Literature Review

2.1 Introduction

Heat transfer mechanism between fluid and a surface may predominantly be natural convection or forced convection depending upon the relative contribution of buoyancy over forced flow. On the other hand, mixed convection finds merit only when buoyancy and forced flow effects are of comparable magnitude. Heat transport due to natural convection is limited due to the paucity of heat transfer. In that sense, forced convection provides some immediate solution. Since, natural convection cannot be ruled out from a system, thus mixed convection is an important topic for a researcher. In this chapter, an attempt is made to briefly review the past work in the area of fin heat transfer.

Literature reveals that the work of Harper and Brown (1922) is the first significant attempt to study heat transfer on single extended surface which is named as a “cooling fin”. Article reports the heat transfer in air-cooled engines equipped with rectangular or wedge like fins. In addition, article introduced the concept of fin efficiency, which later turned out to be the expression “fin effectiveness”. In 1945, Gardner presented mathematical expressions for fin temperature distribution and fin efficiency (ratio of heat transfer from an actual fin to that of an isothermal fin) involving geometrical parameters and heat transfer coefficient. Isothermal fin may be considered as a fin of infinite thermal conductivity. Fin temperature distribution and fin efficiency are expressed in terms of hyperbolic, power-law and Bessel’s functions depending upon the surface profile. Gardner (1945) introduces an additional measure of fin performance parameter, $\varepsilon_f$, which is conventionally termed as fin effectiveness. Effectiveness of fin is defined as the ratio of actual heat transfer from the fin surface to that from the true base surface area in absence of fin under same thermal condition.
2.2.1 Heat transfer by natural convection

2.2.1.1 Vertical fin on a vertical base

Pioneering study on free convection in a vertical channel is made both analytically and experimentally by Elenbaas (1942) maintaining the wall at uniform temperature. Nusselt number is correlated with the governing parameters, namely, Rayleigh number and geometric parameters by a relation, which finds good comparison with the subsequent articles on vertical channels. In addition, article provides a correlation for optimum channel width. Further, it is observed that the optimum Nusselt number is 1.25.

Welling and Wooldridge (1965) summarize the experimental findings of natural convection from a vertical rectangular fin array, which is almost isothermal. Results of fin heat transfer are found to lie between the heat transfer results of vertical plate and the square duct/infinite parallel plate. The effect of boundary layer interference involving lower heat transfer is observed for the case of closely spaced fins that caused reduced air motion over the fin surface. Maximum heat transport is found for an optimum fin height to spacing ratio.

Shroud, above the fin-tip, may have influence on fin heat transfer, especially in the case of vertical fin on a vertical base due to chimney effect. This is realized in the report of Karki and Patankar (1987), in which computation is performed on the developing laminar natural convective heat transfer from a vertical rectangular shrouded isothermal fin array on a vertical base. The base is elevated to a fixed temperature higher than the ambient, while the shroud wall is treated to be insulated. Air is chosen as the cooling medium. Depicted results indicate that bulk temperature increases and Nusselt number decreases with the axial location. Local Nusselt number shows a fully developed value after a certain axial distance far away from the entrance. It is observed further that, for each inlet induced velocity, there exists an optimum axial location, at which dimensionless pressure is maximum and dimensionless pressure is
lower on either side of it. For fixed shroud to fin tip clearance, overall Nusselt number is decreased with the increase in fin length.

Fisher and Torrance (1999) provide an evidence of chimney effect resulting from natural convection heat transfer over the vertical fin array on a vertical base. Results indicate the existence of optimal geometry. Chimney in combination with heat sink produces significant heat transfer enhancement.

An experimental study on natural convection from a vertical rectangular short fin array glued to a rectangular vertical base is found in Güvenç and Yüncü (2001). Report reveals that, for a fixed fin spacing, heat transfer increases monotonically with the fin height as well as with the excess temperature between base and ambient. On the other hand, for a given fin-height and excess temperature, heat transfer rate shows an optimum with the fin-spacing. Dependence of optimum fin-spacing with the fin-height as well as with the excess temperature is found to be weaker. An optimum value of fin-spacing of 7 mm is observed for a range of fin-height from 5 mm to 25 mm.

2.2.1.2 Vertical fin on a horizontal base

One of the earliest experimental studies on the performance of rectangular fin arrays glued to a rectangular base plate undergoing laminar natural convection is reported by Starner and McManus (1963). Article covers four geometric sets of fin arrays with different orientations, namely, horizontal, vertical, and $45^\circ$ with the horizontal, to find the effects of fin height and fin spacing on heat transfer. It is indicated that heat transfer rate from vertical fin arrays on a vertical base is less than that of parallel vertical plate reported by Elenbass (1942). It is noticed further that heat transfer is reduced for the case of base inclined at $45^\circ$ with vertical compared to base directed towards vertical. Heat transfer from horizontal base position shows remarkable influence of end effects. Surrounding cold fluid induces to the
channel through the ends. Shallow horizontal channels, made of fin array, are reported to perform better compared to $45^\circ$ vertical channels.

Harahap and McManus (1967) present experimental finding on the free convection heat transfer from a vertical fin array attached to a horizontal base. To observe the fluid motion, Schlieren-shadowgraph technique coupled with smoke injection is used. Maintaining same fin spacing and same fin height, length of fin is varied. Results enlighten the fact that shorter fin length induces single chimney flow drawing air from both the ends. This is effectively enhanced the heat transfer. On the other hand, larger fin length induces more chimney flow, which invites the flow reversal in the intermediate section along the length of fin. Heat transfer performance is reduced for larger fin length, since flow reversal in the intermediate section draw hot fluid from the chimney. In addition, article provides average heat transfer coefficient correlation with the governing parameters of the problem.

In 1970, Jones and Smith report experimental findings of the optimum fin spacing for vertical rectangular fins on a horizontal base undergoing natural convective heat transport. Local heat transfer coefficient is determined by measuring temperature gradient of fluid with the aid of interferometer. Average heat transfer coefficient of the fin array is then determined by integrating the measured local heat coefficients. Heat transfer coefficients measured thus is solely due to convection component precluding the radiation component, if any. The fin spacing is turned out to be the key geometric parameter, which affects the heat transfer performance. Authors also propose a correlation based on the value of properties evaluated at the average temperature. Further, it is informed that the proposed heat transfer correlation of Harahap and McManus (1967) is valid only for the low range of base temperature.

Induced laminar natural convective heat transfer from vertical parallel plate fins is experimentally performed by Filtzroy (1971). Maximum convective heat transfer is found to
decrease with the decrease in the excess temperature between the fin and the surrounding. Finally, optimum fin spacing is reported for maximum fin heat transfer.

An experimental investigation is made by Leung et al. (1985) on the heat transfer performance of rectangular fin attached to both vertical and horizontal rectangular base. Three different configurations are considered: vertical rectangular fins on a horizontal base, vertical rectangular fins on a vertical base and horizontal rectangular fins on a vertical base. Base temperature is elevated to in a range from 40°C to 80°C, while the fin height is varied in a range from 32 mm to 90 mm. Results highlight that the effect of fin spacing on heat transfer is more significant for vertical rectangular fins on a vertical base. Compared to fin spacing, fin height shows hardly any influence on heat transfer performance of vertical rectangular fins glued to both vertical and horizontal base. It is identified that vertical fins attached to a vertical base is indicated to be the preferred configuration among all positions from the perspective of heat transfer. Experimental investigation on free convection from the vertical rectangular fins protruding from both horizontal as well as vertical rectangular base is extended by Leung et al. (1986) keeping base temperature constant. The effect of fin length on optimum fin spacing is reported. It is revealed that heat dissipation rate per unit base area decreases with the increase in fin length for both vertical and horizontal base positions. Additionally, the optimal fin spacing on horizontal base is found to increase with the increased fin length. Thus, it may be concluded that fin length play an important role on heat transfer performance.

The experimental findings of natural convective heat transport from a single isolated vertical fin as well as vertical fin array attached to a horizontal base are summarized in Sobhan et al. (1990). Differential interferometer is used to measure thermal gradient. Nusselt number for both the cases is correlated separately. For the case of isolated fin, fin conductivity parameter nominally influences the Nusselt number. However, the same is not
true for the case of fin array, since fin conductivity is having definite influence on Nusselt number. As noted in experiments, there exists major difference in the heat flux distribution near the fin-tip between the two cases mentioned above. This variation is attributed to the significant influence of surrounding fins to the central fin, since any variation of surrounding fin temperature distribution in conjunction with its associated boundary layer influences the central fin as well.

A comprehensive experimental study of simultaneous natural convection and radiation from horizontal fin arrays is found in Rao and Venkateshan (1996). In general, emissivity of conventional fin material is low. But with suitable surface treatment, emissivity of conventional fin material may be improved, as shown by Rao and Venkateshan (1996). Article reports the effects of fin surface emissivity, fin spacing, fin height and base temperature. Heat transfer correlation for both fin and base are also provided in terms of governing dimensionless parameters. It is concluded further that radiation-convection interaction should be studied simultaneously, instead of studying the convection and radiation individually and adding them later. Additionally, it is noted that fin temperature variation along the fin-height is significant especially for the larger fin-height and the influence of fin spacing on fin temperature distribution is difficult to ignore.

An experimental study of buoyancy induced convective heat transfer from rectangular vertical rectangular fin array on a horizontal rectangular base is reported in Yüncü and Anbar (1998). Fins used are short and thick having a fin-length of 100 mm and fin-thickness of 3 mm. Fin height is chosen to vary from 6 mm to 26 mm, while the fin-spacing is varied from 6.2 mm to 83 mm. It is highlighted that convective heat transfer coefficient of base-fin system primarily depends on fin-height, fin-spacing and imposed base-to-ambient temperature difference. For a fixed value of base-to-ambient temperature difference, heat transfer rate from the fin array indicates a maximum, which depends on the fin-spacing as well as with the
A numerical study of natural convection heat transfer from an array of rectangular vertical fin on horizontal surface is carried out by Baskaya et al. (2000). A commercial CFD package PHOENICS is used to solve the three-dimensional elliptic governing equations to identify the effects of fin-spacing, fin-height, fin-length and excess temperature between the fin and the surrounding on the free convection heat transfer from horizontal thin fin arrays. All the variables, namely, fin-spacing, fin-height, fin-length and excess temperature participate significantly to influence the overall heat transfer. The overall heat transport is found to enhance with the increase in fin-height, while the same is decreased with the fin-length. In addition, it observes optimum value of fin spacing for which heat transfer is maximum. Moreover, Nusselt number is correlated by the correlation provided in Harahap and McManus (1967).

A three-dimensional numerical study on natural convection heat transfer from longitudinal horizontal rectangular fin array is performed by Mobedi and Yüncü (2003). Three-dimensional Navier-Stokes and continuity equations are converted to vorticity-vector potential equations, which are then solved numerically. Depending on the fin geometry, two different types of flow patterns are found to exist. Fin array with narrow inter-fin spacing, fluid enters the channel from the ends. On the hand, fin array with wide fin spacing, fluid can also enter the channel from the middle section of the inter-fin region and leaves the channel along the fin surface. Computation reveals further that the larger fin-height reduces the amount of the air flow through inter-fin region. The boundary layer, which develops along the length of channel, reduces the heat transfer coefficient with the increase in fin length. As depicted by the authors, the increase of fin spacing increases heat transfer coefficient, since
fresh fluid enters through the middle in addition to the possibility of reduced boundary layer interference.

A numerical study on laminar natural convection from an array of vertical rectangular fin glued to horizontal base is conducted by Dialameh et al. (2008). Fins used are relatively thick (3 mm to 7 mm), but shorter in length (≤50 mm). Three-dimensional elliptic governing equations of flow and heat transfer are solved using finite volume scheme (i.e., ‘SIMPLE algorithms outlined by Patankar (1980)) developed by the authors. Convective heat transfer coefficient enhances with fin-spacing, but reduces with fin-length. Average heat transfer coefficient is hardly influenced by the fin-thickness and the fin-height. Optimum fin-spacing for maximum heat transfer is turned out to be 7 mm for $H/L \leq 0.24$. Two correlations are proposed for two range of Rayleigh numbers, one being $Ra < 1500$ and other being $Ra > 1500$. Proposed correlations find reasonably good agreement with the experimental results of Harahap and McManus (1967), but deviate significantly with the correlation of Leung and Probert (1998).

2.2.1.3 Shrouded vertical fin on a horizontal base

Natural convection heat transfer from the shrouded vertical rectangular fin array on a horizontal base is experimentally examined by Naik et al. (1987). Optimum fin-spacing for maximum heat transfer is ensued when dimensionless clearance ($C/H$) is more than unity. Overall Nusselt number is correlated with the Grashof number and the dimensionless geometric parameters of the problem.

Steady state natural convection heat transfer from rectangular vertical shrouded fin array on a horizontal rectangular base is computationally studied by Yalcin et al. (2008). A finite volume based CFD package is used to solve the three-dimensional elliptic governing equations. In the computation, the various parameters of fin-height ($H$), fin-spacing ($S$), fin-length ($L$), clearance ($C$) and the temperature of base-fin assembly are varied. The optimum
condition is obtained for a configuration having a maximum non-dimensional clearance \((C/H)\) of around 2, the shortest fin-length, the highest ratio of fin-height to length and a fin-spacing of around 8 mm. Overall Nusselt number is correlated, following the relation developed by Naik et al. (1987), with an altered constant, which varies with the length of fin. Results of flow pattern highlight the efficient utilization of fin array, for which enhanced heat transfer coefficient is possible.

### 2.2.2 Heat transfer by forced convection

Article of Sparrow et al. (1978) provide the first numerical study of laminar fully-developed forced convection over the shrouded vertical non-isothermal fin array. In the computation, two different thermal boundary conditions are imposed at the base: (i) isothermal base and (ii) heat flux condition. Shroud is treated as the adiabatic wall. Maximum heat transfer occurs at the fin-tip in the presence of clearance, while the same occurs at the intermediate zone between fin-base and fin-tip without any clearance. Over the fin-height, there exists significant variation of local Nusselt number, which is presumably different from the often-used constant convective heat transfer coefficient in the analytical study of fin. With zero clearance, local Nusselt number assumes zero value at the base of the fin, increases along the fin-height, reaches a maximum, and thereafter it starts decreasing towards the fin-tip. For the case of very low conductance parameter with zero or lower clearance, variation of local Nusselt number is quite different along the fin-height. Variation along fin-height shows initial increase, and thereafter it shows a cross-over from positive to negative value. In this connection, authors argue the non-applicability of the conventional definition of the fin heat transfer coefficient (i.e., \(h_f = q_f / (T_f - T_b)\)), since \(T_b\) is too remote to serve as a proper participant in \(\Delta T\), which drives the flow.
Sparrow and Chyu (1982) report a numerical study of coupled conduction and forced convection heat transfer from a single plate fin projecting downwards from a horizontal base. It reveals that local convective heat transfer coefficient decreases monotonically with the increase in convection-conductance parameter, \( N_{ce} = (kL/k_f t) \text{Re}_{L}^2 \), where: \( \text{Re}_{L} = \) Reynolds number, \( k = \) fluid thermal conductivity, \( k_f = \) fin thermal conductivity, \( L = \) fin-length, \( 2t = \) fin-thickness. Unlike natural convection (Sparrow and Acharya (1981)), local convective heat transfer coefficient in forced convection varies monotonically.

Sunden (1985) extends the study of Sparrow and Chyu (1982) and incorporates the effect of Prandtl number on forced convection heat transfer from a rectangular fin. Results are presented for Prandtl number 0.021, 0.7 and 5 and convection-conduction parameter (in the range 0 to 5. Significant effects of both the parameters are obtained. Further, it is concluded that estimated fin efficiency is comparable to simple conventional fin theory, albeit the fin temperature variation and local heat flux distribution are wrongly predicted. Similar observations are also made by Sparrow and Chyu (1982).

Sparrow and Kadle (1986) experimentally examine forced convective turbulent flow heat transfer from a shrouded rectangular fin array. Apart from fin tip clearance, fin height and fluid flow rate are also varied. It is noted that, with the increase in clearance up to 30\%, there is a decent decrease in the heat transfer coefficient up to 36 \%. It is observed further that heat transfer rate varies little with airflow rate. Moreover, depicted results highlight that thermal development rate near the entrance is decreased in the presence of clearance.

The report of Kadle and Sparrow (1986) enlightens a numerical and experimental study of turbulent air flow over the shrouded longitudinal rectangular non-isothermal fin array attached to heated base. Numerical study is made assuming fully-developed turbulent flow. Local heat transfer coefficient is found to vary along the fin height as well as along the base.
surface. Minimum value is observed at the junctions formed by fin and base, and fin and shroud. Fluid at these corner points get saturated resulting in a lower heat transfer at the corners. Numerically obtained fin efficiencies show a decreasing trend with the Reynolds number as well as with the ratio of fin-height to fin-width. A comparable agreement is found between the computational results and the experimental results.

The experimental study by Writz et al. (1994) show the effect of flow bypass on the heat transfer from an array of longitudinal fin heat sink, frequently attached to electronic packaging. It reveals that approach flow is bypassed the heat sink up to 60% resulting in a reduction in heat transfer up to 50%. A relation is developed relating the heat sink flow rate to approach flow rate and fin density of heat sink to quantify the bypass. Further, it is observed from the effectiveness result that overall heat transfer rate increases with increase in heat transfer surface area up to a limit, beyond which it decreases.

Experimental study of El-Sayed et al. (2002) on the turbulent flow heat transfer over the shrouded longitudinal rectangular fin array highlights the effects of different geometries parameters and Reynolds number. Mean Nusselt number of fin is increased with the increase in Reynolds number, dimensionless fin-spacing ($S/H$), dimensionless fin-thickness ($t/H$), and dimensionless fin length ($L/H$). Mean Nusselt number is correlated with the Reynolds number and the geometric parameters of the problem.

Al-Sarkhi (2005) depicts a numerical finding of variable height shrouded non-isothermal rectangular fin array undergoing laminar forced convection. The product of friction factor ($f$) and Reynolds number ($Re$), for variable fin height, is turned out to be lower than that for constant fin height. Moreover, variable fin height predicts higher convective heat transfer coefficient as compared to constant fin height. Additionally, it reveals: the higher the fin spacing, the higher Nusselt number within the range of parametric studies. But, larger clearance results lower Nusselt number. A possibility of optimum design is inferred in the
report using variable height shrouded fin array with the use of large fin-spacing coupled with small clearance.

An experimental study by Elshafei (2007) provides the flow bypass effect on the thermal performance of the shrouded rectangular fin array. Turbulent flow regime is considered in the problem. Flow bypass is enhanced with the increasing fin density. The total pressure markedly affected by the fin density. The effect of fin density decreases as the clearance increases. Average heat transfer coefficient is found to decrease with the increase of clearance. Overall Nusselt number is correlated with governing parameters, namely, Reynolds number, fin-height, fin spacing, clearance spacing, of the problem.

Jouhara and Axcell (2009) report an analytical and computational study of laminar heat transfer through a fully shrouded heat sink. Computational study involves CFD simulation of fluid flow over the non-isothermal fin. Near the inlet, Nusselt number decreases along the axial length, but attains a fully-developed value after certain axial distance. On the other hand, fin efficiency increases along the axial length.

2.2.3 Heat transfer by mixed convection

2.2.3.1 Mixed convection from finned horizontal channel

Acharya and Patankar (1981) computationally examine fully-developed laminar mixed convective over the vertical rectangular isothermal fin array glued to a horizontal base treating air as the cooling media for a limited geometric parameters. Both hot and cold fins are treated in the article. Performance of hot fin turns out to be superior to that of cold fin. Frictional resistance of cold fin is found to be higher than that of hot fin. Since the problem undertaken by the authors is for fully-developed flow, thus results may not be extended to the entry region flow. Additionally, it may be mentioned that the treatment of isothermal fin is away from the reality, as fin will be having finite thermal conductivity.
A numerical study of fully developed laminar mixed convection over the shrouded isothermal fin array protruding upwards from a rectangular horizontal base is depicted by Maughan and Incropera (1990a). Two separate thermal boundary conditions are specified: (i) in one case, base is exposed to a fixed temperature, while the shroud is treated cold, (ii) in the other case, base is imposed to iso-flux condition and shroud is treated adiabatic. Heat transfer enhancement is noted with increasing fin-height and decreasing fin-spacing. Mixed convection causes cross-stream vortices, which enhance the heat transfer. Decreasing fin spacing, however, decreases the strength of cross-stream vortices. Therefore, there exists an optimum fin-spacing that maximizes the Nusselt number. In a companion paper, Maughan and Incropera (1990b) report experimental finding of mixed convection heat transfer from vertical rectangular fin array on a horizontal base. Two clearances (i.e., $C^*=1$, and 0.11) along with a range of fin-spacing and Rayleigh number are considered in the problem. No secondary flow is found on the un-finned plate till the Rayleigh number exceeds a value of 1708. Longitudinal vortices are developed and persisted till Rayleigh number reaches 20,000. Albeit, fins enhance heat transfer, yet they are found to weaken vortices and delay enhancement caused by vigorous secondary flow. In general, agreement between the experimental measurements and the numerical predictions for fully developed mixed convection is reasonably good. However, sharp disagreement is observed for $C^*=1$ and $S^*=0.5$ with a Rayleigh number greater than 5000. This is attributed to the formation of asymmetrical vortices which breaks the symmetrical nature of flow.

Recently, Dogan and Sivrioglu (2009) perform natural convection dominated mixed convective heat transfer from longitudinal fin array placed inside a horizontal channel. Reynolds number is maintained at a fixed value of 250, while the Rayleigh number is varied in a range from $3\times10^7$ to $6\times10^8$. Fin-height and fin-spacing influence the mixed convection heat transfer. There exists an optimum fin-spacing for which maximum heat transfer is
observed. This optimum is found to be 8 mm to 12 mm. Excess temperature difference between inlet fluid and the base does not affect the optimum fin-spacing. However, optimum spacing shows its dependence on Rayleigh number as well as on fin-height. Smaller fin height and relatively higher Rayleigh number shows higher value of optimum fin-spacing compared to higher fin height. It indicates that cross-stream secondary eddies may not develop at high Rayleigh number if the fin-height is below the desired value. Mixed convection study from rectangular fin array is extended further by Dogan and Sivrioglu (2010) for higher Reynolds number (i.e., Re=1500). Optimum fin spacing is turned out to be in a range from 8 mm to 9 mm. Dogan and Sivrioglu (2009, 2010) correlated the overall Nusselt number with the governing parameters of the problem. The effect of clearance on mixed convection from finned horizontal channel is reported by Dogan and Sivrioglu (2012). Combined experimental and numerical approaches are made to see the clearance effect. Results indicate higher heat transfer coefficient for low clearance.

### 2.2.3.2 Mixed convection from finned vertical channel

The article of Zhang and Patankar (1984) reveals the effect of buoyancy on the fully developed laminar forced convection heat transfer over shrouded vertical isothermal fin array on vertical base. Results indicate that flow through the lower fin-spacing is highly affected by the presence or absence of the clearance. Presence of clearance directs the fluid to follow the least resistive path, namely, the clearance region and thereby, it degrades the heat transfer performance. The phenomenon of flow reversal is noted at a fixed value of critical Rayleigh number, which depends on the geometry of the problem. The value of critical Rayleigh number decreases with the clearance as well as with the fin-spacing.

Recent communication by Al-Sarkhi et al. (2003) is the parametric extension of the study of Zhang and Patankar (1984) on combined free and forced convection over shrouded vertical isothermal fin array. Variation of Nusselt number (Nu) and product of friction factor ($f$) with
Reynolds number (i.e., $fRe$) with different geometrical parameters like clearance, fin-spacing are examined. Higher clearance causes more fluid bypass resulting in a degradation of heat transfer. The effect of Nusselt number is more pronounced at higher clearance even at very low Rayleigh number. There exists significant effect on Nusselt number only at a value of Rayleigh number of around $10^6$ for the case of zero clearance.

Mixed convection studies mentioned above are for fully-developed condition, which reduces the three dimensional problem into a two-dimensional problem. In practice, fully developed condition requires large length to achieve. Hence, these investigations are not applicable to the fins having smaller length. In addition, it may be appropriate to mention here that in most of the earlier studies on mixed convection, inlet velocity is considered as a characteristic velocity for Reynolds number evaluation. But under the situation of mixed convection for vertical fluid flow, contribution of inlet velocity is mainly due to two factors: one is for fan velocity and another is for induced velocity resulting from buoyancy. In the analysis, these two velocities should be segregated by some suitable means. It should be remembered here that induced velocity component due to buoyant force does not contribute to the pressure drop across the length of the duct. Therefore, it is important to decouple the two velocity components to find out the exact fan velocity which is required to evaluate fan power. Moreover, for a fixed inlet velocity induced velocity component increases with the increase in length of the fin due to chimney effect and hence pressure drop across the length of the duct consisting of fin-base and shroud decreases with the duct length till it reaches zero value. Therefore, friction factor never reaches a fully-developed value as evaluated and depicted by the previous investigators (Zhang and Patankar (1984), Al-sarkhi et al. (2003)) and also it does not depend on the Rayleigh number of the problem. Thus, length of the duct in mixed convection is an important parameter.
2.2.4 Temperature dependent thermo-physical property with non-Boussinésq fluid

Close examination of the existing literature enlightens that in most of the computational studies involving mixed convection, fins are treated isothermal assuming infinite thermal conductivity. In practice, fin material will have a finite value of conductivity. In addition, all the previous computational studies on fin heat transfer are made involving Boussinésq approximation. These effects are naturally incorporated when the studies of fin heat transfer are made experimentally. However, the same is not true for the computational study. Deviation associated with Boussinésq assumption with constant thermo-physical properties from practice is enlightened below.

According to Gray and Giorgini (1976), for an excess temperature between the hot and cold surfaces smaller than 30 K, the results of the natural convection obtained by the Boussinésq assumption are well consistent with the practical situation. However, in many important natural convection problems excess temperature are often higher than several hundred degrees. Under the high temperature difference, inapplicability of the Boussinésq assumption causes the factor of compressibility of fluid to change significantly, which introduces mathematical complexity in the problem.

Leal et al. (2000) apply Generalized Integral Transform Technique (GITT) to the classical problem of differentially heated square cavity, assuming both the cases of variable and constant thermo-physical properties of fluid to recognize the applicability limits of the Boussinésq hypothesis for a fluid of Prandtl number 0.71 involving imposed Rayleigh number in a range $10^3$ to $10^5$. It reports the substantial effects of variation of properties on the distributions and transient evolution of Nusselt numbers well within the conventional region of applicability of the Boussinésq hypothesis. It is also observed that limits of the Boussinésq approach may be reasonably acceptable for the most steady-state calculations, albeit it may not represent so accurately during the transient period.
In 2005 Toh et al. report the numerical results in micro-channel flow and heat transfer with temperature dependent thermal properties. The results indicate lower frictional loss due to decrease in viscosity with the increase in water temperature associated with the heat input, particularly at lower Reynolds numbers. However, little discussion was made about the effects of property variations on the characteristics of heat transfer. Later Herwig et al. (2006) computationally extend the study of thermo-physical properties variation to estimate the local Nusselt number variation by employing dimensional analysis using commercial CFD code. The results highlights the increase of local Nusselt number with the increase in heating power caused by the variations of thermal properties of viscosity and thermal conductivity.

Recently a conjugate analysis is made by Li et al. (2007) to examine the effects of thermal property variations on the liquid flow and heat transfer in a micro-channel heat sink. Results are compared adopting three different methods, namely the ‘Inlet property’, the ‘Average property’ and the ‘Variable property’ in treating the thermal properties. The methods of ‘Average’ and ‘Variable’ properties indicate significantly lower apparent frictional ($f_{app}$) coefficient than the ‘Inlet property’ method. Compared to the method of ‘Average property’, the method of ‘Variable property’ shows higher $f_{app}$ at the beginning of the channel, while the lower $f_{app}$ is observed at the later section of the channel. This observation is credited to the variation of viscosity along the channel. Both the local $f_{app}Re$ based on the ‘Inlet property’ and the ‘Average property’ finds good agreement with the simple theoretical results in the hydraulic entrance region. With the development of the flow field in the down-stream, $f_{app}Re$ value based on the method of constant property approaches towards an asymptote of the theoretical fully developed value, while the $f_{app}Re$ value based on the method of variable property is likely to show a value below this theoretical limit. In addition, both the methods of ‘Average property’ and the ‘Variable property’ have shown larger convective heat transfer
coefficient \((h)\) and Nusselt number \((Nu)\) than the method of ‘Inlet property’ presumably due to the lower viscosity \((\mu)\) and the higher fluid conductivity \((k)\).

In 2010, Reddy et al. examine numerically conjugate free convection in a vertical annulus formed by vertical heat generating rod placed centrally in vertical cylinder treating Non-Boussinésq fluid with variable property. The heat is transported from heat generated rod by the coupled conduction and natural convection modes to the outer wall. The specific heat of the cooling medium is assigned to be constant, while the other transport properties are assumed to be the function of temperature. For the chosen range of parameter values, the Boussinésq model predict heat transfer adequately up to a Grashof number of around \(10^9\), beyond this value non-Boussinésq model is more reliable. The Boussinésq model indicates relatively more intensive circulation and greater stratification than the model with non-Boussinésq fluid with variable transport properties. Average Nusselt number at the interface between solid and fluid enhances with the Grashof number, while the maximum dimensionless temperature decreases with Grashof number. It is noted that in view of the simplified equation of state, the Boussinésq approximation unable to predict the average pressure variation. On the other hand, the non-Boussinésq model reveals increasing average pressures with the Grashof number. Further, the non-Boussinésq model indicates higher Nusselt numbers and lower temperatures as compared to the Boussinésq fluid for a Grashof number value of \(10^{10}\).

From the aforesaid literatures, it is identified that Boussinésq assumption with constant thermo-physical properties deviates considerably depending upon the range of parameters of problem. Further, it may be noted that there exists hardly any literature on fin heat transfer, which consider non-Boussinésq fluid with property variation.

Till this point, review of literature is made on the heat transfer from extended surface and also the effect of non-Boussinésq fluid is highlighted. Along with the study of heat transfer
Second law analysis is important, since this provides the viability of irreversible limit on heat transfer. A brief review of Second law is described next.

### 2.2.5 Entropy generation in heat transfer in a channel

Augmentation of heat transfer rate is always associated with the larger rate of entropy generation. However, that does not prevent one to study the conditions of lower entropy generation to transport same amount of heat. The study of this kind can only reveal the optimum geometry under which entropy generation is minimum.

Very limited articles on plate finned surfaces focus studies on entropy generation. Vertical rectangular plate-finned heat sinks attached to a vertical plate resembles parallel plate channel when fin height is large compared to the inter-fin spacing. Flow through vertical channel under turbulent mixed convection is computationally visited by Balaji et al. (2007) for different set of heat flux and aspect ratio. A minimum total entropy generation rate is found at an optimal inlet fluid velocity for each set. It is concluded further that optimum inlet velocity increases with the heat flux but depends hardly on the buoyancy as well as aspect ratio. Moreover, it is noticed that Bejan number is limited to a range from 0.14 to 0.22 under optimum condition.

Andreozzi et al. (2006) report entropy generation from a uniformly heated vertical channel undergoing laminar natural convection and highlight the effect of Rayleigh number and aspect ratio on both local and global entropy generation. Global entropy generation shows the higher value with the increase of Rayleigh number as well as aspect ratio, albeit local entropy generation indicated dissimilar behavior in the different region of flow with Rayleigh number. Therefore, it is imperative to evaluate entropy generation on a local level along with the global to identify clearly its source and location.
Cheng and Huang (1989) compute the distribution of entropy generation for laminar forced convective flow via 2-D Computational Fluid Dynamic (CFD) analysis over a pair of transverse fin placed in the entrance region of a parallel plate channel. Similar study of CFD analyses are made by Cheng et al. (1994) to estimate the entropy generation and heat transfer from laminar mixed convection flows through vertical channel with transverse fins placed at a regular interval on one side of the channel wall. Cheng et al. (1994) highlight the significant enhanced heat transfer through the channel with transverse fins as compared to un-finned channel at the expense of merely slight increase in entropy generation. These observations, therefore, recommends the use of transverse fins with a fin-height to channel width ratio in a range from 0.1 to 0.3.

Sasikumar and Balaji (2002) examine entropy production analysis from a convecting-radiating vertical fin on horizontal rectangular duct. Based on maximum heat transfer rate and minimum entropy generation rate, optimum fin height is observed for different fin profile. It is concluded further that convective heat transfer superseded radiative heat transfer. Study is made with a consideration of constant convective heat transfer coefficient, which is taken from the available correlation Churchill (1975). It is quite familiar that radiative heat transport influences the convective heat transport. Therefore, it is important to use local convective heat transfer coefficient for more accurate analysis, which requires CFD analysis along with analysis of entropy generation. In fact, this had been done later by Rakshit and Balaji (2005) on rectangular vertical fin attached on horizontal rectangular duct in combined approach of CFD and genetic algorithms. Optimum fin height is observed in the study.

Additional study on vertical plate-finned heat sink on a horizontal base subjected to both horizontal and vertical cooling stream under laminar forced convection may be seen in Shih and Liu (2004), who examines optimal design methodology based on entropy generation. It is found that horizontal cooling stream renders better performance than vertical cooling stream
based on heat transfer and pressure drop data observed by Teertstra et al. (1999). A study on entropy generation on the same configuration subjected to turbulent flow is made by Jian-Hui et al. (2009) using a combined approach of CFD and combined optimization technique. Optimum fin density for simultaneous maximum heat transfer and minimum entropy generation is reported by them. Recently, Mishra et al. (2009) provide the Second law based optimization in a cross flow plate-fin heat exchanger design using genetic algorithm. Basak et al. (2013) report the role played by entropy generation in the problem of thermal management in a porous trapezoidal enclosure having both isothermal and non-isothermal wall heating.

Least energy optimization of forced convection heat transfer from plate-finned heat sinks has been studied by Culham and Muzychha (2001), and Iyenger and Bar-Cohen (2002). Entropy generation is calculated based on the inlet temperature of fluid flowing over the surface, since heat transfer and pressure drop data are taken from other sources. It would be more justified to calculate entropy based on the temperature at which heat transfer occurs. Heat transfer occurs only when the extended surfaces are subjected to higher or lower temperature than the ambient. Therefore, entropy generation based on inlet fluid temperature is under/overestimated whenever extended surfaces are elevated to lower/higher temperature. Thus, the results of entropy generation minimization in these studies need to be assessed thoroughly by evaluating local entropy generation along with global entropy generation as shown by Andreozzi et al. (2006). One solution to find exact entropy generation of over the extended surface is to do CFD simulations of the whole flow domain that calculates the local entropy generation based on the relation provided by Bejan (1979, 1982, 1996).

Cited literature reveal that most of the earlier studies on entropy generation over the extended surfaces have been based on the heat transfer and pressure drop correlation suggested by previous studies. Investigations purely based on CFD simulations are scarce in literatures. Moreover, entropy generation analysis from vertical plate-finned heat sinks
attached to a vertical base under mixed convection is not paid due attention inspite of its relevance in various fields, albeit there exist significant amount of studies on mixed convection heat transfer (Acharya and Patankar (1981), Zhang and Patankar (1984), Al-Sarkhi et al. (2003), Dogan and Sivrioglu (2009, 2010), Giri and Das (2012)). Therefore, it is essential to analyze entropy production caused by both thermal and viscous dissipation from a hydraulically developing laminar mixed convection flow over vertical shrouded fin arrays attached to vertical base, so that an optimal design might be found. This optimal condition can be achieved either by the usual mathematical optimization techniques, or by the direct scanning of finite CFD solution set. Direct scanning of finite CFD solution set may not be as disadvantageous as it sound, rather it is more similar to a sensitivity study of different parameters to find optimal conditions based on both local and global entropy generation.

2.2.6 Heat transfer enhancement by secondary flows

The horseshoe vortex, which finds several applications, is an example of a secondary flow. It arises out very frequently around the protrusion introduced in the flow and extends out of the hydrodynamic boundary layer. In 1973, Sedney reports a comprehensive review highlighting the effects of small protuberances in a flow. It is noted that a three-dimensional surface bump produces qualitatively similar effects in both laminar and turbulent flows. Vortices are formed very near the disturbance that bends around the protuberances to be carried far downstream in a pattern similar to a horseshoe. Interestingly, aforesaid secondary flow will originate irrespective of the shape of the disturbance. But the location and the size of the protuberance are highly important. The height of protuberance should be comparable to the thickness of local displacement boundary layer. Longitudinal vortices that are produced by surface bump persist in the downstream in a length more than 100 times the height of protrusion. Protrusions designed to generate vortices are commonly termed as ‘vortex generators’.
Use of winglet vortex generator with 20° angle of attack in a channel is reported by Biswas and Chottopadhyay (1992), who showed enhancement of spanwise average local Nusselt number as good as 34% even at a non-dimensional axial location of 8.4. Increasing the angle of attack from 20° to 26°, an additional 10.6% enhancement is reported by the authors. Even, changing the angle of attack from 20° to 30° much higher enhancement (26.5% over the case of 20°) is noted. By changing the Reynolds number from 500 to 1815, 98.38% augmentation is noted at exit location (= 8.4) of the channel.

An attempt has been made by Deb et al. (1995) to study the heat transfer characteristics with associated flow structure in the regime of laminar and turbulent flows in a rectangular duct having built-in vortex generators using full Navier-Stokes and energy equations. Each pair of winglet invites longitudinal vortices in the downstream. It is revealed that longitudinal vortices formed by the introduction of winglet-pair swirl the motion of fluid around the axis parallel to the streamwise direction, which strongly augment the exchange of fluid between the core and the wall resulting in a high heat transfer. To accommodate high fluid velocity, computation of turbulent flows is made and an effort is also made to compare the accuracy of the computed results with the well documented experiments results.

Delta winglet is a commonly used geometry to generate vortex. Jacobi and Shah (1995) present a review on the enhancement of heat transfer in presence of longitudinal vortices. Very recently, Joardar and Jacobi (2008) reports an increase in heat transfer coefficient of 16.5–44% by introducing a series of vortex generators on the surface of the fin positioned in between the tubes for the Reynolds number in the range from 220 to 960. Chang et al. (1997) study the effect of vortex generator placed in a channel formed by flat tube fin. But, the results are not compared without vortex generator.

From the survey of literature, it is found that mixed convection heat transfer from the extended surface is not highlighted enough for the case of vertical fin on a vertical base. In
this configuration, computational studies of heat transfer are made assuming fully-developed flow over isothermal fin array. Moreover, as mentioned earlier, decoupling of forced convection velocity from the induced velocity gets little attention, albeit it is highly important to know exactly the fan velocity from the design point of view. Further, all the past computational efforts on fin heat transport are made without considering non-Boussinésq fluid with property variation. In addition, mixed convection heat transports are mostly studied computationally treating fin with infinite conductivity. But in practice, fins are having finite thermal conductivity. In the literature, very little attempt is made on the Second law analysis of fin heat transfer. Second law analysis measures the systems’ departure from the reversible limit. Therefore, the optimum condition of heat transfer based on Second law needs special attention. Fast heat removal from the system is also very much relevant due to the miniaturization of equipment, which looks for additional resort in addition to extended surface. This may be done by introducing vortex generator. Thus the present work is an attempt to study computationally the heat flow from a vertical shrouded fin array under mixed convection with and without vortex generator. The shroud is used to direct the fluid flow through the inter fin spacing to reduce the flow by-pass. This work is motivated by practical applications example being heat removal from solar heater, cooling of electrical transformer and cooling of automobile engines, etc.

2.3 Objectives of the present study

The objective of the present work is to predict the flow and temperature distribution in the symmetric channel made by two consecutive vertical fins, a vertical base and a shroud by decoupling the inlet velocity into two components, i.e., the fan velocity and the induced velocity resulting from buoyancy. Following objectives are identified
i. To find the effect of different parameters like fin spacing, fin height, fin length, clearance spacing, Reynolds number, Grashof number and Prandtl number on the mixed convection fin heat transfer and to correlate results with governing parameters.

ii. To optimize the different input variables for the best possible heat transfer using entropy analysis.

iii. To find the enhancement of heat transfer using vortex generator.