Chapter 1

Introduction

1.1. Inventory

The word 'inventory' means simply a stock of idle resources of any kind having an economic value. In other words, inventory means a physical stock of goods, which is kept in hand for smooth and efficient running of future affairs of an organization. It may consist of raw materials, work-in-progress, spare parts, finished goods; human resources such as unutilized labour; financial resources such as working capital, etc. It is not necessary that an organization has all these inventory classes. But whatever may be the inventory items, they need efficient management as generally, a substantial amount of money is invested in them.

The basic inventory decisions include:

→ How much to order?
→ When to order?
→ How much safety stock should be kept?

So, profit can be maximized or total cost can be minimized.

The problems faced by different organizations have necessitated the use of scientific techniques in the management of inventories known as inventory control.
Inventory control is the technique of maintaining stock items at desired levels. In other words, it is concerned with the acquisition, storage, and handling of inventories so that the inventory is available whenever needed.

**Inventory models can be classified according to the following factors:**

1.1.1 **Inventory related costs**

Inventory related costs are classified as

a. Purchase cost
b. Ordering cost
c. Carrying cost (Inventory holding cost)
d. Shortage cost (Stock – out cost)

a. **Purchase (or production) cost:**

It is the cost at which an item is purchased, or if an item is produced, it is the direct manufacturing cost. In many practical situations, the unit purchase price depends on the quantity purchased so the purchase price is of special interest when large quantities are bought or when large production runs may result in a decrease in the production cost.

b. **Ordering (or replenishment or set up) cost:**

The cost incurred in replenishing the inventory is known as ordering cost. It includes all the costs relating to administration (such as salaries of the persons working for purchasing, telephone calls,
computer costs, postage, stationeries, etc.), transportation, receiving and inspection of goods, processing payments, etc. If a firm produces its own goods instead of purchasing the same from an outside source, then it is the cost of re-setting the equipment for production. This cost is expressed as the cost per order or per set up.

c. **Carrying (or inventory holding) cost:**

The cost associated with maintaining the inventory level is known as inventory holding cost. It is directly proportional to the quantity to be kept in stock and the time for which an item is held in stock. It includes handling cost, maintenance cost, storage cost, depreciation, insurance, warehouse rent, taxes, etc.

This cost may be expressed either as per unit of item held per unit of time or as a percentage of average rupee value of inventory held.

In several practical situations, the carrying cost might not be directly proportional to the inventory level. For example, the rent of a warehouse will not change day to day by the change in inventory level.

d. **Shortage (or stock out) cost:**

It is the cost, which arises due to running out of stock (i.e., when an item can not be supplied on the customer's demand). It includes the cost of production stoppage, loss of goodwill, loss of profitability,
special orders at higher price; overtime/idle time payments, expediting, loss of opportunity to sell, etc.

1.1.2 Demand

It is an effective component which is related with a particular time, price, and quantity. The demand pattern of a commodity may be either deterministic or probabilistic. In case of deterministic, it is assumed that the quantities needed in future are known with certainty. This can be fixed (static) or can vary (dynamic) from time to time. To the contrary, in case of probabilistic, the demand over a certain period of time is uncertain, but its pattern can be described by a known probability distribution.

1.1.3 Ordering cycle

An ordering cycle is defined as the time period between two successive placements of orders. The order may be placed on the basis of following two types of inventory review systems:

Continuous review: In this case, record of the inventory level is updated continuously until a specified point (known as reorder point) is reached; at this point a new order is placed. Sometimes, this is referred to as the two-bin system. The inventory is divided into two parts (two bins). Initially, items are used only from one bin, and when it becomes empty, a new order is placed. Demand is then satisfied from the second bin until the order is
received. After receiving the order, the second bin is filled to make up the earlier total. The remaining items are placed in the first bin.

**Periodic review**: In this case, the orders are placed at equally spaced intervals of time. The quantity ordered each time depends on the available inventory level at the time of review.

### 1.1.4 Time horizon

This is also known as planning period over which the inventory level is to be controlled. This can be finite or infinite depending on the nature of demand.

### 1.1.5 Lead time or delivery lag

The time gap between the moment of placing an order and actually receiving the order is referred to as lead time. The lead time can be deterministic, constant or variable, or probabilistic. If there is no such gap, then we say that lead time is zero. If the lead time exists (i.e., it is not zero), then it is required to place an order in advance by an amount of time equal to the lead time.

### 1.1.6 Reorder Level

The level of inventory between minimum and maximum stock which suggests ordering for the next replenishment, is known as reorder level.
1.1.7 Economic Order Quantity (EOQ)

Economic Order Quantity is a size of order, for which the total inventory cost is minimum.

1.1.8 Buffer (or safety) stock

Normally, demand and lead time are uncertain and can not be predetermined completely. So to absorb the variation in demand and supply, some extra stock is kept. This extra stock is known as buffer stock.

1.1.9 Number of items

Generally, an inventory system involves more than one commodity. The number of items held in inventory affects the situation when these items compete for limited floor space or limited total capital.

1.1.10 Deterioration

Deterioration is defined as any process that prevents an item from being used for its intended original use. Deterioration occurs because of spoilage or decay or degradation or loss of potency of the commodity.

1.1.11 Government's policy

For items to be imported as well as for other items like explosive, highly inflammable, and other essential items, the Government has laid down some policy norms. All these affect the level of inventories in an organization.
1.1.12 Supply Chain Management (SCM):

The term 'supply chain management' was first used in the early 1980s to refer to the notion that manufacturing firms should think of their own internal operations as an integrated whole, rather than as separate departments such as purchasing, stores, production, finished good warehouse, distribution and so on. It was quickly extended to cover relationships with suppliers and with immediate customers - the idea being that working more closely and cooperatively with these counterparts would enable a kind of integration and coordination that would lead to reduced inventory, better quality and delivery performance and reduced cost for everyone involved. Today, supply chain management is a prominent concern in large organizations, and is among the most active areas of research in the academic operations management community.

SCM systems are used to coordinate the movement of products and services from suppliers to customers (including manufacturers, wholesalers, and retailers). These systems are used to manage demand, warehouses, trade logistics, transportation, and other issues concerning facilities, and movement and transformation of materials on their way to customers.

Components of SCM include supply chain optimization, and supply chain event management. SCM also comprises warehouse management, radio frequency identification (RFID), and transportation management.

Other modules of SCM solutions include functionality for international trade and logistics, demand management, supplier relationship management and service parts planning.
1.2. Literature survey

Inventory Modeling is one of the most developed branches of operation research and much space is provided to this field in management science, operation research and practitioner – journals. Mathematical modeling for deteriorating items is very interesting part of inventory management. Deterioration is defined as any process that prevents an item from being used for its intended original use. The processes such as: (i) Spoilage, as in perishable foodstuffs, fruits and vegetables; (ii) Physical depletion, as evaporation of volatile liquids such as petrol, alcohol; (iii) Decay, as in radioactive chemicals, (iv) Degradation, as in electronic components, (V) Loss of potency, as in photographic films and pharmaceutical drugs etc.

Deterioration can be categorized in two parts. First, there are some situation in which all the items in inventory becomes simultaneously obsolete at the end of planning horizon, such as style goods in fashion merchandized or outdated newspapers etc. Second, there are some situations in which items deteriorate thought their planning horizon. Further it can be divided in to two parts: (i) items with a fixed shelf life as blood, drugs etc. (ii) items with continuous decay(random lifetime) such as radioactive chemicals, petrol etc.

Significant research has been done on inventory problems for deteriorating items by many researchers from time to time. Research in this area started with the work of Whitin (1957) who considered fashion goods becoming out of fashion at the end of some period. An exponentially decaying inventory model is developed first by Ghare and Schrader (1963) who
observed that some items shrink with a time by a proportion which can be approximated by a negative exponential function of time. This observation lead to the development of modeling of the inventory item with the deteriorating processes by the differential equation,

\[
\frac{dI(t)}{dt} + \theta I(t) = -f(t)
\]

Where \( \theta \) = constant rate of deterioration

\( I(t) \) = the level of inventory at time \( t \),

\( f(t) \) = the demand rate at time \( t \) / constant

Fries (1975) and Nahmias (1975) have studied optimal stocking policies for items with a fixed life time under the assumption of FIFO – issuing policy and fresh supply. Covert and Philip (1973) extended Ghare and Schrader’s model and obtained an EOQ model with a variable rate of deterioration by assuming a two parameter Weibull distribution for the life of an inventory. The inventory system is governed by differential equation,

\[
\frac{dI(t)}{dt} + \theta(t)I(t) = -f(t), \text{ where deterioration is given by } \theta(t) = \alpha \beta t^{\beta-1}.
\]

Here \( \alpha \) is scale parameter and \( \beta \) is shape parameter of the distribution and \( \theta(t) \) can be analogous to the hazard function in the reliability theory.

Then Philip (1974) extended the above model where deterioration rate is given by Weibull distribution of three parameters. Tadikamala (1978)
introduced same model with use of Gamma and Weibull distribution. In all the above models, the average level of inventory is assumed to be linear.


Another scenario of inventory model is about stock-outs. Sometimes shortages occur in the inventory system. It is observed that because of good reputation of the retailer, some customers are willing to wait for new stocks arrival if the wait will be short, while other may go elsewhere. Abad (1996, 2001) derived a pricing and ordering policy for a variable rate of deterioration and partially backlogging. The partial backlogging was assumed to be exponential function of waiting time till the arrival of next replenishment. The assumption of exponential backlogging is unrealistic in developing countries. Also, Abad’s article does not include backordered cost and cost due to lost sales in the formulation of the objective function which influences service level to his customers. Dye et al. (2007) took into account the backorder cost and lost sale.

Wilson’s classical economic order quantity (EOQ) model is derived under the assumption that the demand of the product is uniform or constant over a time. But in practice demand of some seasonal goods like weather selected cloths, blood during riots or accidents, Seasonal paddy grains, air-fare etc. during vacations, some electronics equipments like A.C., Gold,
seasonal fruits and food items, fire crackers, Christmas trees etc. decreases after a practical phase is over. Also these classical EOQ models assumes that the retailer settle the accounts as soon as the items are received in inventory system. Now a days in real practice, a supplier uses credit period as a promotional tool to attract more customers. The supplier offers a permissible credit period to the retailer if outstanding amount is paid within a given credit period and order quantity is larger than a pre – specified quantity by supplier. Here credit period is considered as promotional tool as it is one kind of price discount because paying later indirectly reduces the purchase cost, and motivates retailer to increase his order quantity. The concept of trade credit was first introduced by Haley and Higgins (1973). They developed model to determine EOQ under condition of permissible delay in payments with known constant deterministic demand, no shortages allowed and with zero lead – time. Goyal (1985) developed an EOQ model under the conditions of permissible delay in payments. He excluded penalty cost due to a late payment in Haley and Higgins model. Shah et al. (1988) extended Goyal’s model by allowing shortages. Mandal and Phaujdar (1989) included interest earned from the sales revenue on the stock remaining beyond the settlement period. Chung and Hung (2003) studied Goyal’s model when replenishment rate is finite. Davis and Gaither (1985) derived optimal policy for the firm that offers a one time opportunity to delay payment for an order of a commodity. Such delayed payments results in a reduction of the effective purchase cost, which is a function of the return available on alternative investments, the number of units of commodity ordered and the length of the extended period. Chung (1998) established the convexity of the total annual variable cost
function for optimal EOQ under conditions of permissible delay in payments. Analytically it is shown that the EOQ under conditions of permissible delay in payments is generally higher than the EOQ given by the Wilson's formula.

The average cost approach does take into account the time value of money. Hence, there is no distinction between out-of-pocket holding cost and opportunity cost due to inventory investments. To overcome this scenario, researchers suggested discounted-cash-flow (DCF) approach for the analysis of the optimal inventory policy under the effect of the trade credit. Chung (1989) discussed inventory model under permissible delay in payments using DCF approach. Shah (1993) and Aggarwal and Jaggi (1995) extended Goyal's model to incorporate deterioration of units in the inventory system. Jamal et al. (2000) further generalized the model to allow shortages. Jaggi and Aggarwal (1994) extended Chung (1989)'s model to formulate inventory model for determining the optimal procurement quantity of deteriorating items when permissible delay period is offered using DCF approach. Hwang and Shinn (1997) jointly optimized retailer's sale price and lot size when the supplier offers delay in payments. Dye (2002) developed inventory model for stock-dependent demand for deteriorating items when partial backlogging is allowed and trade credit is offered. Teng (2002) argued that it is economically advantageous for a retailer to order less quantity and take benefits of trade credit more frequently. Chang et al. (2004) developed an inventory model for deteriorating items with instantaneous stock-dependent demand and time value of money when permissible delay in payments is offered. Teng et al. (2005) developed the optimal pricing and lot
sizing under permissible delay in payments by considering the difference between selling price and purchase quantity and demand to be price sensitive. Goyal et al. (2007) formulated an EOQ model for a retailer when the supplier offers a progressive interest scheme, and provided an easy to use closed form solution to make the decision. Rachmadugu (1989) established that the best order quantity is an increasing function of allowable delay period.

Using principles of financial analysis, Dallenbach (1986, 1988), Ward and Chapman (2006), Chapman and Ward (1988) argued that if trade credit has the characteristic of a renewable source of capital, then the usual assumption that the value of the inventory investment opportunity cost made by the traditional inventory theory is correct. Chung et al. (2005) computed the EOQ under conditions of permissible delay time in payments depends on the quantity ordered when the order quantity at which the delay in payment is allowed, the payment for the item should be settled immediately. Otherwise, the fixed credit period is allowed.


The most of above cited researcher have not considered influence of the inflation on inventory policy. However, from a financial point of view, an inventory represents a capital investment and must compete with other assets for a firm’s limited capital funds. Chang (2004), Buzacott (1975), Bierman and Thomas (1977) and Mishra (1979) discussed the inventory decisions under an inflationary condition for the EOQ model. One can read Brahmbhatt (1982), Chandra and Bahner (1985), Datta and Pal (1991), Gor et al. (2008), Huang (2003) and their references.

Most of all the aforementioned citations assumed that the customer must pay for the items as soon as the items are purchased from the retailer. Now – a – days in most of the business transactions, the supplier offers a credit period to the retailer passes some credit period to his customers. Huang (2003) analyzed an inventory model when retailer offers a credit period to his customer which is smaller than the credit period offered by the supplier, in order to increase the demand. In all above citation, the effect of credit period is studied on the objective function. The impact of credit period on demand is ignored. In practice, it is observed that demand of an item does depend upon length of the credit period offered by the supplier to the retailer or retailer to the customer. Jaggi et al. (2008) gave idea of credit – linked demand function to determine the retailer’s optimal credit and replenishment policy when both the suppliers as well as retailer offers the credit period to stimulate end – user demand.
In some of above stated articles, it is assumed that the supplier would offer the retailer a credit period. During this credit period, the retailer will earn interest on the accumulated revenue. Thus, it is assumed that the customer will pay for the goods purchased immediately. That is, the supplier offers credit period to retailer but the retailer does not pass it to the customers. This is termed as “one – level” trade credit, which is unrealistic in competitive business world. Huang (2003) assumed that the retailer will offer the trade credit to boost his customer’s demand to develop supply chain inventory model. Huang and Hsu (2008) derived supply chain model under the assumption that the retailer receives credit period from supplier and passes it partially to his customer. This is termed as “two – level” trade credit. The above stated both models were developed for the market of constant demand which rarely happens.

Supply chain is a network of various stages which involves continuous flow of information, goods or services between different stages. The main purpose of existence of supply chain is to meet the need of end customers.

In the network of supply chain, supply is moving from supplier to the customer and requirements information is moving customer to supplier. In fact in structure of supply chain information flows along both the direction of chain. The structure of supply chain involves various stages like supplier, manufacturer, wholesaler, retailer and customer.

All the stages may not be present in supply chain. Design of supply chain depends upon customer’s need. The success of supply chain should be
measured in terms of the cost and not in terms of the cost of an individual. i.e. total profit of supply chain must be shared across all players in supply chain.

In a supply chain, for continuous flow of information, goods a systematic controlling system is required which generates cost in supply chain system. Thus, success of supply chain is an appropriate management of these flows between the players. Supply chain management involves the management of flows between and among different stages in a supply chain to minimize overall joint cost. Success of supply chain management depends upon decisions which relate flow of information or goods or services in supply chain. They can be categorized as (1) supply chain design (2) supply chain planning and (3) supply chain operation. Supply chain design focuses on structure, location and feasibility of different stages. In supply chain planning, different operating policies are decided before implementation; e.g. manufacturing policies, inventory policies etc. The objective of supply chain operations is to define and implement the operation policies to handle the need of very next player in supply chain in the best effective manner. Supply chain decisions have a large impact on the success or failure of each firm because the significantly influence the performance of system and hence total joint cost of chain. Successful supply chain manages flow of information, product or services to provide a high level of product availability to the customer at the lower cost. The efficiency and performance of supply chain depend on facilities, inventory, transportation and information, which are known as drivers of supply chain performance.
Inventory plays a very important role in supply chain to drive the supply chain performance. Inventory is an excess amount of stocks stored to meet unpredicted variation in demand. Inventory is spread through the supply chain from raw materials to work in progress to finished goods that suppliers, manufacturers, distributors, and retailers hold. Inventory is a major source of the cost in a supply chain and it has huge impact on responsiveness. Inventory also has a significant impact on the material flow time.

By holding large amounts of inventory, a high level of responsiveness can be achieved in the supply chain. Conversely, less inventories can lead to more efficient supply chain. Thus, it is a trade of between responsiveness that results from more inventories and the efficiency which results from the reduction in inventory within the supply chain.

If the product is not stored properly in inventory then it may spoil or deteriorate in quality. In such situation, it decreases the level of responsiveness and increases the integrated cost in supply chain. In supply chain, the integrated cost is the total cost to be shared across all players of the supply chain.

Inventory control and management helps to take actions that allow decreasing the level of inventory in the system without increasing cost. The costs which are associated in inventory control and management can mainly be classified as purchasing cost, cost of ordering, holding cost, stock out cost, and cost of deterioration.

Nowadays, during this recession period when demand is a decreasing function of time, the management of inventory is a critical issue for the
managers. The next most worrying issue is of transfer batching, the integration of production and inventory model as well as the purchase and shipment of items. It is assumed that on the receipt of the delivery of the items, retailer stocks some items in the showroom and rest of the items in warehouse. Clearly, floor area of showroom is limited and well furnished with modern techniques, so inventory holding cost in showroom must be higher as compared to that in warehouse. The problem is how often and how many items are to be transferred from warehouse to the showroom which maximizes the total profit per unit time. Goyal (1977), for the first time, formulated single supplier – single retailer integrated inventory model. Banerjee (1986)’s model. It is assumed that the numbers of shipments are equally sized and production of the batch had to be finished before the start of shipment. Lu (1995) allowed shipments to occur during the production period. Goyal (1995) derived a shipment policy in which, during production, a shipment is made as soon as the buyer is about to face stock out and all the produced stock manufactured up to that point is shipped out. Hill (2000) developed an integrated multi – lot – size production inventory model for deteriorating items. Law and Wee (2006) derived an integrated production – inventory model for ameliorating and deteriorating items using DCF – approach. Yao et al. (2007) argued the importance of supply chain parameters when vendor – buyer adopts joint policy. The interesting papers in this areas are by Hill (1997, 1999); Sarker et al. (2000a, 2000b), Vishwanathan (1998); Goyal and Nababe (2000), Chiang (2001), Kim and Ha (2003), Nieuwenhuyse and Vandaele (2004), Siajadi et al. (2006) and their
cited references. The aforesaid articles are dealing with integrated vendor–buyer inventory model when demand is deterministic and known constant.

Sometimes the continuous usage of machine for production decreases the quality of items. It is observed that at the start of production, system is ‘in–control’ state and items produced are of 100% quality. But after some time, system may shift to an ‘out–control’ state from ‘in–control’ state and produces defective items. This shows that the flexible manufacturing system helps the manufacturing industries to reduce the rate of production to avoid pilling of inventories and defective items. Rosenblatt and Lee (1986) considered the time of shift from ‘in–control’ state to ‘out-of-control’ state follows an exponential distribution with a mean $\frac{1}{\mu}$, assuming $\mu$ to be very small. They derived closed form solution for Economic Manufacture quantity using second order of Maclaurin series expansion of the exponential function. Lee and Rosenblatt (1987, 1989) derived an optimal production run-time and optimal inspection policy simultaneously to observe the production process. Cheng (1991) have derived a closed form expression for the optimal demand to satisfy order quantity and process reliability while the demand exceeds supply and the production process is imperfect. Khouja and Mehrez (1994) assumed that the elapsed time until the production process shifts to an ‘out–of–control’ state to be an exponentially distributed random variable. Hariga and Ben–Daya (1998) and Kim and Hong (1999) have extended Rosenblatt and Lee (1986) model by considering the general time required to shift distribution and an optimal production run time shown to be unique. Makis (1998) derived several properties of the optimal production and inspection
policies in imperfect production process. Some of the above cited references assumed that defective items are reworked instantaneously. Wang (2004) discussed an imperfect Economic Manufacturing Quantity model for production which are required and sold under free – repair warranty policy. Shiu and Chen (2004) derived a lot size model to determine the level preventive maintenance for an imperfect production control. Lee (2005 a – c) has extended the model to increase the service level and reduce the defective items in imperfect production system. Chen and Lo (2006) formulated an imperfect production system with allowable shortage and items are sold with free minimal repair warranty. The probabilities of defective items in both the states (out – of – control and in – control) are considered to be different. Lee (2006) investigated the investment model with respect to repetitive inspections and measurement equipment in imperfect production process. Sana et al.(2007 a) analyzed Economic Production Lot – size model for production system producing items of perfect as well as imperfect quality. The probability of imperfect quality items increases with increase of production run time because of continuous usage of machines. They assumed that the demand rate of perfect quality item is constant whereas the demand rate of defective items which are not repaired is a function of reduction rate. Sana et al. (2007 b) derived a volume flexible inventory model with an imperfect production system where demand rate of good quality items is a random variable and the demand rate of defective items is a function of a random variable and reduction rate. Giri and Doni (2007) developed the inspection scheduling in an imperfect production process when machine shifts from ‘in – control’ state to an ‘out – of – control’ state. They assumed that the shift time
follows an arbitrary probability distribution with increasing failure rate and the products are sold with a free minimal repair warranty. The inspection during production period reduces the number of imperfect quality products. Liao (2007) analyzed an imperfect production processes that requires production corrections and imperfect maintenance. Lo et al. (2007) extended an production – inventory model for a varying rate of deterioration, partial back – ordering, inflation, imperfect production process and multiple deliveries. Panda et al. (2008) formulated an economic production lot size model for imperfect items in which production rate is constant and the demand rate is probabilistic under certain budget and shortage constraints. They have assumed that the percentage of defective items is stochastic and the nature of uncertainty in the constraints is stochastic or fuzzy. Here, the percentage of defective item is independent of production rate and production – run – time. Lee (2008) developed a maintenance model in multi – level multi – stage system. In this model, the investment in preventive maintenance is to reduce the variance and the deviation of the mean from the target value of the quality that reduce the proportion of defectives and increase reliability of the product. The proportion of defectives can be linked to the cost of manufacturing, cost of inventory and loss of profit. The total cost comprise of the cost of manufacturing, Set up cost, holding cost, loss of profit and warranty cost. Sana (2009) derived model with assumptions that the percentage of defective items varies with production rate and production – run – time, and demand to be deterministic and constant.
1.3. Outline of the Thesis:

The proposed thesis has been divided into seven chapters on the basis of the characteristics of different models which are given as under.

- Chapter 1: Introduction
- Chapter 2: Deteriorating Inventory Models with Time Dependent Shortages in Demand Declining Market
- Chapter 3: Optimal Policies when Supplier Credits are linked to Order Quantity
- Chapter 4: Partial Trade Credit Scenario in Supply Chain for Demand Declining Market
- Chapter 5: Supply Chain Inventory Models for Deteriorating Items under Two – Level Credit Policy in Declining Market
- Chapter 6: Transfer–Ordering Strategy for Deteriorating Items in Declining Market
- Chapter 7: Production Schedule in Declining Market for an Imperfect Production System

The following is a chapter-wise discussion of inventory models which deals with the proposed thesis.
Chapter 1 deals with introductory ideas that lie behind Inventory Management. We define the terms which are necessary to be explained to understand concepts of Inventory management, also some basic notations, as well as some basic assumptions by use of which we had developed some economic order quantity (EOQ) models for different kind of situations. Aim of this chapter is to build up the platform for later chapters, taking an overview before moving on to more detailed discussion.

In Chapter 2, three models are discussed. Viz. Model (2.1), Model (2.2), Model (2.3).

Model (2.1) “Deteriorating inventory model for waiting time partial backlogging” defined on an inventory system in which demand is deterministic and inventory level depletes with constant rate of deterioration, Shortages are allowed and some shortages are partially backlogged which depends upon the time. Model is defined to minimize total cost of an inventory system under the above assumptions.

Model (2.2) “Optimal ordering policy for deteriorating items in demand declining market and time dependent partial backlogging” discussed on an inventory system in which demand is declining with time and inventory level depletes with constant rate of deterioration, shortages are allowed and some shortages are partially backlogged depending upon time. Model is defined to minimize total cost of an inventory system under the above assumptions.
Model (2.3) “Optimal ordering and pricing policies for deteriorating inventory in demand declining market and time dependent partial backlogging” analyzed for an inventory system in which demand is a decreasing function depending upon time and selling price of an inventory per unit per time, inventory level depletes with constant rate of deterioration, shortages are allowed and some shortages are partially backlogged depending upon time. Model is defined to maximize total net profit of an inventory system under the above assumptions.

In all three model, inventory depletion is affected by deterioration (Deterioration is defined as decay, spoilage, loss of utility of a product. e.g. deterioration is observed in volatile liquids, beverages, medicines, dairy items etc.) so deteriorating parameter is introduced in to objective function. Shortages are allowed which is partially backlogged and remaining shortages are considered as lost sales. So backlogging parameter depending upon time, and unit cost due to backlogging as well as due to lost sales are introduced in to objective function.

In mathematical derivation differential equations governing inventory system has been solved and objective functions are designed by use of different cost components and/or revenue function. Using a mathematical software Maple 10.0, and by use of calculus objective function is optimized for the test values of different parameters and sensitivity analysis is done with respect to different parameters like deterioration rate, fixed demand, rate of change of demand, backlogging parameter from which some useful conclusion are derived.
In Chapter 3, three models are discussed. Viz. Model (3.1), Model (3.2), Model (3.3).

**Model (3.1)** “Optimal ordering policy in demand declining market under inflation when supplier credit linked to order quantity” is a lot size inventory system in which demand is decreasing function depending upon time under inflation and supplier offers to the retailer a credit period for a larger ordered quantity that is greater than or equal to pre-specified quantity (i.e. if ordered quantity of retailer is less than pre-specified quantity than retailer is supposed to settle the account immediately for the items received.) Shortages are not allowed and effect of inflation is incorporated. This model serves an easy-to-use flow chart to find out optimal replenishment time and order quantity which helps retailer to have a static decision by which he can minimize the total cost of an inventory system.

**Model (3.2)** “Deteriorating inventory model in demand declining market under inflation when supplier credits linked to order quantity” is an upgradation of Model (3.1) with assumption that inventory level is affected by deterioration. And all other assumptions are same as Model (3.1).

**Model (3.3)** “Deteriorating inventory model for two-level credit-linked demand in permissible delay in payments” is somehow more sensible than Model (3.2) as in factual practice it is unrealistic that supplier offers a fixed credit period to retailer and retailer does not offer any credit to his customer. Because of global competition a retailer may use this credit period
as a promotional tool and offer a credit period to his customer to boost his own demand. Here demand function is considered to be a function depending upon the credit period given by retailers to his customers to settle the accounts. The units of inventory system deteriorate with a constant rate. Also deteriorated units can neither be repaired nor replaced during a cycle time. And shortages are not allowed. This model is designed to get an easy – to – use algorithm to determine optimal credit period and replenishment policy for the retailer.

Here all the three models discuss the concept of credit period by supplier to retailer and/or retailer to customers. That is, the credit period is considered as a promotional tool. Supplier gives credit period to retailer to have an order of large number of units and retailer gives credit period to attract his customers and boost his demand in the market. An easy – to – use algorithm is developed to optimize the objective function (i.e. total cost function must be minimized or profit function is maximized). Also numerical analysis is carried out for the test data and sensitivity analysis is done for different parameters.

In Chapter 4, two models are discussed. Namely Model (4.1), Model (4.2).

Model (4.1) “Supply chain inventory model for retailer partial trade credit policy in demand declining market” is evaluated for an inventory system in which retailer is dominant decision maker and when supplier offers retailer a credit period, retailer offers partial trade credit period to his customer to
boost his own demand. Here demand is a decreasing function of time and shortages are not allowed. A computational algorithm is derived to minimize the total cost function.

Model (4.2) “Supply chain inventory model for optimal ordering and pricing policies under retailer partial trade credit scenario in declining market” is defined for an inventory system in which retailer is dominant decision maker of supply chain and when supplier offers retailer a credit period, retailer offers partial trade credit period to his customer. The demand is declining and depends upon time. Shortages are not allowed. Here objective of this model is to maximize retailer’s total profit with respect to selling price and ordering quantity.

Total cost of inventory system in Model (4.1) and total profit of a retailer in Model (4.2) are affected by some other cost parameters like customer’s fraction of the cash down payment at the time of placing an order offered by the retailer, interest earned per $ per year, interest charged per $ in stock per year by the supplier. Here computational algorithms are derived to optimize the objective function in both cases. Algorithms are supported by numerical calculations for test data and sensitivity analysis is carried out for different parameters like customer’s fraction of cash down payments, credit period offered by supplier to retailer, credit period offered by retailer to his customer, selling price, rate of change of demand by use of which some useful conclusions are derived.
In Chapter 5, two models are discussed. Namely Model (5.1), Model (5.2).

Model (5.1) “Supply chain inventory model for deteriorating items under two level credit policy in declining market” is developed for an inventory system when demand is a decreasing function of time and shortages are not allowed. The two - level trade credit is taken under consideration. i.e. Supplier offers credit period to the retailer which is in turn, partially offered to the customers by the retailer. It is assumed that the retailer is a powerful decision maker. The objective of this model is to minimize total cost of an inventory system from the retailer’s end.

Model (5.2) “Deteriorating inventory model for optimal ordering and pricing policies under retailer partial trade credit in supply chain when demand decreases” defined under the consideration of two-level partial trade credit in supply chain inventory system. It is assumed that inventory level depletes at a constant rate of deterioration, and demand is declining with respect to time. The objective of this model is to maximize total profit of a retailer with respect to sales price and ordering policy.

An easy – to – use computational algorithms are developed for both models to optimize their objective functions, along with a supportive numerical examples for test data. Sensitivity analysis is performed under the variation in different parameters like in Model (5.1) customer’s fraction of cash down payment, credit period offered by retailer to his customer, selling price, deterioration rate, rate of change of demand, and in Model (5.2) customer’s fraction of cash down payment, credit period offered by retailer to his
customer, deterioration rate, rate of change of demand. From this sensitivity analysis, some useful conclusions are obtained.

Chapter 6 deals with a Model Namely Model (6.1).

Model (6.1) “Optimal transfer – ordering strategy for deteriorating items in declining market” defined to determine retailer’s procurement quantity and number of transfers from the warehouse to the display area. Here demand is decreasing due to recession and inventory level depletes due to constant rate of deterioration. The objective of this model is to maximize total profit of a retailer. An algorithm is developed to determine optimal policy for a retailer with support of a numerical example. Sensitivity analysis with respect to parameters like the maximum allowable number of units of inventory in the showroom (floor space), Ordering cost per order, fixed cost associated with per transfer from warehouse to showroom, rate of change of demand, by which some important conclusion are carried out.

Chapter 7 deals with a Model Namely Model (7.1).

Model (7.1) “Optimal production schedule in declining market for an imperfect production system” is somehow more realistic than classical economic production lot size model. As we know that sometimes continuous usage of a machine leads production process from ‘in – control’ state to ‘out – of – control’ state after some time duration. This model is developed to study the optimal production when a certain percent of total production is of
imperfect quality. These items are reworked to maintain quality of products. The demand is assumed to be decreasing function of time. The objective of this model is to minimize total cost of an inventory system. The development of model is validated by a supportive numerical example and sensitivity analysis is carried out for different parameters like eta, alpha, beta, delta, total labour/energy cost per unit time, inventory holding cost per unit per time, cost of rework for one unit, set up cost per set up, fixed demand, rate of change of demand.

From this sensitivity analysis some managerial insights are carried out. List of papers published, communicated, presented are given. The thesis ends with bibliography.