Chapter 6

Transfer–Ordering Strategy for Deteriorating Items in Declining Market

6.0. Introduction

This chapter deals with a mathematical model which provides optimal transfer–ordering strategy for deteriorating items during recession under scenario of declining market.

6.1. Optimal Transfer – Ordering Strategy for a Deteriorating Inventory in Declining Market

In this section, the retailer’s optimal procurement quantity and the number of transfers from the warehouse to the display area is determined when demand is decreasing due to recession and items in inventory is subject to deterioration at a constant rate. The objective is to maximize the retailer’s total profit per unit time. The algorithms are derived to find the optimal strategy by retailer. Numerical examples are given to illustrate the proposed model. It is assumed that during recession when demand is decreasing, retailer should keep a check on transportation cost and ordering cost. The display units in the showroom may attract the customers.
6.1.1 Assumptions and Notations

The mathematical model is based on the following assumptions and notations.

6.1.1.1 Assumptions

1. The inventory system under consideration deals with a single item.
2. The planning horizon is infinite.
3. Shortages are not allowed. The lead time is negligible or zero.
4. The maximum allowable item of displayed stock in the showroom is ‘L’.
5. The time to transfer items from the warehouse to the showroom is negligible or zero.
6. The units in inventory deteriorate at a constant rate ‘θ’, 0 ≤ θ < 1. The deteriorated units can neither be repaired nor replaced during the cycle time.
7. The retailer orders Q – units per order from a supplier and stocks these items in the warehouse. The items are transferred from the warehouse to the showroom in equal size of ‘q’ units until the inventory level in the warehouse reaches to zero. This is known as retailer’s order – transfer policy.
6.1.1.2 Notations

$L$ Maximum allowable number of displayed units in the showroom

$I(t)$ Inventory level at any instant of time $t$ in the showroom, $I(t) \leq L$

$D(t) = a(1 - bt)$ Demand rate at time $t$. where $a, b > 0$, $a \gg b$. $a$ denotes constant demand and $0 < b < 1$ denotes the rate of change of demand due to recession.

$\theta$ Constant rate of deterioration, $0 \leq \theta < 1$

$h_w$ Unit inventory carrying cost per annum in the warehouse

$h_d$ Unit inventory carrying cost per annum in the showroom,

with $h_d > h_w$

$P$ Unit selling price of the item

$C$ Unit purchase cost, with $C < P$

$A$ Ordering cost per order

$G$ Known fixed cost per transfer from the warehouse to the showroom

$T$ Cycle time in the warehouse, (a decision variable)

$n$ Integer number of transfers from the warehouse to the showroom per order (a decision variable)

$t_1$ Cycle time in the showroom (a decision variable)
Q Optimum procurement units from a supplier (decision variable)

q Number of units per transfer from the warehouse to the showroom,

\[ 0 \leq q \leq L \] (a decision variable)

R Inventory level of items in the showroom regarding the transfer of \( q \) – units from the warehouse to the showroom

6.1.2 Mathematical Model

6.1.2.1 The total cost per cycle in the warehouse

The retailer orders \( Q \) – units per order from a supplier and stocks these items in the warehouse. The \( q \) – units are transferred from the warehouse to the showroom until the inventory level in the warehouse reaches to zero. Hence, \( Q = nq \) The total cost per cycle during the cycle time \( T \) in the warehouse is the sum of (1) the ordering cost; \( A \), and (2) the inventory holding cost,

\[
h_w \left[ \frac{n(n-1)}{2} q \right] t_1
\]

6.1.2.2 The total cost per unit cycle in the showroom

Initially, the inventory level is \( L_0 \leq L \) due to the unit’s transfer from the warehouse to the display area. The inventory level then depletes to \( R \) due to time – dependent demand and deterioration of units at the end of the retailer’s cycle time, ‘\( t_1 \)’. A graphical representation of the inventory system is exhibited in Figure 6.6.1.1
The differential equation representing inventory status at any instant of time $t$ is given by,

$$\frac{dl(t)}{dt} = -D(t) - \partial l(t), \quad 0 \leq t \leq t_1,$$  \hspace{1cm} (6.6.1.1)
with boundary condition \( l(t_{1}) = R \). The solution of equation (6.6.1.1) is,

\[
l(t) = R e^{\alpha(t_{1}-t)} + a \left( \frac{e^{\alpha(t_{1}-t)} - 1}{\theta^2} (\theta + b) - \frac{b t e^{\alpha(t_{1}-t)} - t}{\theta} \right), 0 \leq t \leq t_{1} \quad (6.6.1.2)
\]

The total cost incurred during the cycle time \( t_{1} \) is the sum of the ordering cost, \( G \) and the inventory holding cost,

where inventory holding cost = \( h_{d} \int_{0}^{t_{1}} l(t) dt \)

\[
= h_{d} \left( - \frac{R}{\theta} + a \left( \frac{b \theta^2 t_{1}^2 - 2 \theta - 2 - 2 \theta^2 t_{1}}{2 \theta^3} \right) - h_{d} e^{\alpha t_{1}} \left( \frac{\theta t_{1} - \theta - b}{\theta^3} - \frac{R}{\theta} \right) \right) \quad (6.6.1.3)
\]

Using equation (6.6.1.2) and \( l(0) = q + R \), we get

\[
q = \frac{R e^{\alpha t_{1}} - a e^{\alpha t_{1}} \theta + a e^{\alpha t_{1}} b - a \theta - ab t_{1} e^{\alpha t_{1}} \theta - R \theta^2}{\theta^2} \quad (6.6.1.4)
\]

The revenue per cycle is

\[
(P - C)q =
\]

\[
(P - C)(Re^{\alpha t_{1}} \theta^2 + ae^{\alpha t_{1}} \theta + a e^{\alpha t_{1}} b - a \theta - ab t_{1} e^{\alpha t_{1}} \theta - R \theta^2) \quad (6.6.1.5)
\]

Then inventory holding cost in the warehouse is

\[
= h_{w} n(n - 1) t_{1} \left( \frac{Re^{\alpha t_{1}} \theta^2 + ae^{\alpha t_{1}} \theta + a e^{\alpha t_{1}} b - a \theta - ab t_{1} e^{\alpha t_{1}} \theta - R \theta^2}{2 \theta^2} \right) \quad (6.6.1.6)
\]
Hence, the total profit, $\Pi_p$ per cycle during the period $[0,T]$ is,

$$\Pi_p = \text{Revenue} - [\text{total cost in the warehouse}] - [\text{total cost in the showroom}]$$

$$= \frac{n(P-C)\left(Re^{a_1}, \theta^2 + ae^{a_1}, \theta + ae^{a_1}, b\right)}{-a\theta - ab - abt_1e^{a_1}, \theta - R\theta^2} - A$$

$$- h_w n(n-1)t_1 \frac{\left(Re^{a_1}, \theta^2 + ae^{a_1}, \theta + ae^{a_1}, b\right)}{-a\theta - ab - abt_1e^{a_1}, \theta - R\theta^2} - nG$$

$$- nh_d \left(\frac{R}{\theta} + a \left(\frac{b\theta^2 t_1^2 - 2\theta - 2b - 2\theta^2 t_1}{2\theta^3}\right)\right)$$

$$+ nh_d e^{a_1}\left(a \left(\frac{\theta t_1 - \theta - b}{\theta^3}\right) - \frac{R}{\theta}\right)$$

During period $[0,T]$, there are $n$ transfers at every $t_1$ time units. Hence, $T = nt_1$. Therefore, the total profit per time unit is, $\Pi(n,R,t_1) = \frac{\Pi_p}{T}$

$$= \frac{n(P-C)\left(Re^{a_1}, \theta^2 + ae^{a_1}, \theta + ae^{a_1}, b\right)}{-a\theta - ab - abt_1e^{a_1}, \theta - R\theta^2} - A - nG$$

$$+ h_w n(n-1)t_1 \frac{\left(Re^{a_1}, \theta^2 + ae^{a_1}, \theta + ae^{a_1}, b\right)}{-a\theta - ab - abt_1e^{a_1}, \theta - R\theta^2}$$

$$- nh_d \left(\frac{R}{\theta} + a \left(\frac{b\theta^2 t_1^2 - 2\theta - 2b - 2\theta^2 t_1}{2\theta^3}\right)\right)$$

$$+ nh_d e^{a_1}\left(a \left(\frac{\theta t_1 - \theta - b}{\theta^3}\right) - \frac{R}{\theta}\right)$$

$$= \frac{n(P-C)\left(Re^{a_1}, \theta^2 + ae^{a_1}, \theta + ae^{a_1}, b\right)}{-a\theta - ab - abt_1e^{a_1}, \theta - R\theta^2} - A - nG$$

$$+ h_w n(n-1)t_1 \frac{\left(Re^{a_1}, \theta^2 + ae^{a_1}, \theta + ae^{a_1}, b\right)}{-a\theta - ab - abt_1e^{a_1}, \theta - R\theta^2}$$

$$- nh_d \left(\frac{R}{\theta} + a \left(\frac{b\theta^2 t_1^2 - 2\theta - 2b - 2\theta^2 t_1}{2\theta^3}\right)\right)$$

$$+ nh_d e^{a_1}\left(a \left(\frac{\theta t_1 - \theta - b}{\theta^3}\right) - \frac{R}{\theta}\right)$$

$$nt_1$$

(6.6.1.8)
6.1.3 Necessary and Sufficient Condition for an Optimal Solution

The total profit per unit time of a retailer is a function of three variables, namely, \( n \), \( R \) and \( t_1 \):

\[
\frac{\partial^2 \Pi(n,R,t_1)}{\partial n^2} = -\frac{2A}{n^3 t_1} < 0
\]  

(6.6.1.9)

Thus, the retailer’s total profit per unit time is a concave function of \( n \) for fixed \( R \) and \( t_1 \).

Next, to determine the optimum cycle time for showroom, for given ‘\( n \)’, we first differentiate \( \Pi(n,R,t_1) \) with respect to \( R \). We get,

\[
\frac{\partial\Pi(n,R,t_1)}{\partial R} = \left( \frac{1-e^{-\theta t_1}}{t_1} \right) \left( -(P-C) + \frac{h_w(n-1)t_1}{2} + \frac{h_d}{\theta} \right)
\]

(6.6.1.10)

Depending on the sign of \((P-C)\theta - h_d\) three cases arise: Define \( \Delta = (P-C)\theta - h_d \)

**Case 1: \( \Delta < 0 \)**

If \( \Delta < 0 \), then \( \Pi(n,R,t_1) \) is a decreasing function of \( R \) for fixed \( R \). It suggests that no transfer of units should be made from the warehouse to the showroom, so put \( R = 0 \) in \( \Pi(n,R,t_1) \) and differentiates resultant expression with respect to \( t_1 \). We have,

\[
\frac{\partial \Pi}{\partial t_1} \bigg|_{R=0} = 0
\]
\[
\begin{align*}
\frac{a(P - C)(1 - bt_i)e^{\alpha_t} - \frac{1}{2} h_w(n - 1)a\theta^2 t_i(1 - bt_i)e^{\alpha_t}}{t_i} \\
\frac{1}{2} h_w(n - 1)a((1 - e^{\alpha_t})(\theta + b) + b\theta t_i e^{\alpha_t}) \\
\left( -\frac{h_a a}{\theta^2} (bt_i - 1)(1 - e^{\alpha_t}) \right)
\end{align*}
\]

\[
\frac{a(P - C)(1 - e^{\alpha_t})(\theta + b) + bt_i e^{\alpha_t} \theta) }{\theta^2}
\]

\[
\frac{1}{t_i^2} + \frac{h_w(n - 1)a((1 - e^{\alpha_t})(\theta + b) + bt_i e^{\alpha_t} \theta)t_i}{2\theta^2}
\]

\[
\left( -\frac{A}{n} - G - \left( \frac{h_a a}{\theta^2} \left( \frac{bt_i(2 + \theta t_i)}{2\theta^2} - \frac{(\theta + b)(1 + \theta t_i)}{\theta^3} \right) \right) \right)
\]

The sufficiency condition is, \( \frac{\partial^2 \Pi(n,R,t_i)}{\partial t_i^2} < 0 \)

\[
\left\{\begin{array}{l}
-4na\theta^3 t_i Pe^{\alpha_t} + 4na\theta^3 t_i Ce^{\alpha_t} + 4n\theta^2 Pa e^{\alpha_t} - 4n\theta^2 Cae^{\alpha_t} \\
+4n\theta Pabe^{\alpha_t} - 4n\theta^2 Pa + 4n\theta^2 Ca - 4nG\theta^3 - 4A\theta^3 \\
-4n\theta Pab + 4n\theta Cab + 4nh_a a\theta + 4nh_a ab - 4n\theta^2 Pab t_i e^{\alpha_t} \\
-4n\theta Cabe^{\alpha_t} + 4n\theta^2 Cabt_i e^{\alpha_t} - 4nh_a a\theta e^{\alpha_t} - 4nh_a ab e^{\alpha_t} \\
+4nh_a ab t_i \theta e^{\alpha_t} + 2na\theta^3 t_i^2 Pe^{\alpha_t} + 2na\theta^3 t_i^2 Pbe^{\alpha_t} \\
-2na\theta^4 t_i^2 Pbe^{\alpha_t} - 2na\theta^4 t_i^2 Ce^{\alpha_t} - 2na\theta^3 t_i^2 Cbe^{\alpha_t} \\
+2na\theta^4 t_i^3 Cbe^{\alpha_t} - n^2 a\theta^4 t_i^3 h_w e^{\alpha_t} + n^2 a\theta^3 t_i^3 h_w be^{\alpha_t} \\
+n^2 a\theta^4 t_i^4 h_w be^{\alpha_t} + na\theta^4 t_i^3 h_w e^{\alpha_t} - na\theta^3 t_i^3 h_w be^{\alpha_t} \\
-na\theta^4 t_i^4 h_w be^{\alpha_t} - 2na\theta^3 t_i^2 h_w e^{\alpha_t} - 2na\theta^2 t_i^2 h_w be^{\alpha_t} \\
+2na\theta^3 t_i^3 h_w be^{\alpha_t} + 4na\theta^2 t_i h_w e^{\alpha_t}
\end{array}\right\} < 0
\]

i.e., \( \frac{1}{2\theta^3 n t_i^3} < 0 \)

Thus, \( \Pi(n,t_i) \), the total profit per unit time is a concave function of \( t_i \) for fixed

'value'. There exists a unique \( t_i' \), denoted by \( t_i'' \) such that \( \Pi(n,t_i'') \) is maximum.
Substituting \( t_i^{*1} \) and \( R^* = 0 \) in to equation (6.6.1.5) to obtain number of units to be transferred (say) \( q^{*1} \) for fixed \( n \).

**Note**: since \( q^{*1} \leq L \) for all \( q \), \( q^{*1} = L \). If \( q^{*1} > L \) then obtain \( t_i^{*1} \) using,

\[
t_i^{*1} = \frac{1}{\theta} \ln \left[ 1 + \frac{L \theta^2}{a(\theta + b)} \right]
\]

**Case 2**: \( \Delta = 0 \)

In this case, we equation (6.6.1.8) as,

\[
\Pi(n, R, t_i) = \left( \frac{h_w R e^{\alpha_R}}{2} + \frac{h_w a e^{\alpha_a}}{2 \theta} + \frac{h_w ab e^{\alpha_{ab}}}{2 \theta^2} + \frac{h_w a}{2 \theta} + \frac{h_w ab}{2 \theta^2} \right)
- \left( \frac{t_i h_w ab e^{\alpha_{ab}}}{2 \theta} + \frac{h_w R}{2} - \frac{G}{t_i} - \frac{A}{nt_i} - \frac{h_w R e^{\alpha_R}}{2} \right)
- \left( \frac{nt_i h_w ab e^{\alpha_{ab}}}{2 \theta} + \frac{h_w a}{2} - \frac{t_i h_w ab}{2 \theta} \right)
\]

(6.6.1.13)

Here, \( \frac{\partial \Pi(n, R, t_i)}{\partial R} = -\frac{h_w}{2} (n - 1)(e^{\alpha_R} - 1) < 0 \)

i.e. \( \Pi(n, R, t_i) \) is decreasing function of \( R \) for given \( n \). So no transfer should be made from the warehouse to the showroom, i.e. \( R = 0 \). So equation (6.6.1.13) becomes

\[
\Pi(n, t_i) = \left( \frac{h_w a e^{\alpha_a}}{2 \theta} + \frac{h_w ab e^{\alpha_{ab}}}{2 \theta^2} + \frac{h_w a}{2 \theta} + \frac{h_w ab}{2 \theta^2} + \frac{t_i h_w ab e^{\alpha_{ab}}}{2 \theta} + \frac{G}{t_i} - \frac{A}{nt_i} - \frac{h_w ae^{\alpha_R}}{2} \right)
- \left( \frac{nt_i h_w ab e^{\alpha_{ab}}}{2 \theta} + \frac{h_w a}{2} - \frac{t_i h_w ab}{2 \theta} \right)
\]

(6.6.1.14)
The optimal value of \( t_{i}^{*2} \) can be obtained by solving,

\[
\frac{\partial \Pi(n, t_{i})}{\partial t_{i}} = \left( \begin{array}{c}
\frac{h_{w} a e^{\alpha_{t_{i}}}}{2} - \frac{t_{i} h_{w} a b e^{\alpha_{t_{i}}}}{2} + \frac{G}{t_{i}^{2}} + \frac{A}{nt_{i}^{2}} \\
- \frac{nh_{w} a e^{\alpha_{t_{i}}}}{2} + \frac{h_{w} t_{i} n a b e^{\alpha_{t_{i}}}}{2} - \frac{h_{w} a b}{2\theta}
\end{array} \right) = 0 \tag{6.6.1.15}
\]

The sufficiency condition is,

\[
\frac{\partial^{2} \Pi(n, t_{i})}{\partial t_{i}^{2}} = \left( \begin{array}{c}
\frac{nh_{w} a \theta e^{\alpha_{t_{i}}}}{2} - \frac{n a b h_{w} e^{\alpha_{t_{i}}}}{2} - \frac{n a b t_{i} \theta h_{w} e^{\alpha_{t_{i}}}}{2} \\
\frac{a \theta h_{w} e^{\alpha_{t_{i}}}}{2} + \frac{a b h_{w} e^{\alpha_{t_{i}}}}{2} + \frac{t_{i} h_{w} a b \theta e^{\alpha_{t_{i}}}}{2} \\
2G + \frac{2A}{t_{i}^{3}} + \frac{2A}{nt_{i}^{3}}
\end{array} \right) < 0, \text{ for } t_{i} = t_{i}^{*2}
\]

Then, \( \Pi(n, t_{i}^{*2}) \) is a concave function of \( t_{i}^{*2} \) and hence \( \Pi(n, t_{i}^{*2}) \) is the maximum profit of the retailer. \( q^{*2} \) can be obtained by substituting value of \( t_{i}^{*2} \) in equation (6.6.1.5)

**Note:** Since \( q^{*2} \leq L \) for all \( q \) then \( q^{*2} = L \). If \( q^{*2} > L \) then obtain \( t_{i}^{*2} \) using,

\[
t_{i}^{*2} = \frac{1}{\theta} \ln \left[ 1 + \frac{L \theta^{2}}{a(\theta + b)} \right]
\]

**Case 3:** \( \Delta > 0 \). There are three sub-cases:

**Sub-case 3.1:** \( \frac{(P - C)\theta - h_{d}}{\theta t_{i}} < \frac{h_{w}(n - 1)}{2} \) then \( \frac{\partial Z(n, R, t_{i})}{\partial R} < 0 \). It is same as Case 1.
The optimal transfer level of units in showroom is zero and there exists a unique \( t_1 \) (say) \( t_1^{*3.1} \) such that \( \Pi(n,t_1^{*3.1}) \) is maximum.

**Note:**
1. \( t_1^{*3.1} \leq \frac{2((P-C)\theta - h_d)}{\partial t_1 h_w(n-1)} \) then \( t_1^{*3.1} \) is infeasible. (6.6.1.2) Because \( q \leq L \) for all \( q \), \( q^{*3.1} = L \). If \( q > L \) then obtain \( t_1^{*3.1} \) using equation (6.6.1.5).

(6.6.1.3) The number of transfers from the warehouse to the showroom must be at least 2.

**Sub-case 3.2:** \( \frac{(P-C)\theta - h_d}{\partial t_1} > \frac{h_w(n-1)}{2} \) Here, \( \frac{\partial \Pi(n,R,t_1)}{\partial R} > 0 \). Therefore, raise the inventory level to the maximum allowable quantity. So from \( L = q + R \) and equation (6.6.1.5), we get,

\[
R = \frac{L\theta^2 - a\theta e^{aR_1} - abe^{aR_1} + a\theta + ab + abt_1\theta e^{aR_1}}{\theta^2 e^{aR_1}} \quad (6.6.1.16)
\]

Then \( R \) is a function of \( t_1 \). Substitute (6.6.1.16) into (6.6.1.8). The resultant expression for the total profit per unit time is function of \( n \) and \( t_1 \). The necessary condition for finding the optimal time \( t_1^{*3.2} \) in showroom is,

\[
\frac{\partial \Pi(n,t_1)}{\partial t_1} =
\]

\[
\left( \frac{Pab - h_dab}{\partial t_1 e^{aR_1}} - \frac{G}{2\theta} + \frac{A}{nt_1^2} - \frac{(P-C)L}{t_1^2} + \frac{(P-C)a}{\partial t_1^2} + \frac{h_dL}{\partial t_1^2} + \frac{h_wab}{2\theta} \right) + \frac{CL}{t_1 e^{aR_1}} - \frac{CL\theta}{t_1 e^{aR_1}} - \frac{h_wa}{2\theta} - \frac{h_dL}{\partial t_1^2} - \frac{h_wab}{2\theta} + \frac{h_wab}{2\theta} \right)
\]
The obtained $t_i = t_i^{3.2}$ maximizes the total profit, $\Pi(n, t_i^{3.2})$, per unit time because 

$$\frac{\partial^2 \Pi(n, t_i)}{\partial t_i^2} = 0$$
Algorithm:

Step 1: Assign parametric values to $A, G, h_d, h_w, P, C, a, b, \theta, L$.

Step 2: If $\Delta < 0$ then go to Algorithm – a.

Step 3: If $\Delta = 0$ then go to Algorithm – b.

Step 4: If $\Delta > 0$ then go to Algorithm – c.

Algorithm – a

Step 1: Set $R = 0$ and $n = 1$.

Step 2: Obtain $t_1^{-1}$ by solving equation (6.6.1.11) with Maple 11 (mathematical software) and $q^{-1}$ from equation (6.6.1.5).

Step 3: If $q^{-1} < L$ then $t_1^{-1}$ obtained in step 2 is optimal otherwise,

$$t_1^{-1} = \frac{1}{\theta} \ln \left[ 1 + \frac{L\theta^2}{a(\theta + b)} \right]$$

Step 4: Compute $\Pi(n,t_1^{-1})$

Step 5: Increment $n$ by 1

Step 6: Continue Step 2 to Step 5 until $\Pi(n,t_1^{-1}) < \Pi((n-1),t_1^{-1})$.

Algorithm – b

Step 1: Set $R = 0$ and $n = 2$. 
Step 2: Obtain $t_i^{*2}$ from equation (6.6.1.14) and $q_i^{*2}$ from equation (6.6.1.5).

Step 3: If $q_i^{*2} < L$ then $t_i^{*2}$ obtained in step 2 is optimal otherwise,

$$t_i^{*2} = \frac{1}{\theta} \ln \left[ 1 + \frac{L\theta^2}{a(\theta + b)} \right]$$

Step 4: Compute $\Pi(n, t_i^{*2})$

Step 5: Increment $n$ by 1

Step 6: Continue Step 2 to Step 5 until $\Pi(n, t_i^{*2}) < \Pi((n-1), t_i^{*2})$

Algorithm – c

Step 1: Set $n = 2$.

Step 2: Solve equation (6.6.1.11) to compute $t_i^{*3.1}$ and determine $q_i^{*3.1}$ from equation (6.6.1.5) and $R = 0$.

Step 3: If $q_i^{*3.1} \leq L$ then $t_i^{*3.1}$ obtained in step 2 is optimal otherwise,

$$t_i^{*3.1} = \frac{1}{\theta} \ln \left[ 1 + \frac{L\theta^2}{a(\theta + b)} \right]$$

is optimal

Step 4: If \( \frac{(P - C)\theta - h_d}{\theta} < \frac{h_w(n-1)}{2} \) then Compute $\Pi(n, t_i^{*3.1})$, otherwise set $\Pi(n, t_i^{*3.1}) = 0$

Step 5: Solve equation (6.6.1.17) to compute $t_i^{*3.2}$
Step 6: If \( \frac{(P - C)\theta - h_d}{\theta t_i} > \frac{h_w(n - 1)}{2} \), then Substitute \( t_i^{3.2} \) in to equation (6.6.1.16) to find \( R \) and Calculate \( \Pi(n, t_i^{3.2}) \), otherwise set \( \Pi(n, t_i^{3.2}) = 0 \)

Step 7: \( \Pi(n, t_i^{3.2}) = \max\{\Pi(n, t_i^{3.1}), \Pi(n, t_i^{3.2})\} \)

Step 8: Increment \( n \) by 1

Step 9: Continue Step 2 to Step 8 until \( \Pi(n, t_i^{3.2}) < \Pi((n - 1), t_i^{3.2}) \)

6.1.4 Numerical Examples

Example 6.6.1.1 Consider the following parametric values in proper units:

\[
\begin{bmatrix}
[a, \theta, h_d, h_w, C, P]
\end{bmatrix} = [1000, 0.10, 0.6, 0.3, 1, 3]
\]

Here, \( (P - C)\theta - h_d < 0 \). We apply Algorithm–a. The variations in demand rate; \( b \), transfer cost; \( G \), Ordering cost; \( A \), maximum allowable units; \( L \) are studied.

<table>
<thead>
<tr>
<th>Table:6.6.1.1 [Variations in ( b )]</th>
</tr>
</thead>
<tbody>
<tr>
<td>[Fixed values ( L = 150, A = 90, G = 10, b = 0.4 )]</td>
</tr>
<tr>
<td>( b )</td>
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<td>---</td>
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<tr>
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<tr>
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<td>0.50</td>
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### Table: 6.6.1.2 [Variations in G]

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<th>n</th>
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<th>$T'$</th>
<th>$q_1'$</th>
<th>$Q'$</th>
<th>$\Pi'$</th>
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</thead>
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<td>148.493</td>
<td>1336.439</td>
<td>1600.113</td>
</tr>
<tr>
<td>20</td>
<td>7</td>
<td>0.151</td>
<td>1.057</td>
<td>147.539</td>
<td>1032.776</td>
<td>1560.089</td>
</tr>
<tr>
<td>30</td>
<td>6</td>
<td>0.138</td>
<td>0.828</td>
<td>135.112</td>
<td>6756.149</td>
<td>1490.671</td>
</tr>
</tbody>
</table>

### Table: 6.6.1.3 [Variations in A]

<table>
<thead>
<tr>
<th>A</th>
<th>n</th>
<th>$t_1'$</th>
<th>$T'$</th>
<th>$q_1'$</th>
<th>$Q'$</th>
<th>$\Pi'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>6</td>
<td>0.149</td>
<td>0.894</td>
<td>145.631</td>
<td>873.786</td>
<td>1679.377</td>
</tr>
<tr>
<td>60</td>
<td>6</td>
<td>0.146</td>
<td>0.876</td>
<td>142.766</td>
<td>856.596</td>
<td>1669.339</td>
</tr>
<tr>
<td>70</td>
<td>5</td>
<td>0.144</td>
<td>0.72</td>
<td>140.854</td>
<td>704.272</td>
<td>1663.394</td>
</tr>
</tbody>
</table>

### Table: 6.6.1.4 [Variations in L]

<table>
<thead>
<tr>
<th>L</th>
<th>n</th>
<th>$t_1'$</th>
<th>$T'$</th>
<th>$q_1'$</th>
<th>$Q'$</th>
<th>$\Pi'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>6</td>
<td>0.138</td>
<td>0.830</td>
<td>135.48</td>
<td>812.94</td>
<td>1635.60</td>
</tr>
<tr>
<td>250</td>
<td>5</td>
<td>0.156</td>
<td>0.778</td>
<td>151.90</td>
<td>759.50</td>
<td>1636.67</td>
</tr>
<tr>
<td>350</td>
<td>5</td>
<td>0.156</td>
<td>0.778</td>
<td>151.90</td>
<td>759.50</td>
<td>1636.67</td>
</tr>
</tbody>
</table>

**Example 6.6.1.2** Consider the following parametric values in proper units:

\[
[a, \theta, h_d, h_w, C, P] = [1000, 0.20, 0.40, 0.10, 1, 3]
\]
Here, \((P - C)\theta - h_d = 0\). Using Algorithm – b, variations in demand rate; 
\(b\), transferring cost; \(G\), Ordering cost; \(A\), and maximum allowable number \(L\) on the decision variables and objective function is studied below:

<table>
<thead>
<tr>
<th>Table: 6.6.1.5 [Variations in (b)]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>[Fixed values (L = 150, A = 90, G = 10, P = 3, C = 1, \theta = 0.2)]</strong></td>
</tr>
<tr>
<td>(b)</td>
</tr>
<tr>
<td>0.4</td>
</tr>
<tr>
<td>0.425</td>
</tr>
<tr>
<td>0.45</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table: 6.6.1.6 [Variations in (G)]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>[Fixed values (L = 150, A = 90, b = 0.4, P = 3, C = 1, \theta = 0.2)]</strong></td>
</tr>
<tr>
<td>(G)</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>12</td>
</tr>
<tr>
<td>14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table: 6.6.1.7 [Variations in (A)]</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>[Fixed values (L = 150, G = 10, b = 0.4, P = 3, C = 1, \theta = 0.2)]</strong></td>
</tr>
<tr>
<td>(A)</td>
</tr>
<tr>
<td>80</td>
</tr>
<tr>
<td>85</td>
</tr>
<tr>
<td>90</td>
</tr>
</tbody>
</table>
Example 6.6.1.3 Consider the following parametric values in proper units:

\[
\begin{bmatrix}
 a, \theta, h_d, h_w, C, P
\end{bmatrix} = [1000, 0.40, 3, 1, 4, 12]
\]

Here, \((P - C)\theta - h_d > 0\). Using Algorithm – c, variations in demand rate; \(b\), transferring cost; \(G\), Ordering cost; \(A\), and maximum allowable number \(L\) on the decision variables and total profit per unit time is studied below:

<p>| Table: 6.6.1.8 [Variations in (L)] |
|-------------------|---|---|---|---|---|</p>
<table>
<thead>
<tr>
<th>(L)</th>
<th>(n)</th>
<th>(t_1^{12})</th>
<th>(T^*)</th>
<th>(q^{12})</th>
<th>(Q^*)</th>
<th>(\Pi^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>22</td>
<td>0.099</td>
<td>2.185</td>
<td>98.31</td>
<td>2162.86</td>
<td>1715.17</td>
</tr>
<tr>
<td>150</td>
<td>10</td>
<td>0.151</td>
<td>1.508</td>
<td>148.43</td>
<td>1484.31</td>
<td>1746.88</td>
</tr>
<tr>
<td>175</td>
<td>8</td>
<td>0.170</td>
<td>1.358</td>
<td>166.76</td>
<td>1334.12</td>
<td>1748.55</td>
</tr>
<tr>
<td>200</td>
<td>8</td>
<td>0.170</td>
<td>1.358</td>
<td>166.76</td>
<td>1334.12</td>
<td>1748.55</td>
</tr>
</tbody>
</table>

<p>| Table: 6.6.1.9 [Variations in (b)] |
|-------------------|---|---|---|---|---|---|</p>
<table>
<thead>
<tr>
<th>(b)</th>
<th>(n)</th>
<th>(t_1^{13})</th>
<th>(T^*)</th>
<th>(q^{13})</th>
<th>(Q^*)</th>
<th>(\Pi^*)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.40</td>
<td>3</td>
<td>0.151</td>
<td>0.452</td>
<td>150.74</td>
<td>452.22</td>
<td>7224.91</td>
</tr>
<tr>
<td>0.45</td>
<td>3</td>
<td>0.145</td>
<td>0.436</td>
<td>145.16</td>
<td>435.47</td>
<td>7195.76</td>
</tr>
<tr>
<td>0.50</td>
<td>3</td>
<td>0.141</td>
<td>0.422</td>
<td>140.16</td>
<td>420.48</td>
<td>7167.68</td>
</tr>
</tbody>
</table>
The following managerial issues are observed from the above tables 6.6.1.1 – 6.6.1.12.
1. Increase in demand rate $b$, decreases $t_1^*$, $q^*$ and $\Pi^*$. It is obvious that retailer’s total profit per unit time, cycle time in the warehouse and procurement quantity from the supplier decreases as the demand decreases.

2. Increase in transferring cost from the warehouse to the showroom increases $t_1^*$, $q^*$ and decreases $\Pi^*$. $\Pi^*$ decreases because number of transfer increases.

3. Increase in ordering cost decreases cycle time in showroom and units transferred from warehouse to the showroom and retailer’s total profit per unit time. The cycle time in warehouse increases significantly.

4. Increase in maximum allowable number in display area increases $t_1^*$ and $q^*$ but no significant change is observed in the total profit per unit time of the retailer. The cycle time in warehouse and procurement quantity from the supplier decreases significantly.

Conclusions

In this chapter, an ordering transfer inventory model for deteriorating items is analyzed when the retailer owns showroom having finite floor space and the demand is decreasing with time. Algorithms are proposed to determine retailer’s optimal policy which maximizes his total profit per unit time. Numerical examples and the sensitivity analysis are given to deduce managerial insights.
The proposed model can be extended to allow for time dependent deterioration. It is more realistic if damages during transfer from warehouse to showroom are incorporated.