Chapter 6

Mixed Convective Heat Transfer from Vertical Fin Array in the Presence of Vortex Generator

6.1 Introduction

To cope up well with the ever-increasing demand, supplementary means may be trusted other than the use of extended surface as discussed in the last three chapters. Adoption of secondary flow over the main motion finds useful applications to enhance the heat transfer. The well-known horseshoe vortex being one of its kinds is usually appeared surrounding the protuberance, which extends out of the hydrodynamic boundary layer resulting in a large transport of energy. An exhaustive review on the effect of small protuberance in boundary layer flows is reported by Sedney (1973). It may be highlighted that qualitatively similar effects for both laminar and turbulent boundary layers are observed by the introduction of three-dimensional surface bump in a flow. Evolution of secondary flows depends strongly on the location and the height of protuberance, but has nominal dependence on the shape of protuberance.

Use of winglet vortex generator placed in a rectangular channel is seen in Biswas et al. (1989), Biswas and Chattopadhyay (1992), Deb et al. (1995). Reported results are highly encouraging, since spanwise average local Nusselt number is enhanced to a value as good as 34% even at a dimensionless channel axial length of 8.4 (Biswas and Chattopadhyay(1992)). Enhancement of heat transfer associated with longitudinal vortices is reviewed by Jacobi and Shah (1995). Joardar and Jacobi (2008) indicate an enhancement of heat transfer in a range from 16.5% to 44% for a flow with Reynolds number in a range of 220 to 960 by the use of protrusion. Enhancement of heat transfer with vortex generator may provide thermal management solution in some area of heat transfer.
To enhance the mixed convection heat transfer from a vertical shrouded fin array on a vertical base, it occurs to us that vortex generator might be useful. Vortex generator in such configuration is hardly considered previously. Moreover, most of the earlier studies placed the vortex generator over the fin surface. In the present study, low conductive protrusion is projected out of adiabatic shroud towards the base and placed in the middle of fin spacing. Further, prior investigations on the similar configuration by Zhang and Patankar (1984), Al-Sarkhi et al. (2003) on mixed convection studies were restricted to isothermal fin. However, in practice, material used for fin will definitely have finite conductivity. In addition, all the computational attempts on fin heat transfer, examined by previous investigators, are based on Boussinésq approximation. Departure of Boussinésq assumption from the reality is enlighten in the literatures of Li et al. (2007), Reddy et al. (2010), in which heat transfer due to convection is estimated using non-Boussinésq fluid. Three main effects: (i) first revolving around non-isothermal fin (ii) second centering on non-Boussinésq fluid with property variation (iii) third involving vortex generator are the central aspects of the present article. Thus present chapter deals with a problem of entry region mixed convective flow over non-isothermal vertical shrouded rectangular plate fin array on a vertical base with vortex generator losing heat by variable property non-Boussinésq fluid flow. Description of physical model is described next.

6.2 Physical model and coordinate system

Physical model of the problem is diagrammed in Fig. 6.1a-c. Rectangular plate fins having length ‘L’, thickness ‘t’, and height ‘H’ are attached vertically to a rectangular hot vertical base plate. Fins maintain a distance ‘S’ from each other. A rectangular adiabatic shroud, that maintains a clearance ‘C’ between the fin-tip and the shroud, is placed in front of the insulated fin-tip. Thus, the assembly of base, shroud and two consecutive fins makes a rectangular duct having a dimension \( L \times S \times (H+C) \). The origin of the coordinate system lies at...
Fig. 6.1. (a) Vertical rectangular fin array attached to a vertical base plate with a vertical shroud placed near the fin-tip; (b) Cross-sectional area of one of the flow passages (computational domain); (c) Side-view of base-fin-shroud assembly.
the corner formed by the left fin and the base. Starting from the left fin, $x$-direction is projected along the base towards the right fin, while the $y$-direction initiating from the base is directed along the left fin normal to the base. Direction of $z$ is treated vertically upwards. Two square sectioned ($t_v \times t_v$) low conductive vortex generators are projected out of shroud towards the base maintaining a direction parallel to $y$-axis and perpendicular to $x$-axis. Each vortex generator is placed along the symmetry line of cross-section (Fig. 6.1b) and maintained a distance $S_1$ from both left and right fins along $x$-direction. First vortex generator is placed at an axial distance of $L_1$ from the entrance, while the second one is placed at a distance of $L_2$ from the first vortex generator (Fig. 6.1c). Both the protrusion maintains a clearance ‘$C_1$’ from the base. Vortex generator is attached to the assembly to enhance the performance of heat transfer. Base is maintained at a higher temperature ($T_w$) than the atmospheric temperature ($T_0$). This sets in natural convection through the ducted fin array. To enhance the heat transfer over and above the induced motion, forced flow is supplemented with the natural convection. Further, base and fin are assumed to be of same material. Contact resistance at the junction of base and fin are treated negligible. Fin is assumed to have finite conductivity. Hence, the present case considers the study of mixed convection heat transfer over the base-fin-shroud assembly in the presence of vortex generator.

6.3. Mathematical formulation

In the present case, density variation is incorporated by assuming density to vary with temperature only. Physical properties of viscosity and conductivity of fluid are also allowed to vary with the temperature of fluid. Equation used to consider the property variation is available in Das and Giri (2014). Viscous dissipation and thermal radiation is considered negligible. Vortex generator is assumed to be of infinite viscosity and very low thermal conductivity. Physical model described above may be described mathematically by the following equations:
Continuity equation:
\[
\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) + \frac{\partial}{\partial z} (\rho w) = 0. \tag{6.1}
\]

Momentum equation
\[
\rho \left( \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left( \mu \frac{\partial u_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( -\frac{2\mu}{3} \frac{\partial u_i}{\partial x_j} \right) + \frac{\partial}{\partial x_j} \left( \mu \frac{\partial u_j}{\partial x_i} \right) + (\rho_0 - \rho) g_i. \tag{6.2}
\]

Energy Equation
\[
\rho C_p \frac{\partial T}{\partial t} + C_p \rho \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = \frac{\partial}{\partial x} \left( k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k \frac{\partial T}{\partial z} \right). \tag{6.3}
\]

Fin Conduction Equation:

(i) Fin at \( x = 0 \)
\[
\frac{k_{\text{fin}}}{k} \frac{1}{2} \left( \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = \frac{\partial T}{\partial x} \bigg|_{x=0}. \tag{6.4}
\]

(ii) Fin at \( x = S \)
\[
\frac{k_{\text{fin}}}{k} \frac{1}{2} \left( \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) = \frac{\partial T}{\partial x} \bigg|_{x=S}. \tag{6.5}
\]

Inlet condition and boundary conditions are described as below.

At the inlet
for \( z = 0 \) and \( 0 < x < S \) and \( 0 < y < (H+C) \)
\[
u = 0, v = 0, w = w_{\text{in}} \text{ and } T= T_0. \tag{6.6}
\]

On the fin surface (i.e., \( 0 \leq y \leq H, x = 0 \) and \( x = S \) and \( 0 \leq z \leq L \))
\[
u = v = w = 0, \text{ and } T= T_f. \tag{6.7}
\]

On the y-z plane of the clearance space (i.e., \( H < y \leq (H+C) \), \( x = 0 \) and \( x = S \) and \( 0 \leq z \leq L \))
\[ \frac{\partial u}{\partial x} = \frac{\partial v}{\partial x} = \frac{\partial w}{\partial x} = \frac{\partial T}{\partial x} = 0. \]  

(6.8)

At the base surface (i.e., \( y = 0 \) and \( 0 \leq x \leq S \) and \( 0 \leq z \leq L \))

\[ \mathbf{u} = \mathbf{v} = \mathbf{w} = \mathbf{0}, \text{ and } T = T_w. \]  

(6.9)

At the shroud (i.e., \( y = H+C \) and \( 0 \leq x \leq S \), and \( 0 \leq z \leq L \))

\[ \mathbf{u} = \mathbf{v} = \mathbf{w} = \mathbf{0}, \text{ and } \frac{\partial T}{\partial y} = 0. \]  

(6.10)

Fin boundary condition

\[ T_f = T_w, \text{ at } y = 0 \text{ and } 0 \leq z \leq L \]  

(6.11)

\[ \frac{\partial T}{\partial y} = 0, \text{ at } y = H \text{ and } 0 \leq z \leq L \]  

(6.12)

\[ \frac{\partial T}{\partial z} = 0, \text{ at } 0 < y < H \text{ and } z = 0 \text{ and } L \]  

(6.13)

Eqs. (6.1) – (6.13) describe the complete mathematical statement of problem undertaken. Using the non-dimensional scheme mentioned in the nomenclature, it is found that present problem involves number of parameters: Prandtl number, Grashof number, Reynolds number, dimensionless fin spacing, dimensionless clearance, dimensionless fin length and fin conductance parameter.

### 6.4. Computational procedure

The aforesaid described problem is elliptic in 3-dimensions and solved using well known numerical finite volume SIMPLER algorithm elucidated by Patankar (1980). At any axial location, \( u- \), \( v- \) and \( w- \) velocities are computed at the staggered location, while the scalar quantities \( T \) and \( p \) are stored at the central location. The combined convective and diffusive terms in cross-stream and streamwise directions (i.e., \( x- \), \( y- \) and \( z- \)directions) are discretised.
with the power law scheme on a staggered mesh. Remaining diffusive terms, which do not mould into power law scheme, are discretised using central difference scheme. The whole flow domain is considered as one region assuming infinite viscosity ($\mu = 10^8$ Pa-s) and very low thermal conductivity ($k = 10^{-7}$ W/m-K) for the protrusion. Assumption of infinite viscosity will produce the zero velocity fields within the protrusion, while the assumption of low thermal conductivity will forbid heat transfer through the protrusion. Rectangular finite difference grids are engendered in $x$-, $y$- and $z$- coordinate directions. Finer grids are employed near the solid boundaries including protrusion. To solve the above described problem a FORTRAN-90 numerical code, which is developed by Das and Giri (2014), is used with suitable modifications. Present results are obtained by using a set of grids $32 \times 48 \times 180$, $32 \times 48 \times 180$ and $42 \times 52 \times 180$ in $x$-, $y$-, and $z$- directions for $S^* = 0.2$, 0.3 and 0.5 respectively. Grid independence tests are performed to check the accuracy of the present computation (Table-6.1). To validate the computation, the results of Sparrow et al. (1978) are compared with the results obtained from the present code (Table-6.2) and present results find reasonably good agreement with Sparrow et al. (1978). It may be recalled that results of Sparrow et al. (1978) are obtained from 2-D developed flow, while the present results are obtained from 3-D developing flow simulation.

Table 6.1. Results of grid independence test.

<table>
<thead>
<tr>
<th>$S^*$</th>
<th>Grid $X \times Y \times Z$</th>
<th>Nu</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>$28 \times 38 \times 180$</td>
<td>7.502</td>
</tr>
<tr>
<td></td>
<td>$32 \times 38 \times 180$</td>
<td>7.536</td>
</tr>
<tr>
<td></td>
<td>$32 \times 46 \times 180$</td>
<td>7.55</td>
</tr>
<tr>
<td></td>
<td>$34 \times 38 \times 180$</td>
<td>7.645</td>
</tr>
<tr>
<td>0.5</td>
<td>$42 \times 40 \times 180$</td>
<td>7.656</td>
</tr>
<tr>
<td></td>
<td>$42 \times 46 \times 180$</td>
<td>7.661</td>
</tr>
</tbody>
</table>
Table 6.2. Comparison of results obtained from the present code with the results of Sparrow et al. (1978).

<table>
<thead>
<tr>
<th>$C^*$</th>
<th>$C_i^*$</th>
<th>$S^*$</th>
<th>$\Omega$</th>
<th>$\text{Nu}_z$ Sparrow et al. (1978)</th>
<th>Present</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.0</td>
<td>0.1</td>
<td>10</td>
<td>14.45</td>
<td>14.34</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>25</td>
<td>22.38</td>
<td>22.28</td>
</tr>
</tbody>
</table>

6.5 Results and Discussions

Geometric parameters for the problem undertaken are summarized below: Fin length is considered as 0.5 m. Fin heights are chosen as 0.04 m and 0.05 m, since these fin height provides the better heat transfer Das and Giri (2014). Dimensionless fin-tip clearance ($C^*$) is maintained to vary from 0.0 to 0.25. $C_i^*$ is assumed a fixed value of 0.1. Non-dimensional fin-spacing ($S^*$) are chosen as 0.2, 0.3 and 0.5, since it is difficult to accommodate a vortex generator lower than the aforesaid spacing. Moreover, higher fin spacing means lower number of fins for same base width, which, in turn, reduces the cost of fin material. Cross-section ($t_v \times t_v$) of the vortex generator is assumed to be 0.004×0.004 (m$^2$). First vortex generator is placed at an axial location of 0.15 m from the entrance for all the computations, while the second one is placed at $L_2 = 0.1$ m from the first one. This corresponds to a ratio of $L_2/t_v = 37.5$, which is sufficient enough to avoid any disturbance due to vortex generator in the flow at the entrance. Fin conductivities ($k_{fin}$) are chosen as 75 W/m-K and 150 W/m-K. Base temperature is assigned a value of 373.15 K, while ambient temperature is assumed to be 293.15 K. Air is considered as the cooling media. Therefore, Prandtl number of the problem is 0.7. Inlet velocities are chosen to vary from 0.6 m/s to 1.0 m/s. This corresponds to the variation of non-dimensional inlet mixed convection velocities from 1280 to 2667, which is equivalent to the Reynolds number variation from 1280 to 2667. Grashof number of
the problem varies from $4.3 \times 10^5$ to $8.4 \times 10^5$ for the chosen parameters. For the purpose of presentation, properties of air are estimated at the mean temperature of base and ambient. However, in the numerical calculation properties are assumed to vary with the temperature. In chapter-3, decoupling of mixed convection velocity is outlined. In the present case, decoupling is done via natural convection analysis. First, mixed convection problem is solved with a given geometry for different set of velocities. Then, natural convection problem is solved for the same geometry. Natural convection velocity component is thus determined. Since, by assumption mixed convection velocity is the supplement of forced convection velocity and natural convection velocity. Reynolds number is obtained by the subtraction of natural convection velocity from mixed convection velocity. Results obtained using the aforesaid parameters are described next.

6.5.1 Effect of property variation

The effect of property variation is compared in Table-6.3. From the table it can be identified that overall Nusselt obtained from Boussinéq assumption coupled with variable properties shows 8% lower than that obtained from Boussinéq assumption with constant thermo-physical properties for fin thermal conductivity of 1000 W/m-K. In chapter-4, variation of fluid thermal conductivity keeping other property constant also yields a maximum deviation of overall Nusselt number around 8% for the case of fin spacing $S^* = 0.5$ with a Grashof number of $8.4 \times 10^5$. In the present case, 8% deviation is noted for $S^* = 0.3$ with a Grashof number $4.3 \times 10^5$. Variable density coupled with variable viscosity maintaining fluid thermal conductivity constant indicates 6% enhancement in overall Nusselt number. Remaining other cases, variation is not so significant. Reducing the fin conductivity from 1000 W/m-K to 75 W/m-K, (i.e., decreasing fin conductivity by 12.33 times the lower value of fin conductivity), results of Nusselt number maintains the aforementioned trends.
However, overall Nusselt number obtained from Boussinêsq assumption coupled with variable properties shows 6% lower than that obtained from Boussinêsq assumption with constant thermo-physical properties for fin thermal conductivity of 75 W/m-K, which is lower by 2% from that obtained for a fin conductivity of 1000 W/m-K. On the other hand, variable density coupled with variable viscosity keeping conductivity fixed shows an enhancement of 4% in overall Nusselt number, which is again lower than the case of 1000 W/m-K fin conductivity by 2%. Once again, it may be mentioned that the effect of property variation is very much problem dependent and cannot be generalized.

Table-6.3 Results of overall Nusselt number showing the property variation

<table>
<thead>
<tr>
<th>$k_{fin}$</th>
<th>$\rho$</th>
<th>$\mu$</th>
<th>$k$</th>
<th>$\text{Nu}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000 W/m-K, $\text{Gr} = 4.3 \times 10^5$, $W_{in,mix} = 2133$, $S^<em>=0.3$, $C^</em>=0.1$</td>
<td>Boussinêsq</td>
<td>Constant</td>
<td>Constant</td>
<td>10.77</td>
</tr>
<tr>
<td></td>
<td>Boussinêsq</td>
<td>Variable</td>
<td>Variable</td>
<td>9.89</td>
</tr>
<tr>
<td></td>
<td>Variable</td>
<td>Constant</td>
<td>Variable</td>
<td>10.56</td>
</tr>
<tr>
<td></td>
<td>Variable</td>
<td>Variable</td>
<td>Constant</td>
<td>11.36</td>
</tr>
<tr>
<td></td>
<td>Variable</td>
<td>Variable</td>
<td>Variable</td>
<td>10.54</td>
</tr>
<tr>
<td>75 W/m-K, $\text{Gr} = 4.3 \times 10^5$, $W_{in,mix} = 2133$, $S^<em>=0.3$, $C^</em>=0.1$</td>
<td>Boussinêsq</td>
<td>Constant</td>
<td>Constant</td>
<td>9.79</td>
</tr>
<tr>
<td></td>
<td>Boussinêsq</td>
<td>Variable</td>
<td>Variable</td>
<td>9.17</td>
</tr>
<tr>
<td></td>
<td>Variable</td>
<td>Constant</td>
<td>Variable</td>
<td>9.75</td>
</tr>
<tr>
<td></td>
<td>Variable</td>
<td>Variable</td>
<td>Constant</td>
<td>10.24</td>
</tr>
<tr>
<td></td>
<td>Variable</td>
<td>Variable</td>
<td>Variable</td>
<td>9.75</td>
</tr>
</tbody>
</table>

6.5.2 Variation of pressure

Local dimensionless pressure ($P^*$) across the section at different axial locations (i.e., $Z = 0.405, 3.592, 6.520$ and $12.346$) is illustrated in Fig. 6.2a-d for a typical case of $S^* = 0.3$ with a clearance space of $C^* = 0.1$. Local dimensionless pressure ($P^*$) in this chapter is nothing but $(p_0 - p_r)H^2 / \rho_0 \nu^2$. Near the entrance, local dimensionless pressure in the vicinity of base
Fig. 6.2. Development of dimensionless local pressure defect across the section for \( k_{\text{fin}} = 150 \text{ W/m-K}, W_{\text{in,mix}} = 1600, \text{Gr} = 4.3 \times 10^5, S^* = 0.3, C^* = 0.10 \), \( x^* = 0.10 \). (a) \( Z = 0.405 \); (b) \( Z = 3.592 \); (c) \( Z = 6.520 \); (d) \( Z = 12.346 \).
/shroud wall is lower (Fig. 6.2a) is presumably due to zero dimensionless velocity to satisfy the no slip condition, while in the inter-fin region dimensionless pressure is higher to maintain the flow in the said region. This will be more evident in the description of axial velocity at section-6.5.3. Map of local pressure just ahead of the first vortex generator is exemplified in Fig. 6.2b. As evident in the figure, local pressure decreases almost linearly away from the base towards shroud and reaches a minimum value very close to shroud. This decrease is estimated to be as high as 24% as compared to the base. Conventional theory assumes sectional pressure, which exists at the passage inlet formed by base and vortex generator. From the present results, it is understood that conventional theory overestimates the pressure to a great extent. Local pressure across the section just after second vortex generator is depicted in Fig. 6.2c. Pressure just behind the vortex generator is maximum presumably due to the formation of standing eddies. This will produce formed drag across duct length other than frictional resistance associated with the flow. Finally, Fig. 6.2d describes the local pressure distribution near the exit section. Near the exit, effect of standing eddies is mitigated. Its variation is very similar to the one obtained for inlet section, but the magnitude of pressure is increased due to the combined effects of skin friction drag and formed drag inherent in the problem.

Variation of dimensionless local average pressure along the axial direction is shown in Fig. 6.3a-d for $S^*=0.3$. Average pressure of fluid increases away from the inlet. Whenever, fluid approaches vortex generator, average pressure decreases just ahead of it. Thereafter, there exists a sudden increase in pressure across the protrusion due to the sudden reduction of flow area coupled with vortex formation, which causes depression of dimensional pressure, i.e., enhancement of dimensionless pressure. This increase in pressure is mitigated downstream of protrusion. Pressure drop across the length is decreased with the clearance is arguably caused by the lower resistance inherited with the larger clearance. Similar
Fig. 6.3. Variation of dimensionless local average axial pressure defect for $k_{fin} = 150$ W/m-K, $S^* = 0.3$. (a) $W_{in,mix} = 1600$, $Gr = 4.3 \times 10^5$; (b) $W_{in,mix} = 1920$, $Gr = 4.3 \times 10^5$; (c) $W_{in,mix} = 1600$, $Gr = 8.4 \times 10^5$; (d) $W_{in,mix} = 2133$, $Gr = 8.4 \times 10^5$. 
observation of clearance effect on pressure is also made in chapter-5 in connection with pumping power ratio (Section-5.3.3). Dimensionless axial local average pressure variation without protrusion is also plotted in Fig. 6.3a-d for the purpose of comparison. It is realized from the figure that total pressure drop across the axial length with vortex generator is increased to a large extent. This increase is as high as 150% as compared to mixed convection without vortex generator for the case of $C^* = 0.1$ and $Re = 1600$. This increase may be bearable in view of the expected enhancement of heat transfer. Increase in inlet velocity invites larger pressure drop, which is highlighted in Figs. 6.3b and 6.3d. Further, higher Grashof number renders lower pressure drop is presumably due to the larger induced velocity associated with the problem, which is identified in section-3.5.1 of Chapter-3.

6.5.3. Velocity development

6.5.3.1 Velocity vector

Fig. 6.4a shows the velocity vector across $X$-$Z$ plane at $Y = 0.583$, while Fig. 6.4b shows the velocity vector across $Y$-$Z$ plane at $X = 0.15$ for $S^* = 0.3$. To examine the flow near the vortex generator, plot is made in a range of $Z$ values from 3 to 7. Vortex generators are identified by the rectangular black lines inserted in the figure. Smooth entry is noted in the flow before the protrusion and therefore entrance flow is not disturbed by the protrusion. To compare heat transfer results with and without vortex generator, this is important. As seen from Fig. 6.4a and 6.4b, velocity of fluid between the fin and the vortex generator as well as between the base and the vortex generator is enhanced due to the insertion of vortex generator to satisfy the continuity equation. Enhanced velocity in the passages between the vortex generator and base surface as well as between vortex generator and fin will alter the growth of boundary layer at the said locations. It is well-known that repeatedly arresting boundary growth by forming new boundary layer enhances the heat transfer. Therefore, enhanced
velocity in the aforesaid passage will have definite bearing on heat transfer. Constricted passage acts like converging-diverging channel, which is realized from the velocity vector shown in Fig. 6.2a. Further, it is identified that standing vortex (Fig. 6.2a) is created behind the vortex generator. Since, standing vortex is created behind the insulated protrusion, therefore it is unlikely to have any impediment to the heat transfer on base and fin at those locations. Rather, enhanced velocity close to the fin and the base will definitely inherit enhanced heat transfer. In addition, it may be noted that there exists no asymmetry of flow in

Fig. 6.4. Velocity vector for $k_{fin} = 150 \text{ W/m-K}$, $W_{in} = 1600$, $Gr = 4.3 \times 10^5$, $S^* = 0.3$, $C^* = 0.10$, $C_i^* = 0.10$. (a) at $Y = 0.5827$ in $X-Z$ plane; (b) at $X = 0.15$ in $Y-Z$ plane.
the X-Z plane. Very small eddies are formed at the corners formed by each protrusion and shroud. However, this is not expected to influence significantly on the overall heat transfer process, since both protrusion and shroud are adiabatic material.

6.5.3.2 \( W \)-velocity

\( W \)-velocity profile is shown in Fig. 6.5a-d for different axial locations. Chosen axial locations are those considered for local pressure distribution. Close to the entrance region, velocity profile is nearly rectangular, which is very much similar to one observed without vortex generator in section-3.5.3 of chapter-3. Protrusion does not alter the entrance condition. As flow moves downstream, fluid encounters protrusion. Even just ahead of protrusion, velocity maintains a profile which is very similar to without protrusion case (Fig. 6.5b). It is clearly identified that axial velocity enhances in the expanse between fin and vortex generator as well as in the region between base and vortex generator (Fig. 6.5c). Velocity profile between base and protrusion along \( Y \)-direction assumes parabolic profile assuming zero velocity at the base wall and attaining maximum velocity in middle. Similar development of velocity is also noted in the expanse between fin and the protrusion. Velocity profile in Fig. 6.5c confirms the negative value near the zone of vortex generator signifying reverse flow identified earlier in Fig. 6.4a. Since axial velocity is enhanced over the heat transfer surface, thus it is expected to augment the heat transfer. This will be realized in the result of local Nusselt number presented in Section-6.5.5. It may be highlighted that velocity profile close to the base (Fig. 6.5d) remains high even though velocity profile is away from the second vortex generator. One reason of higher velocity near the wall is presumably due to buoyancy effect and other could be due to the evolution of cross-stream motion of fluid in the presence of vortex generator. The Effect of buoyancy may be identified in the development of fluid temperature profile in section-6.5.4.2. Described velocity profile will have definite bearing on heat transfer, which will be realized in Section-6.5.5.
Fig. 6.5. Axial development of $W$-velocity profile across the section for $k_{\text{fin}} = 150 \text{ W/m-K}$, $W_{\text{in,mix}} = 1600$, $Gr = 4.3 \times 10^5$, $S^* = 0.3$, $C^* = 0.10$, $C_1 = 0.10$. (a) $Z = 0.405$; (b) $Z = 3.592$; (c) $Z = 6.520$; (d) $Z = 12.346$. 

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6.5.4 Temperature

6.5.4.1 Fin temperature distribution

Fin temperature distribution in Y-Z plane is exemplified in Fig. 6.6a-d for different fin conductivities for $S^*=0.3$. It is noted that fin temperature distribution is identical for both left and right fin. Results indicate significant variation of fin temperature along Z-direction (i.e., along the fin length) as well as in Y-direction (i.e., along the fin height). This development is completely different from the one noted in section-4.5.2 chapter-4. Near the zone of vortex generator, larger drop of fin temperature along the fin-height is presumably caused by higher fluid velocity, which is developed due to presence of vortex generator. However, fin temperature distribution at the downstream of second vortex generator indicates variation mostly along Y-direction, which is again similar to the one presented in chapter-4. Influence of inlet velocity is seen in Fig. 6.6b, which indicates higher fin temperature drop across the fin height. With a very close look of Fig. 6.6a-b, it is realized that fin temperature drop along Y-direction as well as Z-direction enhances with the inlet velocity. With increased velocity, heat removal rate from the fin surface increases, which introduces larger temperature gradient in the fin to accommodate higher heat removal rate. Further, decreasing the fin conductivity from 150 W/m-K to 75 W/m-K (i.e., decreasing the fin conductivity by 100%), higher drop in fin temperature is observed in both Y- and Z-direction (Fig. 6.6c-d). Observations on the influence of velocity for lower conductivity is similar to the one obtained for higher conductivity. Evidently, it is highly unexpected to unfold these temperature variations by simple one-dimensional fin theory assuming constant heat transfer coefficient. Similar development of fin temperature distribution is also noted for a fin spacing $S^*=0.5$. Finally, it may also be argued that the assumption of isothermal fin under such a case is not fully compatible with realistic system and needs greater care. Evidently, fin temperature distribution will influence the fluid temperature profile as well as bulk fluid temperature.
Fig. 6.6. Fin temperature distribution for $\text{Gr} = 4.3 \times 10^5$, $S^* = 0.3$, $C^* = 0.10$, $C_{fi} = 0.10$. (a) $k_{\text{fin}} = 150$ W/m-K, $W_{\text{in,mix}} = 1600$; (b) $k_{\text{fin}} = 150$ W/m-K, $W_{\text{in,mix}} = 2133$; (c) $k_{\text{fin}} = 75$ W/m-K, $W_{\text{in,mix}} = 1600$; (d) $k_{\text{fin}} = 75$ W/m-K, $W_{\text{in,mix}} = 2133$. 
6.5.4.2 Temperature profile

Developments of temperature profile over the cross-section are shown in Fig. 6.7a-d for $S^* = 0.3$. Temperature profile, presented here, are for the same axial location as those used for $W$-velocity. Temperature profile near the inlet is very nearly rectangular (Fig. 6.7a). Similar to $W$-velocity profile, temperature profile near the entrance is unaffected by the insertion of protrusion. Even before the vortex generator, temperature profile (Fig. 6.7b) is very much similar to the case of without protrusion. Fluid in the clearance zone is hardly participated in

Fig. 6.7 Axial development of dimensionless temperature profile across the section for $k_{fin} = 150 \text{ W/m-K}$, $W_{in,mix} = 1600$, $Gr = 4.3 \times 10^5$, $S^* = 0.3$, $C^* = 0.10$, $C_1^* = 0.10$. (a) $Z = 0.405$; (b) $Z = 3.592$; (c) $Z = 6.520$; (d) $Z = 12.346$. 
the heat transport process before the protrusion. As fluid moves down-stream, temperature variation assumes almost parabolic profile along $X$-direction, while in $Y$-direction it does not follow any regular trend possibly due to the presence of cross-stream mixing of fluid (Fig. 6.7). However, there exists a sharp fall of temperature close to the base wall along $Y$-direction. Moreover, it may be noted that fluid near the central region, especially over vortex generator, remains relatively cold is presumably caused by the presence of cold standing vortices. Fluid in the clearance region also remains relatively cold and its value rises near the exit section (Fig. 6.7d).

### 6.5.4.3 Bulk temperature

The dimensionless bulk temperature ($\theta_b$) of the fluid at any $Z$ is defined as:

$$\theta_b = \frac{T_b - T_0}{T_w - T_0},$$  \hspace{1cm} (6.14)

where, $T_b - T_0 = \int_0^S \int_0^{H+C} \rho c_p (T - T_0) w dx dy / \int_0^S \int_0^{H+C} \rho c_p w dx dy$. Axial development of bulk fluid temperature exhibits in Fig. 6.8a-b. An interesting phenomenon is revealed in the vicinity of the vortex generator, where bulk temperature shows a decrease indicating cooling of fluid in place of heating. At first glance, it may seem impossible to occur such situation. But closer look of fin surface temperature profile reveals that fin temperature varying sinusoidally along $Z$-direction in vicinity of vortex generator. This means that as fluid moves downstream, it encounters relatively colder temperature and thereby upstream hot fluid releases heat to fin instead of getting heated. The same incidence is repeated in the vicinity of second vortex generator. The very entrance region remains unaffected by such incidence. Inspite of reverse flow of heat from fluid to fin, bulk temperature with vortex generator shows higher value at the exit section as compared to without vortex generator. Since, bulk temperature is a direct indicator of total heat transport from the base-fin system, thus heat
Fig. 6.8. Axial development of dimensionless bulk fluid temperature for $Gr=4.3\times10^5$, $C_1^*=0.1$. (a) $k_{\text{fin}}=150$ W/m-K, $S^*=0.3$, $C^*=0.10$; (b) $W_{\text{in,mix}}=2133$, $k_{\text{fin}}=75$ W/m-K.
transport is higher with the inclusion of vortex generator. The higher the inlet velocity, the lower the bulk temperature and it is expected, since higher inlet velocity is accompanied by higher mass flow rate/higher heat capacity of fluid. A comparison of Fig. 6.8a with Fig. 6.8b reveals that the higher the fin conductivity the higher the bulk fluid temperature corroborating the work of Das and Giri (2014) on non-isothermal fin array. Increase of fin spacing reduces the bulk fluid temperature indicating higher mass flow rate through the section.

6.5.5. Local Nusselt number

It is always interesting to understand the local heat transfer coefficient, since this may provide desired information to improve the performance and design of heat transfer device. To evaluate the local heat transfer coefficient, it is presumed that heat transport is being made due to the difference of the base temperature and the bulk fluid temperature, since it is the true driving force. Thus at any axial Z-location, total local heat transfer \( Q_{t,z} \), local average heat flux \( q_{av,z} \) and local average convective heat transfer coefficient \( h_{av,z} \) can be expressed as:

\[
Q_{t,z} = Q_{fin,z} + Q_{base,z} = \left( -\int_0^H k \frac{\partial T}{\partial x} \bigg|_{x=0} \, dy + \int_0^H k \frac{\partial T}{\partial x} \bigg|_{x=s} \, dy \right) + \left( -\int_0^S \frac{\partial T}{\partial y} \bigg|_{y=0} \, dx \right)
\]

\( (6.15) \)

\[
q_{av,z} = \frac{Q_{t,z}}{2H + S} ; \quad h = \frac{q_{av,z}}{(T_w - T_b)}
\]

\( (6.16) \)

Therefore, non-dimensional local Nusselt number at any axial Z location may be expressed as:

\[
Nu_z = \frac{h \frac{H}{k}}{2 + \frac{S}{H}} = \frac{1}{(1 - \theta_b)} \left[ -\int_0^1 \frac{\partial \theta}{\partial X} \bigg|_{X=0} \, dY + \int_0^1 \frac{\partial \theta}{\partial X} \bigg|_{X=S} \, dY - \int_0^S \frac{\partial \theta}{\partial Y} \bigg|_{Y=0} \, dX \right].
\]

\( (6.17) \)

Axial variation of local Nusselt number is plotted in Fig. 6.9a-f. Local Nusselt number decreases monotonically along axial direction near the entrance, which resembles heat
transfer coefficient in the entrance region of a duct. Thereafter, local Nusselt number in the vicinity of vortex generator shows peak, which is arguably due to the presence of vortex generator that enhances the axial velocity near its location (Fig. 6.5). As revealed in Fig. 6.9a-d, clearance plays a significant role on the development of local Nusselt number. There exists an increase of local Nusselt number with the increase in clearance till the value of clearance reaches close to 0.1 (solid line in Fig. 6.9a-d). Above the aforesaid value of clearance, local Nusselt number decrease with the clearance. Thus, the clearance close to 0.1 is turned out to be the superior and this will be clearly evident in the results of overall Nusselt number. However, local Nusselt number, in the absence of vortex generator, follows monotonic decrease which is identified in Fig. 6.9a-d by the star symbol for clearance spacing $C^*=0.1$. Therefore, enhancement of heat transfer is evident in Fig. 6.9a-d in the presence of vortex generator. The figure unveils further that, in the vicinity of vortex generator, local Nusselt number enhancement is predicted to be as high as 200% as compared to without vortex generator. This enhancement is of special interest, since any cheap insulating material can be used as a vortex generator which can enhance the heat transport locally as well as globally.

Influence of inlet velocity is identified by comparing Fig. 6.9a with Fig. 6.9b. The higher the inlet velocity, the higher is the local Nusselt number, which is very much in agreement with the reported result. Moreover, local Nusselt number with vortex generator is always higher than that of without vortex generator even at the exit for the same geometry ($C^*=0.1$).

The effect of fin conductivity is realized in Fig. 6.9e-f. Near the entrance, lower fin conductivity (i.e, $k_{fin} = 75$ W/m-K) coupled with lower Grashof number (Fig. 6.9e) indicates slower axial development of heat transfer as compared to the case of higher fin conductivity ($K_{fin}=150$ W/m-K) coupled with higher Grashof number (Fig. 6.9d). This slower development of heat transfer for the case of lower fin conductivity is compensated after the encounter of fluid with the vortex generator, while that for the case of higher fin conductivity is relatively
Fig. 6.9. Axial variation of local Nusselt number $S^* = 0.3$. (a) $k_{fin} = 150$ W/m-K, $W_{in,mix} = 1280$, Gr = $4.3 \times 10^5$; (b) $k_{fin} = 150$ W/m-K, $W_{in,mix} = 1920$, Gr = $4.3 \times 10^5$; (c) $k_{fin} = 150$ W/m-K, $W_{in,mix} = 1600$, Gr = $8.4 \times 10^5$; (d) $k_{fin} = 150$ W/m-K, $W_{in,mix} = 2133$, Gr = $8.4 \times 10^5$; (e) $k_{fin} = 75$ W/m-K, $W_{in,mix} = 2133$, Gr = $4.3 \times 10^5$, $C^* = 0.10$; (f) $k_{fin} = 75$ W/m-K, $W_{in,mix} = 2667$, Gr = $8.4 \times 10^5$. 
mitigated. This behavior can be understood from the fact that heat absorbing capacity of fluid for the higher fin conductivity is reduced after the higher rate of heat transfer near the entrance. On the other hand, relatively lower rate of heat transfer near the entrance from lower fin conductivity increases the possibility of much higher rate of heat transfer in the vicinity of protrusion. Further, it is observed that truncating the heated section immediately after the second vortex generator (i.e., at $Z = 8.75$) by adding insulating material till end of duct length (i.e., $Z = 12.5$), local Nusselt number curve remains unchanged. Overall Nusselt number, which will be presented in next section, is found to increase as high as 20% with the case of truncated duct length. Without truncating the duct length, enhancement of overall Nusselt number with vortex generator is 16% (circle and solid line in Fig. 6.9e represents truncated and without truncated case respectively) as compared to without vortex generator. Earlier Biswas and Chattopadhyay (1992) observed a maximum enhancement of 34% in local Nusselt number with the use of winglet type vortex generator even at a channel of length $Z = 8.5$ (non-dimensional). In the present case, axial length is normalized with the duct height (i.e., larger dimension of the domain cross-section) not with the duct width, while Biswas and Chattopadhyay (1992) did otherwise.

6.5.6 Overall Nusselt number

It is always desirable to know the average Nusselt number of the base-fin system for the practitioner. The overall Nusselt number, which is based on the temperature difference between the base-fin and the environment, is defined as follow:

$$Nu = \frac{1}{(2 + S')L} \int_0^{L'} \left[ -\int_0^L \frac{\partial \theta}{\partial X} \bigg|_{X=0} dY + \int_0^L \frac{\partial \theta}{\partial X} \bigg|_{X=-S'} dY - \int_0^{S'} \frac{\partial \theta}{\partial Y} \bigg|_{Y=0} dX \right] dZ. \quad (6.18)$$

Variation of overall Nusselt number at different clearance spacing is shown in Fig. 6.10a-b for fin conductivity of $k_{\text{fin}} = 150$ W/m-K. As revealed from the figure, the value of overall
Fig. 6.10. Variation of Overall Nusselt number with clearance spacing. (a) $k_f m=150 \text{ W/m-K, } Gr=4.3\times10^5, \gamma=1.10$; (b) $k_f m=150 \text{ W/m-K, } Gr=8.4\times10^5, \gamma=1.10$. 
Nusselt number for higher Reynolds number increases with the clearance till a peak value is attained and immediately thereafter it decreases with clearances. Increase in heat transfer with clearance is presumably due to larger mass flow associated with higher clearance. But, increasing clearance beyond the limiting value causes significant flow by-pass inducing lower heat transfer. Similar observations are also made by Giri and Das (2012) in the study of entry region mixed convective heat transfer from isothermal fin. This observation was obscured by Zhang and Patankar (1984) and Al-Sarkhi et al. (2003), since the clearance value chosen by the authors all had values far away from zero clearance. Therefore, their results were not amenable to capture the present findings. Fin heat transfer associated with $S^* = 0.3$ is turned out to be higher than $S^* = 0.5$ for both the Grashof numbers. This optimum clearance is turned out to be 0.05 at higher inlet velocity.

Overall Nusselt number for a case of fin conductivity, $k_{fin} = 75\ W/m-K$ is presented in Fig. 6.11, wherein the results of $S^* = 0.2$ is compared with $S^* = 0.3$. Clearly superior heat transport performance is evident for $S^* = 0.2$ at higher Grashof number. As fin spacing, $S^* = 0.2$ introduces additional surface area than the other higher fin spacing, therefore it could be a choice for a designer especially at higher Grashof number. However at the lower Grashof number, heat transfer performance of $S^* = 0.3$ is marginally higher than $S^* = 0.2$. As said above, $S^* = 0.2$ provides much higher surface area than $S^* = 0.3$, thus $S^* = 0.2$ conclusively indicates to be the preferable choice. Having observed the choice of a designer, it is always encouraging to examine the case of without vortex generator. A comparison of results with and without protrusion (Fig. 6.11) indicates maximum overall heat transfer enhancement with vortex generator is turned out to be as good as 19%. Das and Giri (2014) observed 18% decrease in heat transfer for the case of heat transfer from non-isothermal fin compared to isothermal fin Giri and Das (2012). Use of vortex generator may compensate this decrease and get the performance very close to isothermal fin heat transfer without vortex generator.
Finally overall Nusselt number is correlated with the governing parameters of the problem. Fig. 6.12a shows such correlation. Following correlation is found to relate 64 data point obtained from the computation:

\[
\frac{Nu}{\Omega^{0.104}} = 0.57 \times Re_m + 0.91
\]  

(6.19)

where, \( Re_m = L^{0.43} \times S^{0.069} \times (0.5 + C^*)^{0.021} \times Gr^{0.14} \times Re^{0.264} \). The parametric range of the correlation is: \( 26 \leq \Omega \leq 65, \ 4.3 \times 10^5 \leq Gr \leq 8.4 \times 10^5, \ 363 \leq Re \leq 1974, \ 0.2 \leq S^* \leq 0.5, \ 0 \leq C^* \leq 0.25, \ 10 \leq L^* \leq 12.5 \). Parity plot for computed and correlated Nusselt number are shown in Fig. 12b to find the deviation. Results found to fall within ±10% error band. Correlation coefficient is turned out to be 0.999, while the standard deviation of \( Nu/\Omega^{0.104} \) is 0.211.
Fig. 6.12. (a) Plot of correlation of overall Nusselt number; (b) Parity plot of computed and correlated Nusselt number.
6.6 Conclusions

A numerical simulation is made to perform mixed convection heat transfer from a shrouded vertical non-isothermal fin array on vertical base in the presence of vortex generator attached to a shroud. Convective medium (i.e., air) is treated as the non-Boussinésq fluid with property variation. Vortex generator causes the pressure drop across the duct length to rise to a value of 150% as compared to without vortex generator. This rise of pressure drop is compensated with the enhancement of heat transfer. Decoupling of mixed convection velocity into forced and natural velocity components is done via natural convection computation. Presence of vortex generator accelerates the fluid velocity, which in turn augments the heat transfer both locally and globally. Fin temperature distribution shows near sinusoidal variation in the vicinity of vortex generator along the axial direction, which is hardly revealed by the conventional one-dimensional conduction theory involving constant heat transfer coefficient. Local Nusselt number decreases monotonically with the axial direction near the inlet, which resembles the result of existing literature in the ducted flows. But the interference of vortex generator causes local Nusselt number to rise to a value as high as 200% near the vortex generator.

Overall Nusselt number shows clear optimum with the clearance especially for higher inlet velocity for the range of parameters considered. This optimum dimensionless clearance is predicted to be 0.05 at higher inlet velocity. Dimensionless fin spacing of 0.2 with lower fin conductivity shows better performance than the other higher fin spacing for higher Grashof number at higher inlet velocity. Overall Nusselt number with vortex generator is augmented to a maximum value as high as 20% as compared to without vortex generator. Finally overall Nusselt number is correlated with the governing parameters of the problem.