CHAPTER VIII

EFFECT OF LOWER HYBRID TURBULENCE ON DRIFT CYCLOTRON LOSS CONE INSTABILITY

1. Introduction:

The injection of high energy neutral beams in mirror machines is of considerable interest as it has been envisaged as an efficient method of fueling and heating the target plasma in future mirror reactors (1,2,3). Theoretical speculations on plausible microinstabilities associated with such systems have also been reported (4). The upshot is that, the most easily excitable instability is that of Lower Hybrid Waves (LHW), which is produced by a relative drift of electrons and ions across the field. The neutral beams in its interaction with the target plasma yields ions and electrons. If then, there is sufficient relative motion between the species because of their different degrees of magnetisation then the lower hybrid waves become unstable.
The threshold for this instability is \( V_0 > U_{\beta i} \) (5) \( (V_0 \) is the beam velocity, \( U_{\beta i} \) is thermal velocity of ions). In mirror machines this condition is very likely to be satisfied as the hot plasma is formed mostly by the charge exchange between the low energy ions of target plasma and the energetic neutral particle of the beam. Recently many groups have reported ion beam driven LHW instability using perpendicular injection (6,7). In Chang’s experiment (8) waves at \( \omega = \omega_{LH} \) \((\omega_{LH} \) is the LH frequency; \( \omega_{LH} = \omega_{p,i} \) ) were observed by perpendicular injection of an ion beam with \( V_0 = 15 U_{\beta i} \). In Burns et al steady state mirror strong radial electric field were produced by electron beam injection. This led to a relative motion between the ions and electrons and consequently strong oscillations at \( \omega \omega_{p,i} \) were observed. Cordey et al (9) have shown that during the initial stages of neutral beam injection the hot particle distribution function is peaked and hence is unstable to perturbations at \( \omega \omega_{p,i} \). They have also shown that the neutral beam injection can give rise to large scale radial electric field which in turn can destabilize LH waves. Quite apart from these considerations, in 2XIIIB the threshold for flute type lower hybrid drift instability \( \left[ \frac{\alpha_i}{L} > \left( \frac{m_i c}{m_i \gamma} + \frac{\omega c_i^2}{\omega_{p,i}^2} \right)^{1/4} \right] \) where \( \alpha_i \) is the ion gyro-radius and \( L \) is the length of the machine) is exceeded hence one expects a continuous spectrum of lower hybrid waves superimposed on a discrete spectrum of ion-cyclotron oscillations (15). Thus it appears that a modest level of LHW turbulence in future neutral beam injected mirror machines seems to be inevitable. The positive aspect of these oscillations is that they provide an efficient mechanism of energy transfer from the beam to the particles and thus lead to a strong
heating of electrons and ions (8). Now it has been shown that in the presence of a background turbulence of these oscillations the normal modes of plasma are drastically modified (10,11). In this Chapter we propose to investigate the effect of a modest level of this turbulence on the most important and the most deterrent normal mode of the mirror plasma i.e. the drift cyclotron loss cone (DCLC) instability in realistic situations. Recently Shaing (12) et al have reported an investigation of the effect of LHW pump on DCLC instability. However, the result of their investigations may not be applicable to the actual experimental situation in the mirror machine for the following reason. They have considered a parametric coupling between a LHW and a DCLC mode, which is a coherent interaction between two waves. But in real situation the interaction is not expected to be coherent. The condition for the saturation of a coherent wave (by particle trapping) is 
\[ \tau_{ac} \sim (1 + \delta) \gg \tau_g \]
where \( \tau_{ac} \) is the auto-correlation time, \( \tau_g \) is the linear growth time, \( K \) is the typical wave number of LHW waves, and \( \delta \) is the spread in the phase-velocity of the waves, while the condition for saturation of broad spectrum is 
\[ \tau_g \sim \tau_{ac} \]
Now \( \tau_g \) for LHW is \( (\omega_B \eta^1_3)^{-1} \) where \( \eta_0 = n_B/n_p \) [\( n_B \) is the beam density, \( n_p \) is the target plasma density] while typically \( \delta = \omega_B \eta^1_3 \) and \( \delta \) is the thermal spread of the beam. Hence 
\[ \tau_{ac} \sim \frac{1}{\omega_B \eta^1_3} \sim \tau_g \] , in which case the saturation of a broad spectrum of LHW by quasilinear diffusion in velocity space is more likely. Indeed in Chang's experiment the observed spectrum of LHW was quite broad. In order to study the incoherent interaction we notice that in experimental situations LHW spectrum is broad while DCLC
wave spectrum is narrow (14) and secondly that the dispersion characteristics of two waves are widely apart i.e. \( \omega_{k} - \omega_{p} \gg \omega_{c} \). \( \omega_{k} \) is the frequency of LHW (DCLC). Hence we will use 'Vedenov technique' which satisfactorily takes into account the incoherent interaction between waves of widely different properties. In this method one treats the high frequency microturbulence as a wave packet with a distribution in \( \hat{K} \)-space that satisfies a wave-kinetic equation. One then studies the motion of these wave packets in a medium varying slowly in space and time, the variation being due to the low frequency long-wave length wave. The reverse influence of the high frequency turbulence on low frequency waves comes through the average electric field pressure \( \nabla E^2 \) which modifies the electron dynamics.

Our studies reveal that in the presence of LHW the DCLC waves are strongly damped. Thus the presence of a lower hybrid turbulence will have a two fold advantage in mirror machines, firstly it will provide an efficient mechanism of the energy transfer from beam to particles for heating purposes and secondly it will help in controlling the DCLC turbulence, and improving the ion lifetime inside the machine.

In Section 2 we have calculated the effect of LHW turbulence on linear growth rates of DCLC waves. In Sec.3 we have discussed our results.

2. Coupling of DCLC Waves to Lower Hybrid Waves:

For a theoretical model we consider a slab geometry with \( Z \)-axis along the mirror axis. The plasma has a density gradient

$$\frac{1}{n} \frac{dn}{dx} = \xi \hat{z}$$

The DCLC-wave-spectrum observed generally in high
density mirror machines like 2XlIB is a narrow spectrum (14), with maximum power in the fundamental mode at \( \omega_c i \). We may approximate this by considering a single DCLC mode in Y-direction with frequency and wave number given by \( \Omega \) and \( \vec{q} \) respectively. We take the LHW turbulence in Y-Z plane with frequency and wave number \( \omega \) and \( \vec{k} \) respectively where \( \vec{k} = k_y \hat{y} + k_z \hat{z} \) and \( \frac{k^2_{\perp}}{k^2} \approx \frac{m_i}{m_e} << 1 \). This turbulence may be either because of injection of energetic neutral beam perpendicular to the field lines or because the threshold for flute types lower hybrid instability has been exceeded in the mirror plasma i.e. \( a_i/l > \left( \frac{m_e}{m_i} + \frac{\omega_c^2}{\omega_{pi}^2} \right)^{1/4} \) (For 2XlIB \( a_i/l \approx 1/2 \) and \( \left( \frac{m_e}{m_i} + \frac{\omega_c^2}{\omega_{pi}^2} \right)^{1/4} \approx 0.1 \). To consider the effect of these modes on DCLC waves we consider a broad spectrum of LHW which has saturated by diffusion in velocity space. The spectrum is taken to be peaked around \( \kappa_0 = \frac{\omega_{lh}}{\nu_0} \) (where \( \omega_{lh} \) is the LHW frequency). The LHW obey the linear dispersion relation

\[
\omega_k^2 = \omega_{lh}^2 \left[ 1 + \frac{k_{\perp}^2}{k^2} \frac{m_i}{m_e} \right]
\]

(1)

where \( \omega_{lh}^2 = \omega_{pi}^2 \left[ 1 + \frac{\omega e^2}{\omega_{ce}^2} \right] \) and \( \frac{k_{\perp}^2}{k^2} \approx \frac{m_e}{m_i} \approx 1 \).

We will now invoke the adiabatic approximation due to Vedenov et al.

In this approximation the evolution of Lower Hybrid (LH) plasmon distribution function \( N_k \) is studied by the following equation (16)

\[
\frac{\partial N_k}{\partial t} + \vec{v}_q \cdot \nabla N_k = \frac{\partial \omega_k}{\partial \vec{k}} \cdot \frac{\partial N_k}{\partial \vec{k}} = 0
\]

(2)
where \( \frac{\partial \omega_k}{\partial k} \) is the group velocity of LHW and from (1)
\[
\vec{V}_g = \omega_{lh} k_{z} / k_{z} \quad \text{as} \quad k_{z} / k_{z} \quad m_{e} / m_{i}.
\]

It should be noted that the wave-kinetic equation (2) is valid only when the spread in the group velocities is so large that the convective term \( \nabla \cdot \nabla N_{k} \) dominates the effect of diffraction i.e. the term containing \( \nabla \cdot \nabla N_{k} / k_{z} \). In brief the effect of LHT arises as follows: The low frequency perturbation creates a perturbation in the plasmon density.

The gradient of this plasmon density gives rise to a ponderomotive force (PF) which reacts back on the low frequency oscillations to modify its characteristics. The effect can also be viewed by considering the group of plasmons as a group of particles (or quasi-particles) on the time scale of low density perturbation. Hence depending upon the slope of plasmon distribution at the resonance point, the Landau resonance between the DCLC wave and plasmon distribution may lead to a growth or damping of the DCLC wave. The PF on ions is \( m_{e} / m_{i} \) times smaller than that on electrons and hence will be dropped. The plasmon distribution function is perturbed as
\[
N_{k} = N_{k=0} \hat{n}_{k} \quad \text{(where} \quad N_{k=0} \quad \text{is the equilibrium distribution function normalised as} \quad \int N_{k=0} d\vec{k} = 1 \quad \text{)}).
\]

From the wave-kinetic equation (2)
\[
\hat{n}_{k} = \frac{\partial \omega_{K} - \partial N_{k=0}}{\partial k} \quad \text{(C, \( \vec{r} \), \( \vec{k} \), \( \vec{r} \), \( \vec{k} \))}, \quad \hat{n}_{k} = \hat{n}_{k} \quad \text{(3)}
\]

The dependence of \( \hat{n}_{k} \) on the low frequency density perturbation comes from the term \( \partial \omega_{K} / \partial k \); for LHW \( \omega_{K} = \omega_{1h} = \omega_{p} \), so that
\[
\partial \omega_{K} / \partial k = i \frac{\omega_{p}}{2} \hat{n}_{1} / \hat{n}_{0} \quad \text{where} \quad \hat{n}_{1} \quad \text{is the low frequency perturbation}.\]
For DCLC modes we will use the model given by Post and Rosenbluth (21). According to this model the DCLC modes are electrostatic flute modes which arise because of resonance between the positive energy electron drift mode and negative energy ion Bernstein mode. The typical frequency and the phase-velocity of these modes as seen in 2XIIB (17) is \( \omega \approx \omega_c \) and \( \nu \approx \Omega_i \) respectively. It should be noted that this model for DCLC Modes ignores certain effects which may be needed for more quantitative accuracy, such as electromagnetic effects to the electron contribution (18) ion drift term (19) temperature gradient (20).

To calculate the PF on an electron the modified equation of motion is

\[
\gamma^* \frac{\partial \vec{V}}{\partial t} + m_e \vec{V} \cdot \nabla \vec{V} = -e \vec{E} - \frac{e}{c} \vec{V} \times \vec{B}_0
\]

(4)

where now in the equation of motion the nonlinear term \( \vec{V} \cdot \nabla \vec{V} \) has been retained. In equation (4) \( \vec{V} = \vec{V}_s + \vec{V}_f \) where \( \vec{V}_s \) is the low frequency part of \( \vec{V} \) sustained by DCLC mode, and \( \vec{V}_f \) is the high frequency part of \( \vec{V} \) sustained by LHJ. The electron sees the effect averaged over many periods of high frequency oscillations. Hence taking the average over many high frequency oscillation period (13)

\[
\gamma^* \left\langle \vec{V}_f \cdot \nabla \vec{V}_f \right\rangle = -e \frac{\vec{E}_s}{c} - \frac{e}{c} \vec{V}_s \times \vec{B}_0
\]

(5)

where \( \vec{V}_f \) is given by the high frequency equation

\[
\gamma^* \frac{d \vec{V}_f}{dt} = -e \frac{\vec{E}_f}{c} - \frac{e}{c} \vec{V}_f \times \vec{B}_0
\]

(6)
In equation (5) \( \vec{E}_s = |E_s| \hat{z} \) is the low frequency perturbed electric field, while in equation (6) \( \vec{E}_f = |E_{fy}| \hat{z} + |E_{fx}| \hat{z} \), \( E_{fy} \ll E_{fx} \) is high frequency perturbed E-field. From equation (5) \( V_{sx} \) can be written as

\[
V_{sx} = \frac{E_s}{B_0} + \frac{<\nabla_f \cdot \nabla V_f> y}{\omega ce} \quad (7)
\]

Here we have neglected the inertial term due to slow mode as \( -\Omega \ll \omega ce \). Then the modified equation of continuity becomes

\[
-i \Omega \eta + \epsilon \eta \left[ \frac{E_s}{B_0} + \frac{<\nabla_f \cdot \nabla V_f> y}{\omega ce} \right] = 0 \quad (8)
\]

the term \( <\nabla_f \cdot \nabla V_f> y \) can be written as

\[
<\nabla_f \cdot \nabla V_f> y = \sum_k \left[ \frac{1}{2} \left( \frac{\partial V_{fy}(\vec{k})}{\partial y} \right)^2 + V_{fx}(\vec{k}) \frac{\partial V_{fy}(\vec{k})}{\partial x} \right] \quad (9)
\]

where \( V_{fy}(\vec{k}) \), \( V_{fx}(\vec{k}) \) are fourier amplitude given by

\[
V_{fx}(\vec{k}) = \frac{\omega ce}{\omega_k^2 + \omega ce^2} \frac{E_{fy}/B_o}{(-\omega_k^2 + \omega ce^2)} \quad (10)
\]

\[
V_{fy}(\vec{k}) = -i \frac{e/\eta}{m_e \omega_k} \frac{E_{fy}(\vec{k})}{\omega_k^2 - \omega ce^2} \quad (11)
\]

Since LHW lie in Y-Z plane \( \partial V_{fy}(\vec{k})/\partial z = 0 \).
Hence we have
\[
\langle \nabla_f \cdot \nabla \tilde{v}_f \rangle_y = \sum_k \frac{1}{2} \frac{e^2}{m^2_c} \omega_k^2 \frac{1}{y} \left| E_y(k) \right|^2 \left( \omega_k^2 - \omega_{ce}^2 \right)^2
\]

Substituting \( E_y(k) = E_f \cos \theta \) where \( \theta \) is the angle between \( Y \) and \( K \) we have
\[
\langle \nabla_f \cdot \nabla \tilde{v}_f \rangle_y = \sum_k \frac{1}{2} \frac{e^2}{m^2_c} \omega_k^2 \frac{1}{y} \left| E_f(k) \right|^2 \left( \omega_k^2 - \omega_{ce}^2 \right)^2
\]

Since \( \left| E_f(k) \right|^2 = N_0 \omega_k \mathcal{M}_{kk} \) where \( N_0 \) is the average LH plasmon density and \( \mathcal{M}_{kk} = N_{k0} + \mathcal{M}_{kk} \). Hence equation (13) can be written as
\[
\langle \nabla_f \cdot \nabla \tilde{v}_f \rangle_y = \sum_k \frac{4 \pi e^2}{m^2_c} \frac{\cos^2 \theta \omega_k^2}{(\omega_k^2 - \omega_{ce}^2)^2} \frac{N_0}{y} \mathcal{M}_{kk}
\]

Substituting \( \mathcal{M}_{kk} \) from equation (3) we have
\[
\langle \nabla_f \cdot \nabla \tilde{v}_f \rangle_y = \sum_k \frac{4 \pi e^2}{m^2_c} \frac{\cos^2 \theta \omega_k^2}{(\omega_k^2 - \omega_{ce}^2)^2} \frac{N_0}{y} \mathcal{M}_{kk}
\]

where \( \mathcal{M} = m_1 e^{(i q y - i \omega t)} \) is that \( \mathcal{M}/y = i q \mathcal{M} \). While evaluating the dispersion relation for DCLC modes we will make use of quasi-neutrality condition. This is consistent with the fact that DCLC instability is for long wavelengths i.e.
\[ b = \frac{\omega_{bi}^2}{\omega_{ce}^2} \frac{1}{Q < a_i >^3} \approx 1 \] and the adiabatic approximation which considers \( Q < a_i > \). Using the quasi-neutrality condition the dispersion relation for DCLC mode in the absence of LHW comes out as

\[ \frac{\omega}{\omega_{ci}} = b \sum_{n=-\infty}^{\infty} \frac{\omega_{ci}}{\omega_{ci}^2 - n^2} \] (16)

where \( b = \frac{\omega_{bi}^2}{\omega_{ce}^2} \frac{1}{Q < a_i >^3} \) and \( \omega_{ci}^* = \frac{\omega_{bi}^2}{\omega_{ce}} |q| \)

Near \( \Omega - \omega_{ci} \) this equation predicts unstable roots for \( b \gg 1 \) with growth rate \( \sim (\omega_{ci} - \omega_{ci}^*)^2 b \). In 2XIIIB (17) the typical unstable mode had following parameter; \( \omega_{bi}^2/\omega_{ci}^2 \approx 10^4, \quad Q < a_i > \approx 3, \quad \omega - \omega_{ci} \approx 3 \times 10^7 \text{ rad/sec} \). For this mode equation (16) would predict a growth rate \( \sim 0.3 \times 10^7 \text{ rad/sec} \), which agrees fairly well with the observed growth rate \( \sim 0.1 \times 10^7 \text{ rad/sec} \). Hence using \( \eta_e = \eta_i \)

in equation (15) and substituting in equation (8) we have the modified electron density perturbation as

\[
\begin{align*}
\eta_e &= \frac{-i \mathbf{E}_s / B_0 \mathbf{c} \eta_i / \Omega}{\left[ 1 - \frac{e_0 \omega^2}{\omega_{ce}} \frac{4 \pi \mathbf{c}^2}{\omega_{ce} m_e \omega^2} \eta_0 \Omega \frac{\omega_{ki}^4}{\left( \omega_{ki}^2 - \omega^2 \right)^2} \frac{1}{\omega_{ki} \Omega} \left( \nabla \mathbf{N} \cdot \mathbf{d} + \mathbf{c} \mathbf{d} \right) \right]}
\end{align*}
\] (17)
where now in the turbulence term the summation has been replaced by an integration. As ions are not affected by the PF, \( \tilde{n}_i \) can be calculated in the way shown by Post et al (21). Then using the quasi-neutrality condition \( \tilde{n}_e = \tilde{n}_i = \tilde{n}_t \) we have the modified dispersion relation for DCLC modes as

\[
\frac{-\omega^2}{\epsilon_2} = b \sum_{n=-\infty}^{\infty} \frac{-\omega}{(\omega - \omega_k)}
\]

(18)

where \( T = \frac{T_{ih}^2}{2} \left( \frac{2}{\omega_{pi}^2} \right) \frac{1}{\omega_{pe}^2} \frac{\omega_k^4}{(\omega_k^2 - \omega_i^2)^2} \). Integration in K-plane has to be performed according to Landau's prescription. Hence the turbulence will give a Cauchy's principle value given by \( T_R \) and a pole term given by \( T_i \) due to resonance of LH plasmons with DCLC modes. The resonance condition is given by \( \Lambda (k) = \frac{-\omega}{\varphi (\omega \lambda^2) \omega} \). As \( \Lambda (k) \) typically \( \sim V_o \) and \( \varphi (\omega \lambda^2) \), the resonance condition will hold and the damping or growth coming from this resonance may be significant. The damping or growth will depend upon the slope of LH plasmon distribution at \( k \) given by \( \Lambda (k) = \frac{\omega_{pe}^2}{\omega_{pi}^2} \). To perform the integration in K-plane we model the LH plasmon distribution function.
In Y-Z plane by a broad two dimensional gaussian spectrum given by

\[ N_{k_0} = \frac{1}{2\pi \Delta^2} \exp \left[ -\frac{(k_\perp - k_{0\perp})^2}{2\Delta^2} - \frac{(k_{11} - k_{110})^2}{2\Delta^2} \right] \]

where \( k_0^2 = k_{0\perp}^2 + k_{110}^2 \) (\( k_{110} \gg k_{110} \)) and \( \Delta \) is the width of the spectrum. Since we have assumed a broad spectrum we take \( \Delta = k_0 \). From equation (19) \( T_Y \) and \( T_\perp \) can be written as

\[ T_Y = \frac{C_0 \beta^2 \theta}{2} q^3 \frac{\Omega^*}{\omega_{pi}} W \frac{\omega_k^4}{(\omega_k^2 - \omega_c^2)^2} \left( \frac{2N_{k_0}}{\partial k_\perp} \right) \int_{-\infty}^{+\infty} dk_{11} dk_{11} \frac{\left( \frac{2N_{k_0}}{\partial k_\perp} \right)}{q \Omega^* (\omega_k^2 - \omega_c^2)} \]

To do the pole integration we make the approximation that \( V_g \) is a function of \( k_\perp \) through \( V_g = \omega_{ce} / k_\perp \). In this case \( k_{11} \) integral can be done directly using equation (20) while \( k_{11} \) integral will have a pole at \( k_{\perp Y} = (\omega_{ce} q / \Omega) \). Hence

\[ T_\perp = -\frac{\pi}{2} \cos^2 \theta q^3 \frac{\Omega^*}{\omega_{pi}} W \frac{\omega_k^4}{(\omega_k^2 - \omega_c^2)^2} \left( \frac{\omega_{pi}}{\Omega^2} \right) \frac{(k_{11Y} - k_{11})}{\Delta^2} \] \[ \exp \left[ -\frac{(k_{11Y} - k_{11})^2}{2\Delta^2} \right] \]

and

\[ k_{11Y} = \frac{\omega_{11} q}{\Omega^2} \quad , \quad k_{110} = k_{0\perp} - \frac{\omega_{11}}{V_0} - \frac{\omega_{pi}}{V_0} \]

In the experimental observations the maximum power is seen to be concentrated in the fundamental mode at \( -\Omega - \omega_{ce} \) (14). To
look for the stability of this mode we put $T = T_y + i T_i$, retain the $n = 1$ term in the summation and solve the resultant quadratic equation for $\Omega$ as

$$\Omega = -\frac{\Omega_x}{2b} + \frac{T_y}{2} + i \frac{T_i}{2} \pm \left[ \frac{(\Omega_x - bT)^2 - 4\Omega_x \omega_k b}{2b} \right]^{1/2} \tag{24}$$

Now to examine the DCLC spectrum in the presence of LHW, we evaluate the relative orders of $T_y$ and $T_i$ as follows. We write $T_y$ as

$$T_y = \frac{C_4 \theta}{2} \frac{\Omega_x}{\omega_{pi}^2} \frac{W_{\Omega_\rho}^2}{\omega_{k5}} \frac{\omega_k^4}{(\omega_k^2 - \omega_{ei})^2} \mathcal{N}_{20} \int \frac{dk_0}{\omega_{ei}^2} \frac{dk_1}{\omega_{ei}} \frac{dk_2}{\omega_{ei}} \mathcal{V}_0 \frac{\omega_{ei}^2}{\omega_{ei}^2}$$

In equation (25) $k_1/k_2 < 1$, $\theta = 0$ and $\cos \theta = 1$. And as $V_0 - V_b > -\Omega_x / \omega_{ei} - U_{thi}$, we may expand the denominator in powers of $-\Omega_x / \omega_{ei}$ and retain the leading term. For $N_{k0}$ we may substitute from equation (20) and perform the $k_\parallel$ integration directly.

For performing $k_\perp$ integration we put $V_0 = \omega_{\Omega_x} \frac{\omega_{\Omega_x}^2}{k_{\perp}} = \frac{\omega_{\Omega_x}^2}{k_{\perp}}$ in which case $T_y$ becomes

$$T_y = \frac{W_{\Omega_\rho}^2 \omega_{\Omega_\rho}^2 \left( -\frac{\Omega_x}{\omega_{pi}^2} \right) \omega_k^4}{(\omega_k^2 - \omega_{ei})^2} \frac{\Omega_x}{\omega_{ei}^2} \tag{26}$$

For typical parameters of present day high density mirrors e.g. 2XIIB, PR-7, etc., $\omega_k > \omega_{ei}$, $-\Omega_x / \omega_{ei} - U_{thi} \sim 10^7$ cm/sec, $n_s > 10^{10}$, $5 \times 10^8$ cm/sec, $\omega_{ci} / \omega_{pi} = 1 / 10^2$ (\omega_{ci} \sim 3 \times 10^7 rad/sec, $\epsilon \sim 1/7$ (cm$^{-1}$) (\omega_{pi} \sim 1.6 \times 10^9 rad/sec for plasma density \sim 10^{13}$ cm$^{-3}$) and a modest
level of LHW turbulence allowed in weak turbulence theory which we are considering i.e. \( W = (m_0/m_1) \) we have \( T = \frac{\omega c}{10} < \omega c \). On the other hand in expression for \( T \) in equation (22) \( \frac{\omega_{hp}}{\nu_0} = \frac{\omega_{hp}}{\omega_{hp}} \) is always \( > \omega_{hp} \) as \( \nu_0 > U_{hp} \) is the threshold for LHW. Hence \( T \) will always bear a negative sign and cause damping (as we will show later). In an order of magnitude sense

\[
T_i = \left( \frac{q_{r/J}}{a} \right)^3 \frac{\omega_{hp}^4}{(\omega_{hp}^2 - \omega_c^2)^2} W \left( \frac{\omega_{hp}q_{r/J}}{\nu_0} \right)^2 \frac{k_{1x} \rho_0}{\nu_0} - \frac{\nu_0}{\omega_c} - \Omega
\]

For \( \Delta = \omega_{hp}/\nu_0 \), \( k_{1x} = \omega_{hp}/\nu_0 \), \( \nu_0 > U_{hp} \), typically DCLC oscillations are excited with \( q_{r/J} \omega_{hp} > 1 \); in 2XIIB \( 2.9 < q_{r/J} \omega_{hp} < 6.1 \). For these wave numbers \( b = \frac{\omega_{hp}^2}{\nu_0^2} \left( \frac{q_{r/J}}{\omega_{hp}} \right)^3 \approx 3 \times 10^3 \). We will now examine the stability of these wave numbers in the presence of a weak LHW turbulence. In equation (24) if we put \( T = a \), we get the dispersion relation obtained by Post and Rosenbluth (21) which is

\[
\omega = -\frac{\omega_{hp} \omega_c}{b} + \frac{1}{a + b} \left[ -\omega_{hp}^2 b - 4 \omega_{hp} \omega_c b \right]^{1/2}
\]

For typical 2XIIB parameters i.e. \( \omega_{hp} \approx 10^{10} \) rad/sec, \( b \approx 3 \times 10^3 \), \( \omega_{hp} \approx 3 \times 10^7 \) rad/sec. Equation (28) gives the real part of DCLC frequency \( -\omega \approx \omega_{hp} \). For these wave numbers

\[
4 \omega_{hp} - \omega \omega \frac{1}{b} > -\omega^2 - \omega_{hp} \omega_c b
\]

under the radical in equation (28), hence they are unstable with a typical growth rate

\[
\gamma = \left( 4 \omega_{hp} - \omega \omega \frac{1}{b} \right)^{1/2}/b
\]

\( \approx 3 \times 10^7 \) rad/sec. The growth rate observed in 2XIIB is \( 0.02 \) to \( 0.03 \omega_{hp} \approx 0.1 \times 10^7 \) rad/sec. The discrepancy of a factor of 3 may be
attributed to the fact the the effects mentioned earlier i.e. electromagentic effects in the electron contribution, ion drift term which are stabilizing are not included. In equation (24) we note that for typical parameters $\frac{\Omega^*}{\beta} > b |\Gamma|$ and $\Omega^* h < 4 \Omega^* \omega c \beta$ hence the modified growth rate in the presence of LHW turbulence is given by

$$\gamma = - \frac{T_i}{2} \pm \left( \frac{\omega c - \Omega^* b}{b} \right)^{1/2}$$

(29)

The damping induced by LHW plasmons is $\gamma = \omega c / \beta \approx 0.15 \times 10^7$ rad/sec which of the same order as $(\omega c - \Omega^* b)^{1/2} / \beta \approx 3 \times 10^7$ rad/sec. Thus we see that a low level of LHW turbulence can bring about a significant stabilization of DCLC modes.

Discussion:

These findings are contrary to the findings of Shiang et al who find stabilization only in the range $\Omega_{\text{LH}} < \Omega^* < \Omega^*$ where

$$\Omega^* = \frac{1}{2} \left[ \Omega + \left( \Omega^2 + \delta (\omega_{\text{LH}})^2 \right)^{1/2} \right]$$

and

$$\Omega^* = \frac{\omega_{\text{LH}}^2}{\omega c}$$

They also find a region of enhancement. In our case, we do not find a region of enhancement. It follows from here that Lower Hybrid plasmon damping has significant stabilizing influence on the observed wave number range of DCLC spectrum. It should be noted that this damping cannot be converted into growth by shifting the turbulent spectrum so that $k_{\perp} \gamma < k_0$. This will require $\gamma \omega c > \Omega_{\text{LH}}$ which violates the condition for the excitation of lower hybrid modes.
Thus it follows that presence of lower hybrid turbulence in mirrors has twofold advantage; firstly, it leads to an efficient heating of ions and electrons and secondly, it helps in reducing the fluctuation level due to DCLC modes which will help in improving the ion lifetime inside the trap.
REFERENCES

(2) F.L. Ribe, Rev. Mod. Phys. 47, 7 (1975).