Chapter 2

Electromagnetic Analysis of Discontinuities in Circular Waveguide Mode Transducers

Discontinuities in waveguides act as mode transducers which excite higher-order modes in oversized waveguides. The content of these higher-order modes have to be minimized or maximized depending on the application sought for. In this chapter, circular waveguides having asymmetrical discontinuities such as radial post and coaxial-line fed probes have been analyzed in the context of multi-frequency ortho-mode transducers (OMT) using moment method based formulation. Scattering parameters for dominant and higher-order modes have been computed as function of geometrical parameters of the discontinuities. Symmetrical waveguide discontinuities have also been analyzed using mode matching technique for step junction. Using this technique a waveguide geometry having tapered section between two cylindrical waveguides and a conical horn geometry have been analyzed. Higher-order mode coupling is studied as a function of flare angle of the tapered waveguide section. The mechanism of excitation of circularly symmetric modes and associated problems is also discussed for microwave rotary joints. The validity of the results is checked through comparison of the data computed using commercially available electromagnetic tools such as TICRA’s CHAMP/FEED for symmetric discontinuities and Ansoft HFSS for asymmetric discontinuities. The experimental and computed data on return loss for post and probe discontinuity problems are also presented.

2.1 Coupling Mechanism and Discontinuities in a Multi-frequency Mode Transducer

A common mode transducer operating over multiple frequency bands is generally preferred as compared to separate mode transducers for each frequency band. These multi-frequency mode transducers can be realized with cascaded circular waveguides having co-axial line fed probes as coupling element as shown in Figure 2.1.
Co-axial line fed probes are used to couple power in the dominant mode of the circular waveguide at individual frequency bands. In the Figure 2.1, straight waveguide sections at each frequency band are terminated at their inputs with tapered sections to allow propagation of signal at all the frequency bands of the multi-frequency mode transducer. The junction between straight and flared waveguide sections act as symmetrical discontinuity exciting undesired higher-order modes along with the fundamental mode. If orthogonal polarizations are required, polarization matched probes have to be used to couple power at different frequency bands. In such cases, the probe in the lower frequency section acts as asymmetrical discontinuity which not only reflects and couples the next incident higher frequency signal but also excites higher-order modes at higher frequencies. The higher-order modes at higher frequency can affect the performance of a horn which is fed with multi-frequency mode transducers. It is of importance to carry out electromagnetic modeling and analysis of symmetrical and asymmetrical waveguide discontinuities to investigate the behavior of fundamental and higher-order waveguide modes as function of physical parameters of the discontinuities. The problems which should be investigated for the device shown in Figure 2.1 are coaxial to circular waveguide junction, scattering from probe or post discontinuities and the discontinuities in the form of step and tapered junctions in circular waveguides.
2.2 Asymmetric Discontinuities in Circular Waveguide

In a probe excited circular waveguide, the impedance matching is achieved at a particular depth (resonant condition) of the probe as reported in [28]. While, the probe excited circular waveguide reported in [28] is perfectly shorted (see Appendix A.1) at its input end, the input ends of the probe excited sections in a multi-frequency mode transducer (Figure 2.1) are terminated with cut off tapers offering a tapered or virtual short. The probe excited circular waveguides having tapered short at its input has not been investigated earlier. An approach of achieving impedance matching in probe coupled circular waveguide having a tapered short at input is described in Appendix A.2 along with the perfect short case for the sake of completeness. To match the impedance offered to the probe in a circular waveguide to that of the coaxial line, the computed probe depth from the expressions in [28] comes nearly quarter of free space wavelength. This depth of probe acts like a metallic post discontinuity and generates a number of higher-order modes in the circular waveguide at various frequency bands. Since, the waveguide junction at lower frequency band is oversized at higher frequency bands, this will support some of the higher-order modes generated at the post or probe discontinuity. These propagating higher-order modes have special radiation characteristics which influence the overall radiation pattern of the feeds excited by the multi-frequency mode transducer. Asymmetric post discontinuities are also used in a variety of microwave filters and tri-mode matched horn feeds [75], [76], [77]. The analysis of a post or probe discontinuity in an oversized circular waveguide which supports higher-order modes has not been reported earlier to the best of author’s knowledge. Thus, it is worthwhile to investigate the scattering characteristics of a post or probe loaded circular waveguide to get an insight about the behavior of higher-order modes which in turn provide more insight into the design of multi-frequency mode transducers presented in Chapter 3.

2.2.1 Post Discontinuity in Circular Waveguide

In this section, the formulation and analysis of scattering characteristics of a radial post discontinuity in an oversized circular waveguide is presented, using Galerkin’s Method of Moments (MoM) considering the entire domain basis functions. Dyadic Green’s function for the radial current has been derived using the approach as given in [28]. The moment
method formulation has been employed by approximating the induced current on the post as a line current and also taking the testing point on the surface of the post as a line. Return loss characteristics and the power coupled to higher-order modes have been found out for various heights of the post. The data on the scattering parameters computed by the present method have been compared with HFSS to justify the validity of the present analysis. Experimental data on return loss of the dominant $\text{TE}_{11}$ mode have been compared with the data computed by the present method. Close agreement has been observed between the experimental and computed data of the return loss.

2.2.1.1 Formulation and Analysis of Post Discontinuity

The geometry to be analyzed is shown in Figure 2.2. It consists of a circular waveguide with a radial post discontinuity. The incident $\text{TE}_{11}$ mode polarized in the direction of the post induces electric current on the post. The induced current on the post generates infinite modes in TE (transverse electric) and TM (transverse magnetic) configurations in both the directions. Depending on the size of the waveguide some modes will be propagating while the other modes will be evanescent.

![Figure 2.2 Circular waveguide loaded with a post discontinuity.](image)

Figure 2.2 Circular waveguide loaded with a post discontinuity.
The tangential components of total electric field \( E^t \) at the post surface should be zero. This is the boundary condition to be satisfied on the surface of the metallic post. With this boundary condition, the electric field integral equation can be written as,

\[
E^{inc} + E^s = E^t = 0
\]

\[
\Rightarrow E^{inc} = -E^s
\]

(2.6)

where, \( E^s \) represents scattered field from the post and is given by,

\[
E^s(\rho, \phi, z) = \int \int G_{pp}(\rho, \phi, z', \rho, \phi, z') \cdot J(\rho) d\phi d\rho
\]

(2.7)

where, \( G_{pp}(\rho, \phi, z, \rho, \phi, z') \) is the dyadic Green's function which has been derived using the approach given in [28]. In [28], the Green's function is derived with one end of the waveguide shorted and the other end terminated at a perfectly matched load. But in the present case the Green's function is derived considering both ends of the waveguide terminated with matched loads. Considering all the \( TE_{mn} \) and \( TM_{mn} \) modes of circular waveguide, the radial component of electric type Green's function \( (G_{pp}^r) \) for \( z > z' \) can be written as [28].

\[
G_{pp}^r(\rho, \phi, z, \rho', \phi', z') = \frac{1}{\omega \mu_0} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} c_m(\rho', \phi', z') \cos(m \phi) J_m(\gamma_{mn} \rho) e^{-\alpha_{mn} z}
\]

\[
+ \frac{j}{\omega \mu_0} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} c_m(\rho', \phi', z') \cos(m \phi) \frac{\partial}{\partial \rho} (J_m(\gamma_{mn} \rho)) e^{-\alpha_{mn} z}; \ z > z
\]

(2.8)

where, \( J_m(\gamma_{mn} \rho) \) and \( J_m(\gamma_{mn} \rho) \) are the Bessel functions of order \( m \).

For \( z < z' \), the expression for radial component of electric type Green's may be obtained by changing the sign of argument of the exponential terms in Equation (2.8).

Using the method described in [28], \( c_{mn}(\rho', \phi', z) \) and \( c_{mn}(\rho', \phi', z') \) have been derived as,

\[
c_{mn}(\rho', \phi', z') = -\frac{\mu}{4 \pi \epsilon_0 \alpha_{mn}^2 \gamma_{mn}^2 \alpha_{mn}} \frac{\partial}{\partial \rho} (J_m(\gamma_{mn} \rho')) \cos(m \phi) e^{-\alpha_{mn} z'}
\]

(2.9)

\[
c_{mn}^i(\rho', \phi', z') = \frac{j \omega \mu_0}{2} \frac{m}{2 \pi \epsilon_0 \alpha_{mn} \gamma_{mn} \alpha_{mn} \rho} J_m(\gamma_{mn} \rho') \cos(m \phi) e^{\alpha_{mn} z'}
\]

(2.10)

From equations (2.8)-(2.10), the expression for the required Green's function can be written as,
\[ G_{\rho\rho}(\rho, \phi, z, \rho', \phi', z') = -\frac{j\omega}{2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \frac{m^2}{2\pi\varepsilon_{\infty}\gamma_m^2 \alpha_m} \frac{J_m(\gamma_m' \rho')} {J_m(\gamma_m \rho)} e^{-\alpha_m z'} \]

\[ \cos(m\phi)\cos(m\phi') e^{i\alpha_m z'} + \frac{j}{2\omega} \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \frac{\alpha_m}{2\pi\varepsilon_{\infty}\gamma_m^2} e^{\alpha_m z'} \left( \frac{\partial}{\partial \rho} (J_m(\gamma_m \rho)) \right) e^{-\alpha_m z'} \cos(m\phi) \cos(m\phi'); \quad z > z' \] (2.11)

\[ \frac{\partial}{\partial \rho} (J_m(\gamma_m \rho)) e^{-\alpha_m z'} \cos(m\phi) \cos(m\phi'); \quad z < z' \] (2.12)

\[ \gamma_m', \alpha_m' \] are the cutoff wave numbers and phase constants respectively for TE_{mn} modes and represented by,

\[ \gamma_m' = \frac{x_{mn}'}{a} \]

\[ \gamma_m'^2 = \alpha_m'^2 + k^2 \]

\[ \alpha_m = j\beta_m = \sqrt{\gamma_m'^2 - k^2} \quad m=1, 2, 3 \ldots \text{ and } n=1, 2, 3, \ldots \]

where, \( x_{mn}' \) is the \( n^{th} \) root of \( J_m(x) = 0 \). Similarly \( \gamma_m ', \alpha_m \) are the cutoff wave numbers and phase constants respectively for TM_{mn} modes and represented by,

\[ \gamma_m = \frac{x_{mn}}{a} \]

\[ \gamma_m^2 = \alpha_m^2 + k^2 \]

\[ \alpha_m = j\beta_m = \sqrt{\gamma_m^2 - k^2} \quad m=0, 1, 2, 3, \ldots \text{ and } n=1, 2, 3, \ldots \]

where, \( x_{mn} \) is the \( n^{th} \) root of \( J_m(x) = 0 \).

The unknown surface current \( J(\rho) \) of equation (2.7) on the post can be expressed in terms of basis functions as,

\[ J(\rho) = \sum_{p=1}^{N} f_p(\rho) \] (2.13)

where \( f_p(\rho) \) is entire domain basis function given by,

\[ f_p = \frac{\sin(pk(\rho - a + h))}{\sin(kh)} \quad p = 1, 2 \ldots N \] (2.14)
Taking the inner product of the equation (2.6) with testing function \( f_q \) (same as the basis function-Galerkin's technique)

\[
<f_q, E^s> = <f_q, -E^m>
\]

(2.15)

where, \( f_q = \frac{\sin(qk(\rho-a+h))}{\sin(kh)} \); \( q = 1, 2, ...., N \)

\[
\int_E f_q(\rho)d\rho = -\int_E m^q f_q(\rho)d\rho
\]

(2.16)

Using equations (2.7), (2.13) and (2.15) we get,

\[
\int \int \sum_{\rho, \rho'} I_{p} f_p(\rho')G_{\rho\rho'}(\rho, \phi, z, \rho', \phi', z')f_q(\rho)d\rho d\rho' \equiv -\int E_{mc} f_q(\rho)d\rho
\]

(2.17)

\[
\sum_{\rho=1}^{N} I_{p} \int \int f_p(\rho')G_{\rho\rho'}(\rho, \phi, z, \rho', \phi', z')f_q(\rho)d\rho d\rho' \equiv -\int E_{mc} f_q(\rho)d\rho
\]

(2.18)

The equation (2.18) can be expressed in matrix form as,

\[
[Z][V] = [V]
\]

(2.19)

where the elements of the matrix \([Z]\) and matrix \([V]\) are,

\[
Z_{pq} = \int \int f_p(\rho')G(\rho, \phi, z, \rho', \phi', z')f_q(\rho)d\rho d\rho'
\]

(2.20)

\[
V_q = -\int_{\rho} E_{mc} f_q(\rho)d\rho
\]

(2.21)

where, \( E_{mc} = \frac{1}{m\rho^4(x'_{11})\sqrt{\pi(x'_{11})^2-1}} \cos(\phi) J_1(\frac{\bar{x}'_{11}}{l}) e^{-j\beta z} \) and \( x'_{11} = 1.841 \).

The integration limits for both \( \rho \) and \( \rho' \) are from \((a-h)\) to \((a)\). Matrix \([I]\) contains unknown coefficients of the current distribution on the post. Solution of matrix equation (2.19) will give the unknown coefficients \((I_p)\) and putting these values of coefficients in equation (2.13), the current \( J(\rho') \) on the post surface can be computed. Once the current is known, the scattered field corresponding to different modes and the other electrical parameters of interest can be found out.
2.2.1.2 Results and Discussions for Post Discontinuity

Using the formulation as discussed in Section 2.2.1.1, a MATLAB based computer program has been developed. The scattering parameters have been computed for a circular waveguide of diameter 32.54 mm with a circular cylindrical post of diameter 1.6 mm. Sixteen point Gaussian integration has been used to solve the integral expressions given in equations (2.20) and (2.21). Sixteen TE and TM modes have been taken into consideration to ensure converged solution. Return loss for the incident TE\textsubscript{11} mode in the circular waveguide is shown in Figure 2.3. The return loss has been plotted for different heights of the post. As shown in these plots, the return loss becomes poor when the post height is increased. At 10 GHz, it becomes -16 dB for a post height of 11 mm as compared to -54 dB when there is no post (i.e., post height 0.0 mm). Ripple like behavior has been noticed in the return loss plot over the 7 to 14 GHz frequency band. The plot has dips at 9 GHz and 12.4 GHz indicating improved return loss performance. These dips occur independent of the height of the post for a fixed size of waveguide. As shown in Figure 2.4, the dips show frequency sensitivity when the size of the waveguide is changed. For larger size waveguide, the response and dips move towards lower frequency and for lower waveguide size they move towards higher frequencies. The frequencies where dips occur can be selected as higher frequency bands for a multi-frequency mode transducer so that they are least affected due to the presence of post at lower frequency. The power carried forward in the dominant TE\textsubscript{11} mode is shown in Figure 2.5. This plot also shows that maximum power is confined in the dominant TE\textsubscript{11} mode where dips occur in the return loss plot. Above 7 GHz, the waveguide becomes oversized and supports higher-order modes. The power coupled to higher-order TM\textsubscript{mn} and TE\textsubscript{mn} modes have been computed and it has been found that the power couples significantly to higher-order asymmetric TM\textsubscript{01}, TE\textsubscript{21}, TE\textsubscript{31} modes and it shows negligible coupling in the TM\textsubscript{1n} and TE\textsubscript{1n} modes. The power coupled to TM\textsubscript{01} mode for different heights is shown in Figure 2.6. The power in TE\textsubscript{21} and TE\textsubscript{31} modes is shown in Figures 2.7 and 2.8 respectively. Scattering performance of the post over wide frequency (6-21 GHz) range for post height of 11 mm is shown in Figure 2.9. The results for higher order modes show that a particular higher-order mode gets coupled only at a frequency where the waveguide size becomes above cut off for that mode. The power in higher-order modes increases with post height. As shown in Figures 2.3-2.8, the results computed from the present method are in close agreement with HFSS results. In order to verify the simulated results, the appropriate hardware has been developed in which two rectangular-
to-circular waveguide transitions are placed back to back with a radial post in the circular waveguide section as shown in Figure 2.10. Using a PNA series network analyzer (E 8363 B), the measurements were carried out for the post height of 11 mm in the frequency range of 7-9 GHz and the results are plotted in Figure 2.11. As shown in Figure 2.11, the measured and the simulated results are in close agreement. Slight deviation above 8.4 GHz may be attributed due to the fact that WR-137 works up to 8.2 GHz, while the measured values are presented up to 9 GHz. The rippled behavior of the measured return loss plot for post discontinuity is due to the reflections in the realized rectangular to circular waveguide transitions. In the absence of the post, the measured return loss of the transitions (put back-to-back) alone is of the order of 21 dB having ripples of ± 3.0 dB as shown in Figure 2.11.

![Figure 2.3](image1.png)

Figure 2.3 Return loss performance for a post discontinuity in a circular waveguide.

![Figure 2.4](image2.png)

Figure 2.4 Return loss performance for different sizes of the waveguide with post discontinuity.

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Figure 2.5  Forward power for a post discontinuity in a circular waveguide.

Figure 2.6  Modal power in TM_{01} mode for post discontinuity.

Figure 2.7  Modal power in TE_{21} mode for post discontinuity.
Figure 2.8 Modal power in TE_{31} mode for post discontinuity.

Figure 2.9 Scattering performance of post of height 11mm over wider frequency range.

Figure 2.10 Hardware of circular waveguide with post discontinuity.
Figure 2.11 Simulated and measured values of return loss for post height of 11 mm.

### 2.2.2 Coaxial Probe Discontinuity in Circular Waveguide

The analysis presented in Section 2.2.1 describes electromagnetic scattering by a post. However, in multi-frequency OMT, the coaxial probe also acts like an asymmetric discontinuity at lower frequency and has to be analyzed considering the magnetic current in the coaxial aperture region and the current induced on the probe. Using these current sources, electric and magnetic field integral equations have to be formulated applying proper boundary conditions on the surface of the probe and over the coaxial aperture respectively. In addition, expressions for electric and magnetic type dyadic Green’s functions have to be found out for both the electric and magnetic current sources at the locations of probe and the coaxial aperture. The probe at lower frequency will not only reflect and couple the incident signal but also excite higher-order modes at higher frequencies.

In this section, the formulation and analysis for a probe discontinuity in a circular waveguide is presented, where two separate equations have been formed by applying boundary conditions on the probe surface and on the surface of the coaxial aperture. In order to formulate the expressions for the scattered field, dyadic Green’s functions have been derived by considering two different current sources. The integral equations are
solved using method of moments technique. Computed data on return loss and power coupling in the dominant and higher-order modes from the present method have been compared with the data computed using Ansoft HFSS. Measured data on return loss is also given.

### 2.2.2.1 Formulation and Analysis of Coaxial Probe Discontinuity

The geometry of a circular waveguide loaded with a coaxial probe is shown in Figure 2.12, where TE_{11} mode has been assumed to be incident at Port-1. The incident TE_{11} mode has the electric field orientation in the direction of the probe. The incident mode induces electric current on the probe and magnetic current over the coaxial aperture. Depending on the size of the waveguide, these current sources may couple power in the different modes corresponding to the incident mode.

![Figure 2.12 Circular waveguide loaded with coaxial probe discontinuity.](image)

Figure 2.12 Circular waveguide loaded with coaxial probe discontinuity.

Let $\overline{E}_{T_{E_{11}}}^{T_{E_{11}}}$ and $\overline{H}_{T_{E_{11}}}^{T_{E_{11}}}$ be the incident electric and magnetic fields corresponding to the dominant TE_{11} mode in the circular waveguide. Let the scattered electric and magnetic fields in the circular waveguide due to electric current source $\overline{J}(\phi)$ on the probe are $\overline{E}_{S,CW}^{T_{E_{11}}}$ and $\overline{H}_{S,CW}^{T_{E_{11}}}$ and due to magnetic current source $\overline{M}(\phi)$ over the coaxial aperture
are $E_{s,cw}^M, H_{s,cw}^M$. Let the scattered electric and magnetic fields in the coaxial waveguide due to magnetic current source over the coaxial aperture are $E_{s,cw}^M, H_{s,cw}^M$. The cylindrical co-ordinate systems are represented as $r, \phi, z$ and $r_x, \phi_x, z_x$ for circular and coaxial waveguides, respectively as shown in Figure 2.12.

In general, the electric field due to an electric current source $J(R')$ [33] is given as,

$$E(R) = j \omega \mu_0 \iint_\Sigma J(R') \cdot \overrightarrow{G}_{el}(R, R') dV'$$  \hspace{1cm} (2.22)

The electric field due to a magnetic current source $\overline{M}(R')$ is given as,

$$\overline{E}(R) = -\iint_\Sigma \nabla \times \overrightarrow{G}_{el}(R, R') \cdot \overrightarrow{M}(R') dS'$$  \hspace{1cm} (2.23)

where, magnetic current $\overline{M}(R')$ is related to aperture field $\overline{E}(R')$ as $\overline{M}(R') = -\overline{n} \times \overline{E}(R')$ and $\overrightarrow{G}_{el}(R, R')$ is an electric dyadic Green's function of first type and $\overrightarrow{G}_{e2}(R, R')$ is electric dyadic Green's function of second type.

The magnetic field due to the electric current source $\overrightarrow{J}(R')$ is given as,

$$\overrightarrow{H}(R) = \iint_\Sigma \nabla \times \overrightarrow{G}_{el}(R, R') \cdot \overrightarrow{J}(R') dV'$$  \hspace{1cm} (2.24)

$$\nabla \times \overrightarrow{G}_{el} = \overrightarrow{G}_{e2}$$

where, $\overrightarrow{G}_{el}(R, R')$ is magnetic dyadic Green's function of second type.

The magnetic field due to a magnetic current source $\overline{M}(R')$ is given as,

$$\overrightarrow{H}(R) = -j \omega \varepsilon_0 \iint_\Sigma \overrightarrow{G}_{e2}(R, R') \cdot \overline{M}(R') dS'$$  \hspace{1cm} (2.25)

$$\nabla \times \overrightarrow{G}_{e2} = \overrightarrow{G}_{m1}$$

where, $\overrightarrow{G}_{m1}(R, R')$ is magnetic dyadic Green's function of first type.

The general expressions for $\overrightarrow{G}_{el}, \overrightarrow{G}_{e2}$ and $\overrightarrow{G}_{m1}, \overrightarrow{G}_{m2}$ for cylindrical and coaxial waveguide are given in [33] and also described in Appendix A.3 for the sake of completeness. From these expressions the required components of the Green's functions have been derived for the present coaxial probe problem and are presented in Appendix A.4.

On the probe surface, the boundary condition is applied on the tangential component of the total electric field. Assuming probe to be a perfect conductor, the tangential component (in
the radial direction) of the total electric field $E'$ at the probe surface should be zero. This boundary condition is expressed as,

$$E_{S,CW}^J + E_{S,CW}^M + E_{mc,CW}^{TE11} = E' = 0$$ (2.26)

Over the coaxial aperture, the boundary condition is applied on the azimuthal component of the total magnetic field. The azimuthal component of the total magnetic field should be continuous across the coaxial aperture. This boundary condition is expressed as,

$$H_{S,CW}^J + H_{S,CW}^M + H_{mc}^{TE11} = H_{S,CW}^M$$ (2.27)

The unknown surface current on the probe can be expressed in terms of basis functions as

$$J(r) = \sum_{p=1}^{N} I_p f_p(r); \text{ where, } f_p(r) \text{ is entire domain basis function.}$$ (2.28)

Taking entire domain basis functions to expand the unknown surface current (assumed as line current at the axis of the of the thin cylindrical probe) as,

$$f_p = \frac{\sin(pk(r-a+h))}{\sin(kh)}; p = 1, 2...N$$ (2.29)

A TEM mode distribution has been assumed for the coaxial aperture field (or magnetic current).

The expressions for field components are derived as,

$$E_{S,CW}^J = \int \int G_{\rho}(r,r')J_r(r')dr'$$ (2.30)

$$H_{S,CW}^J = -H_x^' \sin \phi_x + H_x^' \cos \phi_x$$

$$- \int \int G_{\rho}(r,r')J(r')\sin \phi_x dr' + \int \int G_{\rho}(r,r')J(r')\cos \phi_x dr'$$ (2.31)

$$E_{S,CW}^M = -\int \int G_{\rho}(r,r')E_{r\rho} \cos \phi_x r_x dr_x d\phi_x + \int \int G_{\rho}(r,r')E_{r\rho} \sin \phi_x r_x dr_x d\phi_x$$ (2.32)

$$H_{S,CW}^M = \int \int G_{\rho}(r,r')E_{r\rho} r_x dr_x d\phi_x$$ (2.33)

$$H_{S,CW}^M = \int \int G_{\rho}(r,r')E_{r\rho} \sin \phi_x r_x dr_x d\phi_x - \int \int G_{\rho}(r,r')E_{r\rho} \cos \phi_x r_x dr_x d\phi_x$$

$$+ \int \int G_{\rho}(r,r')E_{r\rho} \sin \phi_x r_x dr_x d\phi_x - \int \int G_{\rho}(r,r')E_{r\rho} \cos \phi_x r_x dr_x d\phi_x$$ (2.34)

$$E_{mc} = \frac{j}{\rho \alpha k_c^2 \mu} CTE_{11} \cos \phi I_1(\frac{x_1}{\alpha} \rho) \cdot e^{(-j\beta_1 y \tan \phi)}$$ (2.35)
\[ H_{CW,\phi}^{mc} = -H_z^{mc} \sin \phi_z + H_\phi^{mc} \cos \phi_z \]
\[ = -\int [G_{\nu}(r, r')J(r') \sin \phi_z \, dr' + \int [G_{\nu}(r, r')J(r') \cos \phi_z \, dr'] \]
\[ H_z^{mc} = CTE_{11} \frac{-1}{j \omega \mu} \beta_{11}^2 J_1(x_1') \cos \phi \cdot e^{(-j \beta \rho \tan \phi)} + k J_1(x_1') \cos \phi \cdot e^{(-j \beta \rho \tan \phi)} \]
\[ H_\phi^{mc} = CTE_{11} \frac{-\beta_{11}}{\omega \mu} J_1(x_1') \sin \phi \cdot e^{(-j \beta \rho \tan \phi)} \]

where, \( x_1' = 1.841 \)
\[ CTE_{11} = \sqrt{\frac{2}{\pi (x_1'^2 - 1)}} \cdot J_1(x_1') \]

By taking inner product of the equation (2.26) with the testing functions, which are same as basis functions, the equation (2.26) can written as,
\[ \langle f_q, E_{S,CW}^j \rangle + \langle f_q, E_{S,CW}^M \rangle = -\langle f_q, E_{TE}^{11} \rangle \]

Similarly, equation (2.27) can written as
\[ \langle 1, H_{S,CW}^j \rangle + \langle 1, H_{S,CW}^M \rangle + \langle 1, H_{TE}^{11} \rangle = \langle 1, H_{S,CW}^M \rangle \]

where,
\[ < f_q, E_{S,CW}^j > = \int [G_{\nu}(r, r') \sum_{p=1}^{N} I_p f_p(r) \, dr'] \]
\[ = \sum_{p=1}^{N} I_p \cdot Z_{11} \]

where,
\[ Z_{11} = \int [f_p(r')G_{\nu}(r, r') \, f_q(r) \, dr'] \]
\[ < f_q, E_{S,CW}^M > = -\int [G_{\nu}(r, r')E_{rz} \cos \phi_z f_q(r)r_z \, dr' \, d\phi_z \, dr] \]
\[ + \int [G_{\nu}(r, r')E_{rz} \sin \phi_z f_q(r)r_z \, dr' \, d\phi_z \, dr] \]

In the coaxial waveguide
\[ E_{rz} = \frac{E_\alpha}{r_x} \]
\[
\langle f_q, E_{S,CW}^M \rangle = - \iint G_{s \phi}(r,r') E_0 \cos \phi_x f_q(r) d\phi_x' d\phi_y' d\phi_z' + \iint G_{s \theta}(r,r') E_0 \sin \phi_x f_q(r) d\phi_x' d\phi_y' d\phi_z' dr
\]  
(2.43)

\[
\langle f_q, E_{S,CW}^M \rangle = E_0 \cdot Z_{12}^q
\]  
(2.44)

where, \( Z_{12}^q = - \iint G_{s \phi}(r,r') \cos \phi_x f_q(r) d\phi_x' d\phi_y' d\phi_z' dr + \iint G_{s \theta}(r,r') \sin \phi_x f_q(r) d\phi_x' d\phi_y' d\phi_z' dr
\]

\[
\langle 1, H_{S,CW,\phi}^J \rangle = - \sum_{p=1}^{N} \int I_p f_p(r') \sin \phi_x r_x^' d\phi_x^' d\phi_y^' d\phi_z^' + \sum_{p=1}^{N} \int G_{s \phi}(r',r') \cos \phi_x r_x^' d\phi_x^' d\phi_y^' d\phi_z^' dr
\]  
(2.45)

\[
Z_{21}^q = - \iint G_{s \phi}(r,r') f_p(r') \sin \phi_x r_x^' d\phi_x^' d\phi_y^' d\phi_z^' + \iint G_{s \theta}(r,r') f_p(r') \cos \phi_x r_x^' d\phi_x^' d\phi_y^' d\phi_z^' dr
\]  
(2.46)

\[
\langle 1, H_{S,CW}^M \rangle = \iint G_{s \phi}(r,r') E_{x_\phi} \sin \phi_x r_x^' d\phi_x^' d\phi_y^' d\phi_z^' d\phi_x d\phi_y d\phi_z
\]

\[
- \iint G_{s \phi}(r,r') E_{x_\psi} \cos \phi_x r_x^' d\phi_x^' d\phi_y^' d\phi_z^' d\phi_x d\phi_y d\phi_z + \iint G_{s \theta}(r,r') E_{x_\theta} \sin \phi_x r_x^' d\phi_x^' d\phi_y^' d\phi_z^' d\phi_x d\phi_y d\phi_z
\]  
(2.47)

\[
\langle 1, H_{S,CW}^M \rangle = \iint G_{s \phi}(r,r') E_0 \sin \phi_x r_x^' d\phi_x^' d\phi_y^' d\phi_z^' d\phi_x d\phi_y d\phi_z
\]

\[
- \iint G_{s \phi}(r,r') E_0 \cos \phi_x r_x^' d\phi_x^' d\phi_y^' d\phi_z^' d\phi_x d\phi_y d\phi_z + \iint G_{s \theta}(r,r') E_0 \sin \phi_x r_x^' d\phi_x^' d\phi_y^' d\phi_z^' d\phi_x d\phi_y d\phi_z
\]  
(2.48)

The azimuth component of magnetic filed in coaxial region is given as,

\[
\langle 1, H_{S,CW}^M \rangle = \frac{2\pi^2 E_0 (r_2 - r_1)(r_2^2 - r_1^2)}{Z_o}
\]  
(2.49)

\[
\langle 1, H_{S,CW}^M \rangle = - \langle 1, H_{S,CW}^M \rangle = \iint G_{s \phi}(r,r') E_0 \sin \phi_x r_x^' d\phi_x^' d\phi_y^' d\phi_z^' d\phi_x d\phi_y d\phi_z
\]

\[
- \iint G_{s \phi}(r,r') E_0 \cos \phi_x r_x^' d\phi_x^' d\phi_y^' d\phi_z^' d\phi_x d\phi_y d\phi_z + \iint G_{s \theta}(r,r') E_0 \sin \phi_x r_x^' d\phi_x^' d\phi_y^' d\phi_z^' d\phi_x d\phi_y d\phi_z
\]

\[
- \iint G_{s \phi}(r,r') E_0 \cos \phi_x r_x^' d\phi_x^' d\phi_y^' d\phi_z^' d\phi_x d\phi_y d\phi_z - 2\pi^2 E_0 (r_2 - r_1)(r_2^2 - r_1^2) / Z_o
\]  
(2.50)
In the above expressions, the expressions for $G_{rr}, G_{r\phi}, G_{\phi r}, G_{\phi \phi}, G_{21}$ and $G_{22}$ have been derived from the general expressions of dyadic Green's functions $\overline{G}_1, \overline{G}_2$ and $\overline{G}_{m1}$, $\overline{G}_{m2}$. These derived expressions are presented in Appendix A.4.

For the incident terms of the matrix

$$V_1 = \int f_0 E_{mc}^{inc}(r) dr$$

$$E_{mc}^{inc} = \frac{j}{\rho \alpha k_0^2 \mu} CTE_{11} \cos \phi I_1 \left( \frac{x_1'}{a} \right) \cdot e^{(-j\beta x\tan\phi)}$$

$$V_2 = \int H_{mc}^{inc} J_{1} \cdot r' dr' d\phi' r dr d\phi$$

Thus equation (2.39) and (2.40) can be written in the matrix form as,

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I \\ E_0 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Equation (2.55) contains unknown coefficients ($I_0$) of the current distribution on the probe and the unknown peak field ($E_0$) over the coaxial aperture. The solution of matrix equation (2.55) will give these unknown coefficients and using these coefficients, the current distribution on the probe, the scattered fields corresponding to different modes, power coupled to the coaxial line and the other electrical parameters of interest can be found out.
2.2.2.2 Results and Discussion for Coaxial Probe Discontinuity

Using the formulation presented in the previous section, a MATLAB based computer program has been developed. The scattering parameters have been computed for a circular waveguide (diameter 32.54 mm) having a co-axial line fed probe of diameter 1.6 mm. The diameter of the inner and outer conductors of the coaxial line were taken as 1.6 mm and 6.5 mm, respectively. Return loss for the incident TE\textsubscript{11} mode in the circular waveguide and the power coupled to higher order modes and to the coaxial line port have been computed and compared with the data from HFSS. The return loss has been computed for the different depths of the probe. The computed data for return loss is shown in Figure 2.13. Improved return loss (-10 dB) has been observed at lower frequency for probe in comparison to the return loss (-3.6 dB) for post (see Figure 2.9) of similar height (11 mm). This may be due to the fact that probe couples part (-3.8 dB) of the incident signal into the coaxial line unlike that of post. Return loss also varies with probe depth (or height) in circular waveguide. For example at 7 GHz, it becomes -10 dB for a probe height of 11 mm as compared to -50 dB when there is no probe (probe height 0 mm). The forward power carried by the dominant TE\textsubscript{11} mode is shown in Figure 2.14. It is observed that the power coupling to the dominant TE\textsubscript{11} mode is increased when probe height is reduced. The probe also acts as asymmetrical discontinuity in circular waveguide exciting higher-order TM\textsubscript{01}, TE\textsubscript{21}, TE\textsubscript{31} modes above 7 GHz. The Figure 2.15 shows the power coupled into higher order TM\textsubscript{01} mode. The power coupled into higher order TE\textsubscript{21} mode is shown in Figure 2.16. The power in higher order modes increases with probe height. The part of the incident power gets coupled to the coaxial probe which is shown in Figure 2.17. The hardware shown in Figure 2.10 has also been used to verify the probe analysis results. In this hardware (Figure 2.18), a coaxial line fed probe of height 11 mm was used in place of the post in a circular waveguide section. Measured return loss and forward power for the incident TE\textsubscript{11} mode in the circular waveguide are shown in Figures 2.13 and 2.14 respectively. The power coupled to the coaxial port with respect to incident TE\textsubscript{11} mode in the circular waveguide is shown in Figure 2.17. Scattering performance of probe of height 11 mm over wider frequency range for few higher order modes is shown in Figure 2.19. The results computed from the present method are in close agreement with the data computed from HFSS and measurements. In the absence of the probe, the measured return loss of the transitions (put back-to-back) itself is of the order of 21 dB having ripples of ±3.0 dB up to 9 GHz. This causes ripples in the measured return loss plot for the probe discontinuity as shown in Figure 2.13.
Figure 2.13  Return Loss of probe loaded circular waveguide.

Figure 2.14  Forward power in the dominant TE_{11} mode for probe discontinuity.

Figure 2.15  Power in TM_{01} mode for probe discontinuity.
Figure 2.16 Power in TE_{21} mode for probe discontinuity.

Figure 2.17 Power coupling to the coaxial line.

Figure 2.18 Hardware of circular waveguide section with co-axial line fed probe discontinuity.
Figure 2.19  Scattering performance of probe of height 11 mm over wider frequency range.

The computed results for probe discontinuity show that the probe deteriorates the return loss performance, couples the part of incident signal and excites higher-order modes at higher frequencies. This study gives a good insight about the nature of coupling and the higher-order modes excited due to the co-axial probe discontinuity. This analysis may be very useful in the design of multi-frequency ortho-mode transducers, where the probe at lower frequency ports acts as asymmetrical discontinuity, thereby affecting the performance of higher frequencies.

2.3 Symmetric Discontinuities in Circular Waveguide

The symmetric waveguide discontinuities in waveguides also act as mode transducers and generate another type of higher-order modes as compared to asymmetric discontinuities. In the multi-frequency mode transducer (Figure 2.1), two consecutive cylindrical waveguide sections are joined with tapered waveguide sections. The tapered section between two smooth wall cylindrical waveguides acts as a symmetrical discontinuity and it may excite higher-order modes. The discontinuities formed with tapered sections can be analyzed in terms of step junctions. A step discontinuity is created at the junction between two uniform waveguides which are of different cross-sections. The step junction discontinuity as shown in Figure 2.20 acts as a mode transducer [10], [11], which excites number of higher order modes depending on the size of the output waveguide with respect to the input waveguide. As described in [35], [42] smooth wall conical horns, corrugated horns and any specific waveguide geometry can always be represented as a series of step junctions. Therefore, if a
single step junction can be modeled and analyzed properly, other mode transducers represented as series of step junctions can also be modeled and analyzed by cascading scattering matrices of the individual step junctions [42].

Figure 2.20  Step junction between two smooth-walled waveguides of different radii.

Modal field matching is a very powerful technique to analyze step junctions. Although, this technique for step junctions in circular waveguide has been presented in detail in [35], [42], it is described briefly in Appendix A.5 for the sake of convenience. A tapered waveguide junction between two waveguides can be analyzed using this technique after representing the tapered waveguide section as series of step junctions as shown in Figure 2.21. The scattering matrix of the individual step junctions can be cascaded to obtain overall scattering matrix of the tapered geometry.

Figure 2.21  Smooth wall conical waveguide approximated by incremental step junctions to apply mode matching using cylindrical modes.
The waveguide geometries such as smooth wall conical horn or corrugated horn can also be represented in the form of cascaded step junction discontinuities. A schematic of a circular to conical waveguide mode transducer in the form of a corrugated horn is shown in Figure 2.22(a). Each corrugation can be represented in the form of a pair of step junctions as shown in Figure 2.22(b), whose scattering matrix can be computed using expression for mode matching technique. The complete horn can be analyzed after computing scattering matrix for each pair of step junction along the horn and subsequently scattering matrices for all the junctions can be cascaded to obtain an overall scattering matrix of the horn.

Using the formulations [35], [42] for mode matching technique as described in Appendix A.5, a computer program was developed using MATLAB. Using this program, the geometry of Figure 2.21 has been analyzed to deduce higher order modes. A unit amplitude TE_{11} wave is assumed to be incident at the plane of discontinuity (z = 0) as shown in Figure 2.20. At the discontinuity higher-order modes are generated part of which is reflected back and part transmitted. Figure 2.23 shows the computed modal amplitudes for $2a_1 = 11$ mm and $2a_2 = 32.54$ mm over the frequency range from 16 GHz to 30 GHz for flare angles 5, 10, 45, and 90 degrees for the tapered section. This figure shows that the maximum power is confined to the dominant TE_{11} mode when flare angle is less than 10 degrees. Power coupling from incident TE_{11} mode to higher order modes increases with flare angle and reaches to maximum when flare angle is 90 degrees. At flare angle of 90 degrees, the geometry of Figure 2.21 changes into the geometry of Figure 2.20, i.e., it becomes a step junction. The power coupling to higher-order modes changes with frequency for fixed flare
angle. These observations will be very useful while designing a multi-frequency mode transducer where purity in the dominant TE$_{11}$ is required. Also this analysis will be useful for designing a multi-mode coupler horn, where higher order modes will be needed for achieving shaped radiation patterns.

Figure 2.23 The modal amplitudes for flare angles. (a) 5 degree. (b) 10 degree. (c) 45 degree. (d) 90 degree.

A horn geometry (Figure 2.22) having 10 corrugations with input diameter ($D_i$) of 45.7 mm and output diameter ($D_o$) of 59 mm (semi flare angle 15 degrees) has also been analyzed using the developed code based on mode matching technique. The pitch (p) of corrugations is 2.5 mm and width ($w$) is 2 mm. The depth of corrugations are optimized to achieve performance from 6 GHz to 12 GHz. The optimized depth of the corrugation at the input (1$^{st}$ corrugation) of the horn is 14.2 mm and at the output (10$^{th}$ corrugation) is 12.6 mm. The depth of all other corrugations are in between the depths of these two
The analysis for circular waveguide geometries shows that the power from the incident TE_{11} mode at a junction gets coupled to the higher order TE_{in} and TM_{in} modes, which have azimuthal dependence of unity. These modes can be exploited to yield shaped symmetric radiation characteristics with low cross-polarization in feed horns.

The return loss plotted in Figure 2.24 shows a very close agreement between the results computed from the developed code and the results from TICRA’s CHAMP/FEED code. The computed modal amplitudes at the aperture of the horn are found within close agreement (±0.03) with the results computed from CHAMP/FEED which is a commercial tool based on mode matching to analyze circular waveguide geometries. The far-field radiation patterns have been computed from these modal amplitudes using far-field expressions given in [63]. The electrical parameters computed for the horn are presented in Table 2.1.

Table 2.1 Electrical parameters of horn.

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>TE_{11} amplitude</th>
<th>TM_{11} amplitude</th>
<th>Return loss (dB)</th>
<th>Cross Pol. (dB)</th>
<th>Beam width (dB)</th>
<th>Beam width (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>0.95</td>
<td>0.31</td>
<td>-25.3</td>
<td>-25.4</td>
<td>46.0</td>
<td>48.6</td>
</tr>
<tr>
<td>8</td>
<td>0.94</td>
<td>0.38</td>
<td>-31.5</td>
<td>-30.0</td>
<td>40.7</td>
<td>41.8</td>
</tr>
<tr>
<td>9</td>
<td>0.92</td>
<td>0.36</td>
<td>-32.0</td>
<td>-31.0</td>
<td>36.8</td>
<td>37.5</td>
</tr>
<tr>
<td>10</td>
<td>0.96</td>
<td>0.22</td>
<td>-38.0</td>
<td>-25.0</td>
<td>30.0</td>
<td>34.0</td>
</tr>
<tr>
<td>11</td>
<td>0.97</td>
<td>0.18</td>
<td>-43.5</td>
<td>-23.0</td>
<td>26.0</td>
<td>30.0</td>
</tr>
</tbody>
</table>
2.4 Mode Transducers to Excite Circularly Symmetric Modes

Mode transducers based on circularly symmetric modes are very useful for realizing microwave rotary joints which are used to deliver uniform transmission of power with rotation. Two such modes in circular waveguide are $TM_{01}$ and $TE_{01}$ modes. In the sequence of modes in a circular waveguide $TE_{11}$ is the first mode followed by $TM_{01}$, $TE_{21}$, $TM_{11}$, $TE_{01}$, $TE_{31}$ modes and all the other higher order modes. $TM_{11}$ and $TE_{01}$ are degenerate modes having same cut off wavelengths. If any particular mode is excited in a waveguide, the other lower order modes get automatically supported. Therefore, the excitation mechanism of a particular mode should be evolved in such a way that all the lower order modes are suppressed or rejected. It would be easier to excite a mode which has less number of lower order modes. Therefore, $TM_{01}$ mode, which has only one lower order $TE_{11}$ mode can be excited easily. $TM_{01}$ mode can be excited in a circular wave guide using a circular coaxial waveguide fed probe mechanism as shown in Figure 2.25. If the $TM_{01}$ in circular waveguide is to be excited from a rectangular waveguide, a transition from rectangular to coaxial waveguide can be used as shown in the Figure 2.25. The diameter of the circular waveguide should be preferably between the cut off diameter of this mode and the cut off diameter of the next higher order mode which is the $TE_{21}$ mode. Although, $TE_{11}$ mode is the lower order fundamental mode, it is unlikely to be excited with such excitation mechanism and pure $TM_{01}$ mode can be obtained.

![Diagram](image)

Figure 2.25 Scheme of $TM_{01}$ mode excitation in circular waveguide using axial probe.

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A geometry of Figure 2.25 was simulated at 13.4 GHz, using a WR-75 rectangular waveguide (19 mm X 9.5 mm), a circular waveguide of diameter 31.5 mm and a coaxial waveguide of outer conductor diameter 4 mm and inner conductor diameter 1.3 mm. The diameter of the probe in circular waveguide was also taken as 1.3 mm. At 13.4 GHz, this waveguide size was oversized and supported the dominant TE_{11} mode and the higher-order TM_{01} mode and TE_{01} modes. It was found from the simulated results that maximum power gets coupled into TM_{01} mode (>-0.1) and the power in the other lower and higher order modes was very low (<-70 dB) for 250 MHz bandwidth. The next higher order circularly symmetrical mode in circular waveguide is the TE_{01} mode. It would be difficult to control the purity of this mode since it has four lower order modes which are TE_{11}, TM_{01}, TE_{21}, TM_{11} respectively. Apart from this, the next higher order TE_{31} mode will also get supported if the waveguide diameter is selected sufficiently above the cut off diameter of the desired TE_{01} mode. The excitation mechanism should be such that the maximum power is coupled to TE_{01} mode only and all the other non desired modes get cancelled or suppressed. TE_{01} mode can be excited in a circular waveguide using slot coupling. Since, the waveguide size to support TE_{01} mode will automatically support lower order modes, a single slot may not give pure TE_{01} mode. Single slot coupling may result into sufficient power coupling in the non desired lower or higher order modes. In order to verify this and to know the power coupling characteristics of undesired modes, an oversized circular waveguide supporting TE_{01} mode was simulated using HFSS at 13.4 GHz. The circular waveguide was excited from rectangular waveguides using longitudinal slots on the periphery of the circular waveguide as shown in Figure 2.26. The diameter (31.5 mm) of the circular waveguide was taken 15% above the cut off diameter for TE_{01} mode. The length and width of slots were 11.5 mm and 2 mm respectively. WR-75 rectangular waveguide (19 mm X 9.5 mm) was used. The computed cut off diameter for TE_{01} mode comes 27.3 mm at 13.4 GHz.

Figure 2.26 Slot coupling of an oversized circular waveguide to propagate TE_{01} mode.
The simulation was carried out to compute the modal power in various modes in the circular waveguide for single slot, two opposite slots and four equally spaced slots, respectively. The rectangular waveguides outputs were combined using 2-way and 4-way waveguide power combiners for the cases of coupling with two and four slots. The simulated results for single slot coupling show that the power couples significantly in TE_{11}, TE_{21}, TE_{31} modes along with desired TE_{01} as shown in Table 2.2. Results for coupling with two slots show that the power couples in only TE_{21} and TE_{31} modes and other modes are cancelled. For the case of coupling using four slots (having uniform amplitude and phase) the power is coupled only in the required circularly symmetric TE_{01} and all the other modes were rejected as shown in Table 2.2.

**Table 2.2 Modal amplitudes in the oversized circular waveguide excited with slot coupled rectangular waveguides.**

<table>
<thead>
<tr>
<th>Mode Number</th>
<th>Mode Name</th>
<th>Modal amplitudes in circular waveguide (Single slot)</th>
<th>Modal amplitudes in circular waveguide (Two slots)</th>
<th>Modal amplitudes in circular waveguide (Four slots)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>TE_{11}</td>
<td>-13.34*</td>
<td>-63.10</td>
<td>-61.06</td>
</tr>
<tr>
<td>2</td>
<td>TM_{01}</td>
<td>-62.02</td>
<td>-81.20</td>
<td>-79.31</td>
</tr>
<tr>
<td>3</td>
<td>TE_{21}</td>
<td>-6.22*</td>
<td>-2.57*</td>
<td>-49.27</td>
</tr>
<tr>
<td>4</td>
<td>TE_{01}</td>
<td>-8.84*</td>
<td>-3.72*</td>
<td>-0.12*</td>
</tr>
<tr>
<td>5</td>
<td>TM_{11}</td>
<td>-47.96</td>
<td>-73.90</td>
<td>-72.04</td>
</tr>
<tr>
<td>6</td>
<td>TE_{31}</td>
<td>-2.632*</td>
<td>-54.80</td>
<td>-51.98</td>
</tr>
<tr>
<td>7</td>
<td>Return loss</td>
<td>-14.76</td>
<td>-16.70</td>
<td>-20.0</td>
</tr>
</tbody>
</table>

* modes in which significant coupling takes place

It is clear from the simulated results that it would be appropriate to use four slots [68] instead of just one, on the periphery of the circular waveguide for pure excitation of TE_{01} mode. Such excitation mechanism (using four slots) is shown in Figure 2.27. All the four slots should be excited with equal amplitude and phase to get maximum power coupling in the desired TE_{01} mode and negligible power in the non desired modes.
The design of single rotary joints would be easier due to the requirement of only one circularly symmetric mode in a circular waveguide. But, the design of dual channel rotary joints would be much more complex due to the requirement of simultaneous excitation of two circularly symmetric modes in a circular waveguide. These modes should be excited in such a way that there is high isolation or minimum cross-talk between channels. In view of this, the details of various excitation mechanisms of mode transducers have been investigated in Chapter 6 in order to achieve the purity of circularly symmetric modes and to realize microwave rotary joints.

2.5 Conclusion

In this chapter various types of discontinuities occurring in multi-frequency mode transducers and mode couplers have been analyzed. Asymmetrical discontinuities such as radial post and coaxial probe discontinuities in circular waveguide have been analyzed for higher-order modes using method of moments technique. Symmetrical step junction discontinuities have also been analyzed using mode matching technique. The characteristics of fundamental and higher-order waveguide modes as function of physical parameters of the discontinuities have been studied in circular waveguides which become overmoded or oversized at higher frequencies. It was found from these studies that step junction, post [78] and probe [97] discontinuities in circular waveguides have profound effect on return loss and power coupling to dominant and higher-order modes. Subsequent chapters deal with the design and development of circular waveguide based mode transducers having multiple
discontinuities. The analysis of discontinuities presented in this chapter will help in the design and realization of mode transducers in terms of controlling non-desired modes which depend on the size and nature of waveguide discontinuities. For example, in the design of multi-frequency mode transducers presented in Chapter 3, the angle of tapered section between waveguides and the probe heights can be selected based on the observations in this chapter so as to confine the power in the dominant $\text{TE}_{11}$ mode at the output of the mode transducer at higher frequencies where waveguide becomes overmoded. The analysis results of step junction in this chapter can help in the design of multi-mode transducers (Chapter 5) such as multimode feeds for controlling power in higher-order modes as function of parameters like taper angle and step size so that shaped and symmetric radiation patterns are achieved. Simulation of an end wall probe excited circular waveguide (Figure 2.25) showed that the pure circularly symmetric $\text{TM}_{01}$ mode can be achieved. The simulation of a slot-coupled oversized circular waveguide shows that the power coupling in the lower-order modes can be avoided and maximum power can be coupled to the desired circularly symmetric higher-order $\text{TE}_{01}$ mode by using a coupling mechanism with four slots (Figures 2.26 and 2.27) on the periphery of the circular waveguide.