In this thesis we have studied some non-linear interactions involving kinetic Alfven waves. In particular we have investigated the non-linear interactions of the kinetic Alfven waves with drift modes and tearing modes. Such interactions have important applications in laboratory and astrophysics. The motivation for the present work arose from the particular relevance of these interactions in Alfven wave heating schemes in tokamak plasmas. Alfven waves are considered excellent candidates for supplementary r.f heating schemes. Theoretical considerations show that near the resonance region these waves have enhanced amplitudes and that several non-linear processes can take place \[\ldots\]. Therefore, a systematic study of the non-linear properties is necessary. In this context, we have
investigated the non-linear interactions of kinetic Alfven waves with two important modes, namely the drift and tearing modes.

In Chapters II and III, we have examined the non-linear interactions of the kinetic Alfven waves with the resistive tearing mode. In Chapter II, we have discussed the resonant excitation of tearing modes through parametric interaction with the kinetic Alfven waves, using a fluid model. The momentum equation and Ohm's law are the basic equations which describe the evolution of the T.M. The non-linear interaction generates additional convective forces in the former and anomalous viscous and resistive effects in Ohm's law. Using variational and asymptotic matching methods, we find that the $m = 1$ and $m = 2$ tearing instabilities are excited by the kinetic Alfven waves with their typical growth rates scaling as $|\phi_e|^{1/3} |\phi_i|^{3/2} \Delta^{-2/3}$ respectively. These excited growth rates fall in the range $10^6 - 10^4 \text{ sec}^{-1}$ for typical tokamak parameters (given in Chapter II) [2]. Several experiments which have been conducted in Alfven wave heating have reported enhanced transport of particles and plasma disruptions [3]. It may be possible that these disruptions are caused by excitation of tearing modes. However, so far, no direct evidence has been obtained.

In Chapter III, we have investigated the non-resonant interaction between kinetic Alfven waves and
resistive tearing modes, in which equilibrium flows generated by the Alfven waves couple non-linearly to the tearing mode perturbations. These non-linear drifts arise due to the interaction between the electric and the magnetic fields of the wave \((\vec{E}_A \times \vec{B}_A)/\beta_+^2\). The drifts have components in the axial, azimuthal and radial directions. The former Doppler shifts the mode frequency, while the latter components give rise to large gradients in the momentum equation. We find weakly growing tearing instabilities with growth rates proportional to the radial drift. The results of our analysis are in agreement with the weakly unstable modes obtained by Pollard-Taylor, Bondeson [4]. For tokamak parameters, these instabilities are found to grow on a longer time scale than the parametrically excited tearing modes. In a tokamak plasma, both the resonant and non-resonant processes could occur simultaneously; the former occurs when the resonant wave matching conditions are satisfied, while the latter is a more general phenomena.

In Alfven wave experiments, the antenna excites several modes which have a single frequency (or a small spread in frequency) simultaneously [3]. These kinetic Alfven waves could undergo non-linear interactions among themselves. Such a mechanism results in the excitation of waves at the sum and difference frequencies due to the presence of non-linear terms in the fluid equations.
have considered a situation where one of the resulting frequency, wave vector combination corresponds to that of the resistive T.M. and resonantly excites it (Chapter IV). The system responds like a driven harmonic oscillator wherein the non-linear interaction between the kinetic Alfven waves act as external forces driving the system at its natural tearing frequency. Using a fluid formalism, we obtain an inhomogeneous third order differential equation describing the evolution of the T.M. This problem differs from earlier investigations (ref. Chapter II) in that the non-linear terms are independent of the tearing mode perturbations. We have obtained solutions in terms of orthonormal basis functions, namely Hermite polynomials. It is found that the solutions are very sensitive to the parity of the driven Alfven waves. For arbitrary wavelengths, in the limit of vanishing pump amplitudes, the earlier results of Paris [5] are recovered. In the presence of the non-linear external forces, the growth rate for the symmetric tearing modes with positive 'm' numbers are enhanced, while for modes with negative 'm' numbers the effect is stabilising. These driven tearing modes (for typical tokamak parameters) are found to grow more slowly than the parametrically excited modes. Excitation of these tearing modes could lead to enhanced transport through destruction of good magnetic surfaces.
Drift waves in sheared magnetic fields have been extensively studied. In laboratory plasmas, they are considered responsible for anomalous transport of particles. In Chapter V, we have investigated the problem of parametric excitation of drift waves by kinetic Alfven waves. The kinetic equations are used to describe the decay of a pump kinetic Alfven wave into a side-band Alfven wave and a drift wave. The quasi-neutrality condition and Ampere's law are used to obtain the coupled equations for the decay process. The dispersion relation was obtained under a local approximation. We find that the calculated growth rate of the excited drift wave is quite large and competes significantly with the growth rate of the ion acoustic wave calculated by Hasegawa-Chen [1]. The ratio of the growth rates of the two processes is found to be $\frac{\omega_s}{k_u c_i} \sim \Omega$. We have also demonstrated that the kinetic Alfven waves could excite temperature gradient drift waves which have larger growth rates. In addition, we have investigated the effects of the back-ground inhomogeneity on the decay process, which calls for the retention of the full differential operators in the coupled equations. Treating the inhomogeneity scale length as a perturbation parameter, and using WKB methods, we have established the conditions under which an absolute instability can occur.
In the high temperature regime, the plasma is basically collisionless and the collisionless version of the tearing and drift modes are believed to play a very significant role in the reconnection mechanism. In Chapter VI, we have investigated two non-linear coupling processes - (1) between the kinetic Alfven waves and collisionless tearing modes 2) between the kinetic Alfven waves and the collisionless drift waves. We have investigated the effect of the P.F. generated by two kinetic Alfven waves on the two modes. The two fluid equations are used to describe the salient features of the kinetic Alfven waves. A generalised Ohm's law and the momentum equation describe the dynamics of the collisionless T.M. The former describes the electron response, while the latter describes the ion motion. The equilibrium P.F. (generated by the kinetic Alfven waves) broadens the electron wave particle response and modifies the conductivity profile in Ohm's law. The eigenvalues of the coupled equations have been obtained using variational methods prescribed by Hazeltine et al [6]. The modifications produced by the P.F. in the collisional and collisionless regimes have been obtained. In the latter, the parallel P.F. \( (F_{\parallel}) \) modifies the growth rate of Laval et al [7]. For \( F_{\parallel} > 0 \), the growth rates are strongly enhanced while for \( F_{\parallel} < 0 \), the effect is stabilising. In parameter
space the condition leads to a relation between the wave vectors of the kinetic Alfvén waves, namely $F_{\parallel b} < 0$ for $\frac{k_x}{k_{b_x}} < \frac{k_y}{k_{b_y}}$ and for the reverse condition $F_{\parallel b}$ is positive. For typical tokamak parameters and Alfvén fluctuations ($|\Psi|^2 \ll 1$) the enhancement factor due to the parallel force is however small. Although the destabilising effects are small, the stabilising effect of the P.F. could be of interest in tokamak plasmas.

In the collisional regime, consistent with expectations, the modifications to the growth rate of Drake and Lee [8] were found to be minimum. The enhancement factor in the growth rate is however of second order in $F_{\parallel b}$ and this factor is too small to be of any significance.

The dynamics of the collisionless drift waves are delicately controlled by the inverse Landau damping of electrons and shear effects. The electron response is therefore modelled by kinetic equations. The parallel P.F., as in the case of T.M., broadens the electron wave particle response, while the perpendicular P.F. Doppler shifts the mode frequency. The motion of the ions is described by the hydrodynamic approximation. The radial eigenmode equation is obtained from the quasineutrality condition and the eigenvalues are derived using the variational principle [9]. It is found that the parallel P.F. has a significant effect
on the linear growth rate of the drift wave and competes with the shear stabilising effect. For $F_{\parallel} > 0$, ($\frac{k_{2}}{k_{x}}, \frac{k_{3}}{k_{x}}, \frac{k_{3}}{k_{y}}$) the parallel force contributes to the shear effect and stabilises the mode. For the reverse inequality, $F_{\parallel} < 0$, and the effect is destabilising. It is found that for typical tokamak parameters, the shear damping is not overcome by the destabilising contribution from the parallel force. However, the stabilisation of the drift modes by external P.F. generated by the interacting kinetic Alfven waves could be of interest in laboratory plasmas, where these modes are known to have deleterious effect on the plasma confinement.