetical Problem:-

Let us take a problem discussed in 5.3.6 where:

<table>
<thead>
<tr>
<th>No.</th>
<th>$\beta$</th>
<th>$x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>0.9</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>0.8</td>
</tr>
<tr>
<td>3</td>
<td>50</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Output of the hypothetical problem is,

<table>
<thead>
<tr>
<th>Item</th>
<th>Demand (Units)</th>
<th>Holding Cost</th>
<th>Short Order Purchasing Cost</th>
<th>Maximum Order Level</th>
<th>Optimal $Q_{\text{opt}}$</th>
<th>Total Cost</th>
<th>Inventory Turnover Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11414</td>
<td>20.00</td>
<td>10.00</td>
<td>88.05</td>
<td>5</td>
<td>2805</td>
<td>28413</td>
</tr>
<tr>
<td>2</td>
<td>10349</td>
<td>20.00</td>
<td>11.00</td>
<td>74.68</td>
<td>6</td>
<td>2468</td>
<td>24992</td>
</tr>
<tr>
<td>3</td>
<td>11026</td>
<td>20.00</td>
<td>12.00</td>
<td>62.27</td>
<td>7</td>
<td>2227</td>
<td>22583</td>
</tr>
</tbody>
</table>

5.3.13. Remark:-

In this case, demand, optimal turnover and total inventory cost are too high and ITOR is small which indicates unexpected market situation. Therefore financial manager of the firm must be taken appropriate steps with respect to the inventories. Also we observe marginal effect on ITOR with respect to setup cost.

5.3.14. Case IV: $D = \beta_1 Q - \beta_2 Q^2$, $C_3 = C_3 + bQ$

$\beta_1$, $\beta_2$ are constants and $Q$ is contract quantity.

The Inventory turnover ratio is defined as:
Differentiate equation (5.24) with respect to \( Q \) and equate it to zero, we get

\[
\frac{\partial I(Q)}{\partial Q} = \{\beta_1 p - 2 \beta_2 Q p\}.
\]

\[
\left[ \frac{C_1 Q^2}{2Q} + \frac{C_2 Q}{2} - C_2 Q_1 + \frac{C_2 Q_1^2}{2Q} + \beta_1 (C_3 + bQ) + \beta_2 (C_3 + bQ)Q \right] - \frac{\beta_1 Q p - \beta_2 Q^3 p}{\beta_1 Q p - \beta_2 Q^3} \left[ \frac{-C_1 Q_1^2}{2Q^2} + \frac{C_2 Q_1}{2Q^2} + \beta_1 b + \beta_2 bQ + 2 \beta_2 bQ \right] = 0
\]

\[
\left[ \frac{C_1 Q^2}{2Q} + \frac{C_2 Q}{2} - C_2 Q_1 + \frac{C_2 Q_1^2}{2Q} + \beta_1 (C_3 + bQ) + \beta_2 (C_3 + bQ)Q \right]^2 = 0
\]

\[
\therefore \beta_2 (C_2 + 2 \beta_2 C_3) Q^3 - 4 \beta_2 (C_2 Q_1 - \beta_1 C_3) Q^2 + \\
\left(3 \beta_2 Q_1^2 (C_1 + C_3) + 2 \beta_1 (C_2 Q_1 + \beta_1 C_3) \right) Q - 2 \beta_1 (C_1 + C_3) Q^2 = 0 \quad \ldots 5.27
\]

and \( \frac{\partial^2 I(Q)}{\partial Q^2} < 0 \)

This implies that,

\[
Q = \left\{ \frac{-2 \beta_2 C_3 Q_1 - 2 \beta_1 C_2 C_3}{1 \pm \frac{\beta_2}{2} \left(1 + \frac{\beta_1}{C_2 Q_1 - 3 C_1 C_3} + 6 \beta_2^2 C_3 Q_1^2 \right)} \right\} \left( C_1 + C_3 \right) + 2 \beta_1 \beta_2 C_2 C_3 (\beta_1 - 2 \beta_2 Q_1) - \frac{4 \beta_1 \beta_2 C_3 Q_1^2}{Q} (C_1 + C_3)
\]
The equation (5.28) indicates that \( Q_{\text{opt}} \) is the optimum turnover and is obtained by successive approximation with the help of computer program. Under certain constraints it is observed that the turnover ratio is negative if second derivative is positive.

5.3.15. Hypothetical Problem:-

Let us take a problem discussed in 5.3.6 where:

<table>
<thead>
<tr>
<th>No.</th>
<th>( \beta_1 )</th>
<th>( \beta_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>70</td>
<td>0.5</td>
</tr>
<tr>
<td>2</td>
<td>80</td>
<td>0.4</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Output of the hypothetical problem is,

```
<table>
<thead>
<tr>
<th>Item</th>
<th>Demand</th>
<th>1</th>
<th>20.00</th>
<th>10.00</th>
<th>60.91</th>
<th>5</th>
<th>1000</th>
<th>91</th>
<th>2405</th>
<th>4.64</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>2</td>
<td>3667</td>
<td>20.00</td>
<td>11.00</td>
<td>51.29</td>
<td>6</td>
<td>1000</td>
<td>129</td>
<td>2748</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>7767</td>
<td>20.00</td>
<td>12.00</td>
<td>42.10</td>
<td>7</td>
<td>1000</td>
<td>210</td>
<td>3657</td>
</tr>
</tbody>
</table>
```

5.3.16. Remarks:-

Here, consumption rate \( \beta_1 \) and \( \beta_2 \). Therefore optimal turnover, total inventory cost and ITOR increases with respect to \( \beta_1 \) and \( \beta_2 \). Thus financial manager of the firm find out optimum order quantity with respect to the situation stated above. Also we observe marginal effect on ITOR with respect to setup cost.
5.4. **Conclusion:-**

Financial Analysis reveals that the turnover plays a very important role in the overall performance of a firm. A low or high turnover is an indication of poor or good management. A low turnover implies too much of inventory being held as obsolete and a high turnover imply incurring shortages, thereby losing the good will of the customers. Thus, usually a balance is struck between these two turnovers.

In this chapter, the problem of ITOR has been considered under the shortages situation to eliminating the waste. Using this analogy the ideal situation of ITOR production and supply is undertaken. Thus, an overall review of the different models formulated reveal that high turnover is an indication of good management leading to the better performance of the firm. Moreover the risk of large stock outs is minimized. Also, with the change in the setup cost there is no large difference in the situation stated above.