Laminar Mixed Convective Heat Transfer from Isothermal Vertical Fin Array

3.1 Introduction

Rapid heat removal from heat generating components is essential for the safe operation of equipment such as gas cooled nuclear reactors, automobile and aerospace vehicles, cooling of transformer and electronic systems. On the other hand, rapid collection of heat by solar heater makes the system more effective. With the development of advanced technology the demand for miniaturization of engineering components is increasing day by day. The miniaturization of components and higher packaging densities however invariably result in higher heat fluxes. Thus, the components are elevated to higher temperature, thereby degrading the performance and reliability of such equipment. Hence there is an ever increasing need for the rapid and reliable heat removal for the safe operation of the components. In many cases, heat removal from the equipment is accomplished by blowing air at moderate velocities. This results in laminar convection involving low values of heat transfer coefficients. In such situations, a common practice is to add fins to augment the heat transfer, since fins increase the available surface area. Applicability of natural convection heat transfer is restricted due to the paucity of heat transfer, while the forced convection provides some resort over natural convection. Again, natural convection is inherent in all the systems. Therefore, studies of combined mixed convection are relevant and justified.

Chapter-2 identifies that Acharya and Patankar (1981), Maughan and Incropera (1990a, 1990b) examined mixed convection from a shrouded vertical fin array on a horizontal base. Recently, Dogan and Sivrioglu (2009) reported combined free and forced convection from the same configuration as used by Acharya and Patankar (1981), Maughan and Incropera
(1990a, 1990b). A configuration, which provides better heat transfer, is the vertical fin array on a vertical base is attempted by Zhang and Patankar (1984) and Al-Sarkhi et al. (2003). However, their studies were limited to fully-developed conditions, which find application only to those cases, in which fin lengths are large since fully-developed conditions acknowledge only the large lengths. Therefore, there was an essence to study the entry region mixed convection to support the case of smaller fin length. Further, it may be noted that in most of the earlier investigations on mixed convection over vertical channel, Reynolds number being one of the most important governing parameters is evaluated based on the inlet velocity. Inlet velocity may be decoupled into two components for vertical mixed convection flow, i.e., (i) the fan velocity and (ii) the induced velocity resulting from buoyancy. In the analysis, suitable means should be identified to find out two velocities. Induced velocity component arising out of buoyancy contribute nominally to the total pressure drop across the ends of the duct. Therefore, the decoupling will help in estimating the exact fan velocity, which is necessary to evaluate the fan power. Further, the higher the natural convection velocity component the higher the fin length due to chimney effect. Thus, for a fixed inlet velocity, forced convection velocity component decreases with the fin length introducing lower and lower pressure drop across the length of the channel made of base-fin-shroud till zero value is achieved signifying pure natural convection. Thus, vertical length of the duct in mixed convection is an important parameter. So the present observation on friction factor defer from the earlier reports by Zhang and Patankar (1984) and Al-Sarkhi et al. (2003), in which friction factor shows strong dependence on Rayleigh number of the problem indicating a fully-developed value with respect to Reynolds number.

In the above context, present chapter describes the study the entry region mixed convection over shrouded vertical rectangular isothermal fin arrays attached to a vertical base by decoupling the inlet velocity into two components, i.e., the fan velocity and the induced
velocity resulting from buoyancy. Finally, pressure drop across the duct, induced velocity and overall Nusselt number have been correlated with the governing parameters of the problem. Description of the physical model is presented in the following section.

3.2 Physical model and coordinate system

Fig. 3.1 shows the physical model of the problem considered here. Rectangular plate fins having length ‘L’, thickness ‘t’, and height ‘H’ are attached vertically to a rectangular hot vertical base plate. Fins are placed ‘S’ distance apart from each other. A rectangular adiabatic shroud, that maintains a clearance ‘C’ in between the fin-tip and the shroud, is placed in front of the fin-tip. Thus, the assembly of base, shroud and two consecutive fins makes a rectangular duct having dimension $L \times S \times (H+C)$. The origin of the coordinate system lies at the corner of base and fin. In the present problem, $x$-direction is considered along the base and directed towards the symmetry line of the channel cross-section (Fig. 3.1b). The $y$-direction is measured from the base in a direction normal to the base and along fin, whereas $z$-direction is considered vertically upwards. Each duct is symmetric about the plane passing through the midpoint of base and parallel to $y$-$z$ plane. Base plate is elevated to a temperature $T_w$, which is higher than the ambient temperature $T_0$. This induces an upward flow through the ducted fin array due to natural convection. To augment the heat transfer, forced flow is added to the induced motion. Thus the present case considers the study of mixed convection heat transfer over the base-fin-shroud assembly. Further, it is assumed that base and fin are made of same material. Contact resistance between base and fin are assumed negligible. The fin is assumed to be at the same temperature as the base plate. Under such a situation, there is an induced upward flow due to natural convection along the fin length. In addition a forced flow is induced by a fan over the natural convection flow to enhance the heat transfer. This makes the present case a mixed convection flow through the shrouded vertical isothermal fin array. In view of the symmetry of the problem only half cross-sectional area of the duct (shaded
area in Fig. 3.1b) is considered for the present investigation. Therefore, present chapter is devoted to ideal isothermal fin, which will definitely provide an upper limit heat transfer.

Fig. 3.1. (a) Vertical rectangular fin array attached to a vertical base plate with a vertical shroud placed near the fin-tip; (b) Cross-section of one of the flow passages and shaded area is the computational domain.
3.3 Mathematical formulation

3.3.1 Governing equations

The physical problem described above is governed by the equations of mass, momentum and energy. Oberbeck-Boussinésq approximation is invoked in the momentum equations, i.e., the density is constant in all the terms of the momentum equations except in the buoyancy term. The density variation in the buoyancy term of the momentum equation is taken into account as the problem considered is laminar mixed convection. Density variation is assumed primarily due to temperature difference alone. By Taylor’s series expansion, density can be expressed as

\[ \rho = \rho_0 + \frac{\partial \rho}{\partial T} \bigg|_{T_0} \Delta T + \text{higher order terms} \]  

Neglecting higher order terms, density variation in the buoyancy term is expressed in z-momentum equation as follow:

\[ \rho = \rho_0 \left[ 1 - \beta \left( T - T_0 \right) \right], \]  

where, \( \beta = -\frac{1}{\rho_0} \left( \frac{\partial \rho}{\partial T} \right) \bigg|_{T_0} \). With the z-axis of the geometry aligned parallel to the gravity vector, the total pressure defect \( p \) (i.e., the excess of the hydrostatic pressure \( p_o \) over the static pressure \( p_s \)) may be expressed as the sum of an average pressure defect, \( \bar{p} = \int_0^{H} \int_0^{0.5S} p(x, y, z) dxdy / A_c \) which varies with \( z \) and a spacewise varying pressure \( p_f \) which is a function of \( x, y, \) and \( z \). However, for the parabolic flow situations such as the present one, \( d\bar{p} / dz \) is usually much larger than \( \partial p_f / \partial z \), so that the \( z \)-directional dependence of the cross-stream pressure \( p_f \) can be neglected. Thus:

\[ p(x, y, z) = \bar{p}(z) + p_f(x, y, z) \]  

(3.3)
The effect of radiation and viscous dissipation is considered negligible. Dogan and Sivrioglu (2010) indicate that radiative transport never exceeds 6% of the total heat transport. Further, negligible diffusion is considered along the stream-wise directions of the momentum and energy equation.

Continuity equation:
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \tag{3.4}
\]

Momentum equation in x-direction
\[
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} + \rho w \frac{\partial u}{\partial z} = -\frac{\partial p_f}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right). \tag{3.5}
\]

Momentum equation in y-direction
\[
\rho u \frac{\partial v}{\partial x} + \rho v \frac{\partial v}{\partial y} + \rho w \frac{\partial v}{\partial z} = -\frac{\partial p_f}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right). \tag{3.6}
\]

Momentum equation in z-direction
\[
\rho u \frac{\partial w}{\partial x} + \rho v \frac{\partial w}{\partial y} + \rho w \frac{\partial w}{\partial z} = \left( \frac{\partial p}{\partial z} + \rho g \right) + \mu \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) + \left( \rho_0 - \rho \right)g. \tag{3.7}
\]

Energy Equation
\[
\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right). \tag{3.8}
\]

3.3.2 Non-dimensionalization

In order to reduce the number of parameters involved in the problem and to obtain solutions which are applicable to a wide range of parameter values, the governing equations are non-dimensionalized with the following scheme.
Inserting the above scheme of non-dimensionlization, governing equations in dimensionless form can be cast as:

**Continuity Equation:**

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} = 0
\]  

(3.15)

**X-momentum Equation:**

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} + W \frac{\partial U}{\partial Z} = -\frac{\partial P}{\partial X} + \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)
\]  

(3.16)

**Y-momentum Equation:**

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} + W \frac{\partial V}{\partial Z} = -\frac{\partial P}{\partial Y} + \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right)
\]  

(3.17)

**Z-momentum Equation:**

\[
U \frac{\partial W}{\partial X} + V \frac{\partial W}{\partial Y} + W \frac{\partial W}{\partial Z} = \frac{\partial P^*}{\partial Z} + Gr \theta + \left( \frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2} \right)
\]  

(3.18)
Energy Equation:

\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} + W \frac{\partial \theta}{\partial Z} = \frac{1}{Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)
\]  

(3.19)

Following Bhoite et al. (2005), dimensionless Z-directional inlet velocity is assumed as a summation of natural and forced convection velocity components. In other words, \( W_{in} \) can be considered as: \( W_{in,mix} = W_{in,nc} + \text{Re} \), where Re is the Reynolds number and is evaluated on the basis of fan velocity component. In view of the above assumption, Re is zero in the case of pure natural convection and hence, \( W_{in} \) becomes \( W_{in,pnc} \). In this connection, it is appropriate to mention here that if Re is estimated on the basis of total inlet velocity it would be nonzero even in the case of pure natural convection and hence identification of different regimes would be difficult. Alternatively, to identify the velocity components of free and forced convection some assumptions are made as follows:

In a situation of pure natural convection, the flow is established entirely by buoyancy with a net zero pressure drop between the duct outlet and the duct inlet. A situation similar to the natural convection is assumed to prevail in the duct for the natural convection velocity component in the present mixed convection study. Therefore, the pressure defect that is being observed is purely due to the component of forced convection. To solve the present problem, first pure forced convection problem with \( W_{in,force} = \text{Re} \), is computed with the different values of Re. The problem of mixed convection is then computed for specified inlet velocity, Grashof number and other parameters of the problem, and the pressure drop is thus determined. Since by assumption the pressure drop of forced and mixed convection are same for a given value of Reynolds number, the value of Reynolds number in the case of mixed convection can be evaluated from the analysis of forced convection. The quantity \( W_{in,nc} \) is then calculated by subtraction, i.e., \( W_{in,nc} = W_{in,mix} - \text{Re} \). Alternatively, \( W_{in,nc} \), resulting in zero
pressure drop, can be termed as the additional contribution to the inlet fan velocity, Re, resulting from the pressure drop.

In most of the existing literature on the problem of mixed convection, Re is calculated on the basis of inlet velocity only. However, in practical situation inlet velocity is the sum of Re and $W_{in,nc}$. Since the pressure drop versus fan velocity characteristic requires only Re, not $W_{in,nc}$, and hence the selection of fan velocity in a practical situation becomes very difficult. Further, the quantity Re, which is a forced convection parameter, will have a non-zero value even under the case of pure natural convection, as $W_{in}$ remains finite. Hence, the decomposition of inlet velocity, as outlined in the present case, seems to be relevant for practical applications.

### 3.3.3 Inlet and Boundary conditions

Eqs.(3.15)-(3.19) are parabolic in nature. The required inlet conditions are stated below:

for $Z=0$ and $0<X<0.5S^*$ and $0<Y<(1+C^*)$

$$U = 0, V = 0, W = W_{in}, \text{ and } \theta = 0.$$ (3.20)

Boundary conditions are described as below.

On the symmetry plane (i.e., $X = 0.5S^*, 0 < Y < (1+C^*)$ and $0 < Z \leq L^*$)

$$U = \frac{\partial V}{\partial X} = \frac{\partial W}{\partial X} = \frac{\partial \theta}{\partial X} = 0.$$ (3.21)

On the fin surface (i.e., $0 \leq Y \leq 1, X = 0$ and $0 < Z \leq L^*$)

$$U = V = W = 0, \text{ and } \theta = 1.$$ (3.22)

On the $Y-Z$ plane of the clearance space (i.e., $1 < Y \leq (1+C^*), X = 0$ and $0 < Z \leq L^*$)

$$U = \frac{\partial V}{\partial X} = \frac{\partial W}{\partial X} = \frac{\partial \theta}{\partial X} = 0.$$ (3.23)
At the base surface (i.e., \(Y=0\) and \(0 < X < 0.5S^*\) and \(0 < Z \leq L^*\))

\[
U = V = W = 0, \quad \theta = 1. \quad (3.24)
\]

At the shroud (i.e., \(Y=1+C^*\) and \(0 < X < 0.5S^*\), and \(0 < Z \leq L^*\))

\[
U = V = W = \frac{\partial \theta}{\partial Y} = 0, \quad (3.25)
\]

### 3.3.4 Bulk temperature

The dimensionless bulk temperature \(\theta_b\) of the fluid at any \(Z\) is defined as:

\[
\theta_b = \frac{T_b - T_0}{T_w - T_0}, \quad (3.26)
\]

where,

\[
T_b = \int_0^{0.5S^*} \int_0^{H+C^*} \rho c_p T_w dx dy
\]

\[
\int_0^{0.5S^*} \int_0^{H+C^*} \rho c_p w dx dy.
\]

### 3.3.5 Local Nusselt number

The local Nusselt number \(\text{Nu}_z\) is calculated based on the temperature difference between isothermal base-fin and the bulk temperature of fluid, since it is the true driving force for heat transfer from the base-fin system. Hence local Nusselt number at any \(Z\) location is expressed as

\[
\text{Nu}_z = \frac{1}{(1 + 0.5S^*) (1 - \theta_b)} \left[ \int_0^1 \left. \frac{\partial \theta}{\partial X} \right|_{Y=0} dY - \int_0^{0.5S^*} \left. \frac{\partial \theta}{\partial Y} \right|_{X=0} dX \right]. \quad (3.27)
\]

### 3.3.6 Overall Nusselt number

One of the important aspects of the study of heat transfer is to use the same in practical application. Since it is not possible to estimate the axially varying bulk temperature, thus it becomes difficult for a practitioner to estimate the actual heat transfer rate from the
local Nusselt number. In view of this, the overall Nusselt number $Nu$, which is based on the temperature difference between the base-fin and the environment, is defined as follow:

$$
Nu = \frac{1}{(1 + 0.5S^*)L} \int_0^L \left[ \int_0^{0.5S^*} \frac{\partial \theta}{\partial X} \mid_{y=0} dY - \int_0^{0.5S^*} \frac{\partial \theta}{\partial Y} \mid_{x=0} dX \right] dZ. \quad (3.28)
$$

Eqs. (3.15)-(3.28) constitute the complete mathematical statement of the problem considered. It can be seen from the present problem that it is governed by a number of parameters: Prandtl number ($Pr$), Reynolds number ($Re$), Grashof number ($Gr$), dimensionless inter-fin spacing ($S^*$), axial length ($L^*$) and dimensionless clearance spacing ($C^*$). The method of solving the system of equations and the results of the parametric studies are presented in the following sections.

### 3.4 Computational procedure

The above described problem is solved using well known numerical SIMPLER algorithm elucidated by Patankar (1980). Grid layout of two consecutive axial locations of $Z$ and $Z + \Delta Z$ is shown in Fig. 3.2. At any axial location, $U$ and $V$ velocities are computed at the staggered location, while $W$, $T$ and $P$ are stored at the central location of the grid as depicted in Fig. 3.2. The combined convective and diffusive terms in cross-stream directions (i.e., $X$- and $Y$-directions) are discretised with the power law scheme on a staggered mesh. Since, $Z$-directional diffusion term is negligible in all the momentum and the energy equations, thus only convection terms remains in the equation. Therefore, equations become parabolic in $Z$-direction. Treatment of such parabolic equations is exemplified in Patankar and Spalding (1971). To incorporate the parabolic treatment outlined by Patankar and Spalding (1971), convective terms in $Z$-direction are discretised using backward difference scheme. Rectangular finite difference grids are generated in $X$-, $Y$- and $Z$- coordinate directions. Finer grids are employed near the solid boundaries. To solve the above described problem a
FORTRAN-90 numerical code, which is developed by Giri et al. (2003), is used with suitable modifications.

**Fig. 3.2** Computational Domain of two subsequent axial locations of $Z$ and $Z + \Delta Z$

**Fig. 3.3** Staggered grid arrangement at any axial location
Numerical experiments are conducted to test the grid sensitivity of the results obtained for the present problem. In the cross stream plane (i.e., in the $X$-$Y$ plane), grids of $20\times35$, $25\times40$, and $20\times46$ for $S^* = 0.1$, and $20\times30$, $30\times40$ and $30\times50$ for $S^* = 0.3$, and another grids of $25\times30$, $35\times40$ and $45\times50$ for $S^* = 0.5$ have been used. In $Z$-direction 100 to 150 grids are employed to cover the entire fin length in vertical direction. Grids employed in all three directions increase in geometric progression. Results for the different grids differ within 1.3%. Results of comparison of different grids with different parameters are presented in Table 3.1.

Table 3.1. Comparison of results of various grids for isothermal fin; $T_w = 373$ K, $T_o = 293$ K, $Gr = 4.3 \times 10^5$.

<table>
<thead>
<tr>
<th>$S^* = 0.1, \ C^* = 0.05, \ W_{in, mix} = 1707, \ Re = 1407$</th>
<th>Grid in $X$ &amp; $Y$</th>
<th>$Z = 3.425$</th>
<th>$Z = 7.275$</th>
<th>$Z = 14.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Nu_z$</td>
<td>$20 \times 35$</td>
<td>31.06</td>
<td>24.18</td>
<td>17.25</td>
</tr>
<tr>
<td></td>
<td>$25 \times 40$</td>
<td>31.145</td>
<td>24.35</td>
<td>17.43</td>
</tr>
<tr>
<td></td>
<td>$20 \times 46$</td>
<td>31.145</td>
<td>24.352</td>
<td>17.428</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$S^* = 0.3, \ C^* = 0.075, \ W_{in, mix} = 2133, \ Re = 1491$</th>
<th>Grid in $X$ &amp; $Y$</th>
<th>$Z = 2.65$</th>
<th>$Z = 5.375$</th>
<th>$Z = 12.2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Nu_z$</td>
<td>$20 \times 30$</td>
<td>14.334</td>
<td>12.48</td>
<td>11.652</td>
</tr>
<tr>
<td></td>
<td>$30 \times 40$</td>
<td>14.354</td>
<td>12.526</td>
<td>11.688</td>
</tr>
<tr>
<td></td>
<td>$30 \times 50$</td>
<td>14.387</td>
<td>12.554</td>
<td>11.705</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$S^* = 0.5, \ C^* = 0.15, \ W_{in, mix} = 1707, \ Re = 746$</th>
<th>Grid in $X$ &amp; $Y$</th>
<th>$Z = 2.6$</th>
<th>$Z = 5.1$</th>
<th>$Z = 14.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Nu_z$</td>
<td>$25 \times 30$</td>
<td>11.25</td>
<td>9.34</td>
<td>7.75</td>
</tr>
<tr>
<td></td>
<td>$35 \times 40$</td>
<td>11.26</td>
<td>9.35</td>
<td>7.76</td>
</tr>
<tr>
<td></td>
<td>$45 \times 50$</td>
<td>11.27</td>
<td>9.36</td>
<td>7.78</td>
</tr>
</tbody>
</table>
In the present investigation, grids of 20×46 for $S^* = 0.1$, 30×50 for $S^* = 0.3$ and 45×50 for $S^* = 0.5$ is used. To verify the correctness of the results, numerical runs were taken for the laminar forced convection until it reaches fully developed value under same configuration as mentioned above. The obtained values were then compared with the results of Sparrow et al. (1978) in Table 3.2.

Table 3.2. Comparison of present results with the results of Sparrow et al. (1978).

<table>
<thead>
<tr>
<th>$S^*$</th>
<th>$C^*$</th>
<th>$\text{Nu}_x$ (Present) (Isothermal Fin)</th>
<th>$\text{Nu}_x$ (Sparrow et al. (1978)) (Isothermal Fin)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>0</td>
<td>34.27</td>
<td>34.02</td>
</tr>
<tr>
<td>0.25</td>
<td>0.605</td>
<td>0.616</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>0</td>
<td>5.920</td>
<td>5.806</td>
</tr>
<tr>
<td>0.25</td>
<td>4.385</td>
<td>4.448</td>
<td></td>
</tr>
</tbody>
</table>

3.5 Results and discussions

Geometric and physical parameters are chosen as follows: dimensional fin length is varied from 0.48 m to 0.58 m, and dimensional fin height is varied from 0.03 m to 0.05 m. Values of dimensionless inter-fin spacing ($S^*$) is chosen to vary from, $S^* = 0.1$ to 0.5. The dimensionless clearance spacing ($C^*$) is varied from 0 to 0.30 for all the fin spacing. Air is taken as working medium. Values of air properties are taken at the mean value of the base wall temperature and the atmospheric temperature. This results in a consideration of the value of $k = 0.0287$ W/m K, $Pr = 0.7$. The base wall temperature is varied from 70°C to 100°C whereas ambient temperature is maintained at a fixed value of 20°C. In view of the above parameters, Grashof number values of the present problem are found to vary from $1.8 \times 10^5$ to $8.4 \times 10^5$. Non-dimensional inlet velocities ($W_{in, mix}$) for mixed convection are varied from 800 to 3000. This corresponds to a Reynolds number variation from 800 to 3000 for forced
convective flows. To evaluate Reynolds number of fan velocity for mixed convective flows, pressure drop across the duct is first computed by solving Eqs. (3.15)-(3.19) with necessary inlet and boundary conditions mentioned above. Then the pressure drops for forced convection flows are computed keeping the same duct length and same duct cross-section as those for mixed convection for a series of non-dimensional inlet velocities. Then the pressure drop of forced convection is compared with the pressure drop of mixed convection and thus, Reynolds number is determined for the mixed convection flow.

3.5.1 Natural convection induced velocity in the presence of mixed convection

Induced inlet natural convection velocity component \( W_{\text{in,nc}} \) in the presence of mixed convection is plotted in Figs. 3.4-3.6. In Figs. 3.4 and 3.6, dimensional fin length is maintained at a value 0.58 m, whereas in Fig. 3.5 dimensional fin length is maintained at a value of 0.48 m. From these figures, it can be seen that induced inlet natural convection velocity component decreases with the increased fan velocity or Reynolds number. This is possibly due to better and faster heat removal from the base-fin system in the presence of higher fan velocity and subsequent reduction in the buoyancy effect. Further, higher Grashof number results in higher induced inlet velocity, which is purely due to enhanced buoyancy that causes higher Grashof number. Fig. 3.4 shows that the induced velocity for \( S^* = 0.1 \) tends to stabilize at a finite value especially for lower Grashof number. Due to lower Grashof number, induced velocity is lower and stabilized to a fixed value. In all the figures, it is found that clearance spacing does not influence significantly the induced velocity within the ranges of parameters considered presently. Higher induced velocity is observed at higher fin spacing for same Re compared to that of lower fin spacing, since higher fin spacing causes lower and slower heat removal compared to smaller fin spacing. Thus, it enhances the buoyancy effect at larger fin spacing.
Fig. 3.4. Variation of $W_{in,nc}$ with Re at different Grashof numbers ($S^* = 0.1$)

Fig. 3.5. Variation of $W_{in,nc}$ with Re at different Grashof numbers ($S^* = 0.3$)
Fig. 3.6. Variation of $W_{in,nc}$ with Re at different Grashof numbers. ($S^* = 0.5$)

Fig. 3.7. Plot of computed and correlated data for $W_{in,nc}$. 
In obtaining a correlation for the natural convection induced velocity with the parameters of the problem, guidance from Churchill (1974) was taken. Following correlation is found to be suitable for all the data computed presently.

\[
\frac{1}{W_{in,nc}} = \left(0.0331Gr_m^{-0.3966} + 0.0265Gr_m^{0.125}\right)^{2.48},
\]

(3.29)

where \(Gr_m = Gr^{-0.5551} Re^{-0.75} (1 - 2.5C^*)^{0.227} L^{-1.5618} (1 - S^*)^{-2.75}\). The correlation given by Eq. (3.29) is made with 192 data points (Fig. 3.7). The correlation coefficient is evaluated as \([1 - (\text{mean square error/total sum of square of variable})]^{0.5}\) and found to be 0.984. The range of validity of the above correlation is \(10^7 \leq Re \leq 2176, 1.8 \times 10^5 \leq Gr \leq 8.4 \times 10^5, 0.05 \leq C^* \leq 0.3, 9.6 \leq L^* \leq 19.3, \) and \(0.1 \leq S^* \leq 0.5\). The standard deviation of the above correlation is found to be 0.01.

3.5.2 Dimensionless axial pressure

Typical variation of dimensionless axial pressure defect against the dimensionless axial distance for different inlet velocities is plotted in Fig. 3.8a for \(S^* = 0.1, Gr = 4.3 \times 10^5\) and \(C^* = 0.05\), in Fig. 3.8b for \(S^* = 0.3, Gr = 4.3 \times 10^5\) and \(C^* = 0.075\), and in Fig. 3.8c for \(S^* = 0.5, Gr = 4.3 \times 10^5\) and \(C^* = 0.15\) for both the cases of mixed and forced convection. Results reveal that pressure defect for the forced convection is continuously rising as the axial length of the fin increases. This is not surprising as with the increase in axial length fluid has to overcome greater frictional pressure drop. Further, comparing Fig. 3.8a-c, it can be noted that significantly higher drop in pressure defect is observed for smaller fin spacing compared to larger fin spacing. This is obviously due to the higher frictional resistance offered by the duct having smaller fin spacing leading to higher fan power.

On the other hand, in the case of mixed convection, variation of pressure defect deviates away from the corresponding pure forced convection pressure defect variation for
the same inlet velocity (Fig. 3.8a-c). It is true that pressure defect due to forced convection should increase with the increase in fin length. At the same time, with the increase in fin length induced velocity component also increases (Karki & Patankar (1987), Giri et al. (2003)). This results in a decrease of forced convection velocity component. Since by assumption, forced convection velocity component is the cause of the pressure defect across the duct length for both forced and mixed convection, thus mixed convection pressure defect decreases with the fin length as compared to pure forced convection case for same inlet velocity. Hence in mixed convection less fan power will be required for same or more heat transfer (Subsection-3.5.6) from the base-fin system. The above statement is prominent in the case of higher fin spacing \((S^* = 0.5)\) with \(W_{in, mix}=1387\), where pressure defect at the exit is almost zero, signifying the domination of natural convection (Fig. 3.8c). In fact, similar observations will be made for other cases also, but it requires larger fin length. With the increase in fin length, a situation will arise where inlet velocity equals induced velocity. Hence length is an important parameter for the case of mixed convection. Therefore, it pertinent to point out that friction factor with the increase in fin length neither reaches a fully-developed value nor depends on the Rayleigh number of the problem, as evaluated and depicted by the previous investigators (Zhang and Patankar (1984), Al-Sarki et al. (2003)).

The overall pressure difference under forced convection is correlated with Re, \(C^*\), \(L^*\), and \(S^*\) of the problem. The following correlation (Eq. (3.30)) is obtained with 263 data points. The correlation coefficient and standard deviation are found out to be 0.999 and 0.02 respectively. Computed and correlated data are plotted in Fig. 3.9.

\[
\left(\frac{1}{\Delta p^*}\right)^{0.1} = 0.92 \text{Re}^{-0.1180} C^*^{0.0221} L^*^{-0.1417} S^*^{0.1442}.
\]

(3.30)

The above correlation is valid in the range of \(80 \leq \text{Re} \leq 2176\), \(0.05 \leq C^* \leq 0.3\), \(9.6 \leq L^* \leq 19.3\), and \(0.1 \leq S^* \leq 0.5\).
Fig. 3.8. Variation of dimensionless axial pressure defect with the axial length for $Gr = 4.3 \times 10^5$. (a) $S^* = 0.1, C^* = 0.05$; (b) $S^* = 0.3, C^* = 0.075$; (c) $S^* = 0.5, C^* = 0.15$. 
Fig. 3.9. Plot of computed and correlated data for overall pressure difference \((\Delta P^*)\) across the fin length.

3.5.3 W-velocity profiles

\(W\)-velocity profiles for the inter-fin spacing \(S^* = 0.1\), \(Gr = 4.3 \times 10^5\), \(C^* = 0.05\) with inlet velocity \(W_{in,mix} = 1707\), are plotted in Fig. 3.10a-d for the axial planes \(Z = 0.075, 1.007, 3.425\) and 14.45. This corresponds to a forced convection component of inlet velocity, \(Re = 1408\). In view of the symmetry, \(W\)-velocity distribution is shown only for the half cross section of the duct. It can be seen from Fig. 3.10a-d that the \(W\)-velocity variation approaches a parabolic profile in \(X\)-direction in the inter-fin region as the value of \(Z\) increases, the velocity being more or less constant along the fin height at all \(Y\). Additionally, it can be observed that in the clearance space between the fins and the shroud, the velocity profile, which is very nearly rectangular near the inlet assumes roughly a parabolic shape in \(Y\)-
direction as \( Z \) increases. Further, it should be noted from the figures that, as fluid moves towards higher axial location, fluid in the inter-fin spacing is pushed into the clearance region and hence the maximum of \( W \)-velocity occurs in the clearance space instead of the fin region. Therefore, it may be concluded that clearance space causes significant flow by-pass for the case of smaller fin spacing. The present results in the region of fully developed flow agree well with the reported results of Al-Sarkhi et al. (2003).

For higher fin spacing \( (S^* = 0.5) \) with \( Gr = 4.3 \times 10^5 \), \( C^* = 0.15 \) and with same inlet mixed convection velocity as the previous case, the development of \( W \)-velocity profile is shown in Figs. 3.10e-h. This corresponds to a forced convection component of inlet velocity \( Re = 746 \). Near the inlet, the velocity profile is nearly rectangular similar to the velocity profile as in the case of \( S^* = 0.1 \). But as fluid moves in the higher axial location velocity profile in \( X \)-direction yet to achieve fully developed parabolic profile even at the exit. But in the \( Y \)-direction velocity profile over the fin length is more or less constant except near the base very close to the exit plane, where velocity profile is higher than other regions of the duct. This may be due to the buoyancy effect closer to the base of the near exit plane. Simultaneously in the clearance region velocity retards near the exit plane may be because of the combined effect of reduced buoyancy and viscosity. This point will be obvious when we look at the dimensionless temperature profile in the following subsection (Section 3.4.4). Hence, it may be argued that at higher fin spacing \( (S^* = 0.5) \), flow by-pass through the clearance space is not significant compared to fin spacing \( S^* = 0.1 \). This observation is important, since this will have a bearing on heat transfer. Development of \( W \)-Velocity profile for \( S^* = 0.3 \) is similar to the case of \( S^* = 0.5 \). Temperature profile development is discussed in the following section.
Fig. 3.10. Development of $W$-velocity profile for $Gr = 4.3 \times 10^5$ and $W_{in,mix} = 1707$. (a) $S^* = 0.1, C^* = 0.05, Z = 0.075$; (b) $S^* = 0.1, C^* = 0.05, Z = 1.007$; (c) $S^* = 0.1, C^* = 0.05, Z = 3.425$; (d) $S^* = 0.1, C^* = 0.05, Z = 14.45$; (e) $S^* = 0.5, C^* = 0.15, Z = 0.075$; (f) $S^* = 0.5, C^* = 0.15, Z = 1.007$; (g) $S^* = 0.5, C^* = 0.15, Z = 3.425$; (h) $S^* = 0.5, C^* = 0.15, Z = 14.45$. 
3.5.4 Temperature profiles

Three dimensional plots of non-dimensional temperature of air over the inter-fin region are shown in Fig. 3.11a-d for the fin spacing, \( S^* = 0.1 \), \( Gr = 4.3 \times 10^5 \), \( W_{in,mix} = 1707 \). These figures correspond to the same axial location and same parametric values of the \( W \)-velocity plots as shown in Fig. 3.10a-d. It can be clearly identified that the most of the fluid is at zero dimensionless temperature near the entrance, with the thermal penetration effect limited to the thin regions adjacent to the base and fin. Temperature profile reaches fully developed value within the inter-fin region much before the exit plane and even in the clearance space temperature profile reaches also fully developed value near the exit plane. The temperature of the shroud remains nearly zero near the entrance and thereafter it begins to rise. Additionally, it may be noted that at axial location away from the inlet, there is a sharp fall of temperature in the clearance space between the fin-tip and the shroud. A similar development of dimensionless temperature profile is shown in Fig. 3.11e-h for \( S^* = 0.5 \), \( Gr = 4.3 \times 10^5 \) and \( C^* = 0.15 \). In the higher fin spacing, temperature rise is much slower in the clearance space which justifies the conclusion of velocity profile development in the clearance space region made in the previous subsection (Section 3.5.3). It should be noted further that the full development of temperature profile is not achieved for both the fin region and the clearance region even very close to the exit plane for higher fin spacing (i.e., \( S^* = 0.5 \)). Results of temperature profile for \( S^* = 0.3 \) reveals similar trend as the case of \( S^* = 0.5 \).

Influence of Grashof number on temperature profile across the section at different axial location may be seen in Fig. 3.12a-d for \( S^* = 0.1 \). Development is similar to one presented in Fig. 3.11a-d. Near the outlet section (\( Z=11.36 \)), fluid in the clearance region Fig. 3.12d is relatively cold as compared to the case of lower Grashof number in Fig. 3.11d at \( Z=14.45 \). Albeit, axial dimensionless values of \( Z \) in Fig. 3.11d and Fig. 3.12d are different, yet their
Fig. 3.11. Axial development of temperature profile for $Gr = 4.3 \times 10^5$ and $W_{in,mix} = 1707$. (a) $S^* = 0.1$, $C^* = 0.05$, $Z = 0.075$; (b) $S^* = 0.1$, $C^* = 0.05$, $Z = 1.007$; (c) $S^* = 0.1$, $C^* = 0.05$, $Z = 3.425$; (d) $S^* = 0.1$, $C^* = 0.05$, $Z = 14.45$; (e) $S^* = 0.5$, $C^* = 0.15$, $Z = 0.075$; (f) $S^* = 0.5$, $C^* = 0.15$, $Z = 1.007$; (g) $S^* = 0.5$, $C^* = 0.15$, $Z = 3.425$; (h) $S^* = 0.5$, $C^* = 0.15$, $Z = 14.45$. 

55
dimensional values are almost same. Non-dimensional values are different, since fin heights are different. It may be highlighted that temperature profile in the inter-fin region almost reaches the value of base temperature. Therefore, it is unlikely to have efficient heat transfer if the fin length is increased further for a fin geometry with $S^* = 0.1$ even for higher Grashof number.

Fig. 3.12. Axial development of temperature profile for $Gr = 8.4 \times 10^5$, $W_{in,mix}=1733$, $S^* = 0.1$, $C^* = 0.05$. (a) $Z = 0.061$; (b) $Z = 0.51$; (c) $Z = 4.0$; (d) $Z = 11.36$.

### 3.5.5 Bulk Temperature

Axial variation of bulk fluid temperature is shown in Fig. 3.13a-c for a $Gr = 4.3 \times 10^5$. For $S^* = 0.1$, it can be clearly seen in Fig. 3.13a that in general bulk temperature rises almost exponentially along the fin length. This is because the rate of heat transfer near the entrance quickly warms up the fluid reducing its heat absorbing capacity further downstream. Moreover, no significant difference in bulk temperature is observed between the forced and mixed convective heat transfer for the same inlet velocity condition. This is probably due to
Fig. 3.13. Axial variation of bulk temperature for Gr = 4.3x10^5. (a) $S^* = 0.1, C^* = 0.05$; (b) $S^*$ = 0.3, $C^*$ = 0.075; (c) $S^*$ = 0.5, $C^*$ = 0.15.
the faster heat removal near the entrance causing insignificant buoyancy effect, which results in less induced velocity indicating the dominance of forced convection. However for higher fin spacing (i.e., $S^* = 0.3$, and $S^* = 0.5$) bulk temperature rises very sharply near the entrance, thereafter it increases almost linearly (Fig. 3.13b and Fig. 3.13c). Additionally, it can be identified that bulk temperature in the case of mixed convection rises at a faster rate than that of forced convection for the same inlet velocity conditions indicating the significance of buoyancy in the higher fin spacing. Hence, the assumption of only forced convection under mixed convection situation can lead to inaccuracies in the prediction of bulk temperature, especially for higher fin spacing.

Influence of higher Grashof number with the axial variation bulk temperature may be examined in Fig. 3.14a-c after comparing the result presented in Fig. 3.13a-c. In general, the higher the Grashof number, lower is the bulk temperature rise. Similar to $Gr = 4.3 \times 10^5$, axial variation of bulk temperature for $S^* = 0.1$ achieve the base temperature within the fin length chosen in the present study. However, sufficiently away from the entrance, the rate of increase in bulk temperature is significantly low. Practically, this is not the zone of effective heat transport. This behavior can be understood from the variation of fluid temperature profile across the section (Fig. 3.12d). Fluid temperature in the inter-fin region gains a value, which is very close to the base temperature. Thus, heat transport from base-fin system to fluid becomes ineffective, thereby reducing the rate of increase in bulk temperature along the axial length. Further, it is noticed that bulk fluid temperature of mixed convection rises almost at the same rate of forced convective heat transport for the case of $S^* = 0.1$, which is similar to the case of lower Grashof number. For the case of larger fin spacing at higher Grashof number (Fig. 3.14b-c), deviation of axial bulk temperature of mixed convection enhances from that of forced convection possibly due to more flow over base-fin system thereby reducing the flow by-pass. Further, it may be identified that the rate of increase of bulk fluid
Fig. 3.14. Axial variation of bulk temperature for $Gr = 8.4 \times 10^5$. (a) $S^* = 0.1, C^* = 0.05$; (b) $S^* = 0.3, C^* = 0.075$; (c) $S^* = 0.5, C^* = 0.15$. 
temperature sufficiently away from the entrance is much faster in larger fin spacing compared to smaller fin spacing.

### 3.5.6 Local Nusselt number

Variation of local Nusselt number ($\text{Nu}_z$) at different clearance spacing is shown in Fig. 3.15a-c. For $S^* = 0.1$ (Fig. 3.15a), $\text{Nu}_z$ decreases with the increase of clearance spacing and this may be because of flow by-pass through the clearance spacing. This flow bypass is obvious (Fig. 3.10a-d) even at the lowest clearance spacing. These results are in agreement with the earlier investigation of Zhang and Patankar (1984). However, the results of higher fin spacing ($S^* = 0.3$ and $S^* = 0.5$) reveal that the ability to dissipate heat from the base-fin system first increases until the clearance spacing reaches a moderate value, where it attains a peak and thereafter begins to decrease with the increasing clearance space. This increase is about 5-7 % from zero clearance spacing. This optimum clearance spacing is found to be 0.075 for $S^* = 0.3$ and 0.15 for $S^* = 0.5$. From low to optimum clearance spacing, the increase in $\text{Nu}_z$ may be due to increased mass flow rate through the inter-fin space and beyond the optimum clearance spacing, the decrease of $\text{Nu}_z$ is possibly due to greater flow bypass through the clearance. It is shown by previous investigators (Zhang and Patankar (1984), Al-Sarkhi et al. (2003)) that Nusselt number for higher fin spacing decreases continuously with the increase of clearance spacing. Optimum clearance spacing is not revealed by Zhang and Patankar (1984) possibly due to the consideration of larger clearance space in their investigation.

Axial variation of local Nusselt number ($\text{Nu}_z$) for different inlet velocities for both mixed and forced convection are compared in Fig. 3.15d, for $S^* = 0.1$, in Fig. 3.15e for $S^* = 0.3$ and in Fig. 3.15f, for $S^* = 0.5$. From all the figures it can be seen that $\text{Nu}_z$ decreases sharply near the entrance and reaches a fully developed value at some length from the entrance. It is noted from these figures that the values of $\text{Nu}_z$ quantity in the mixed convection
Fig. 3.15. Axial variation of local Nusselt number for \( Gr = 4.3 \times 10^5 \). (a) \( S^* = 0.1, W_{in,mix} = 1707 \); (b) \( S^* = 0.3, W_{in,mix} = 1707 \); (c) \( S^* = 0.5, W_{in,mix} = 1707 \); (d) \( S^* = 0.1, C^* = 0.05 \); (e) \( S^* = 0.3, C^* = 0.075 \); (f) \( S^* = 0.5, C^* = 0.15 \).
is always higher than the forced convection values for same inlet velocities and this is more pronounced in higher fin spacing case. This may be interpreted as the effect of buoyancy, which creates flow redistribution near the solid surfaces, namely the fin and the base surface resulting enhanced heat transfer. Further, near the inlet, the higher values of $\text{Nu}_e$ quantity are observed for higher inlet velocity in both the cases of mixed and forced convection, since higher velocity causes smaller thermal and momentum penetration causing higher heat transfer near the inlet.

### 3.5.7 Overall Nusselt number

Fig. 3.16 shows the variation of $\frac{Gr^{0.1817}}{Nu}$ with $\text{Re}^{0.4639} S^{*0.7072} C^{*0.2537} L^{*-0.8064}$. From the figure, it can be seen that all the data points collapse on to a single curve reasonably well. The computed data is correlated as follow:

$$\frac{Gr^{0.1817}}{Nu} = \left(2.065 \text{Re}_{m}^{-1.15} + 0.227 \text{Re}_{m}^{0.69}\right)^{\frac{1}{n}},$$

(3.31)

where, $\text{Re}_{m} = \text{Re}^{0.4639} S^{*0.7072} C^{*-0.2537} L^{*-0.8064}$ and the value of ‘$n$’ is found out to be 0.92. The correlation is made with 192 data points. Overall correlation coefficient is found out to be 0.994. This correlation is valid in the range of Re number $10^7 \leq \text{Re} \leq 2176$, clearance spacing in the range $0.05 \leq C^{*} \leq 0.3$, fin spacing in the range of $0.1 \leq S^{*} \leq 0.5$, fin length in the range of $9.6 \leq L^{*} \leq 19.3$ and Grashof number in the range of $1.8 \times 10^5 \leq \text{Gr} \leq 8.4 \times 10^5$. The standard deviation of data from the correlation is found out to be 0.196. For the sake of comparison, the experimental results (24 data points with diamond symbol) of Dogan and Sivrioglu (2009, 2010) for dimensionless clearance spacing up to 1 and Rayleigh variation $2 \times 10^7$ to $6 \times 10^8$ are also plotted along with the present correlated data (Fig. 3.16). The experimental results show reasonable agreement with the present correlation even though the range of parameters is different.
3.6 Conclusions

A computational study of laminar mixed convection over shrouded vertical rectangular isothermal fin array attached to a vertical base is performed. The natural convection inlet velocity component in the presence of mixed convection is obtained by subtraction of the forced velocity component from the total inlet velocity. The clearance spacing, fin spacing and Reynolds number are varied from 0 to 0.3, 0.1 to 0.5 and 107 to 2176 respectively. Grashof number is varied in a range from $1.8 \times 10^5$ to $8.4 \times 10^5$ and Prandtl number is kept at 0.7.

Mixed convection pressure defect decreases continuously with the fin length compared to forced convection due to increased induced velocity. It has been observed that smaller fin spacing requires higher fan power compared to higher fin spacing for the same Reynolds number. Local Nusselt number falls down sharply near the entrance along the length of fin and reaches a fully developed value after certain length of the duct. Fully-developed local Nusselt number does not vary with Reynolds number. Further, it may be noted that there
exists an optimum clearance spacing for which fully-developed local Nusselt number shows a
clear maximum for the larger fin spacing. These values of clearance spacing are found to be
0.075 and 0.15 for the fin spacing of 0.3 and 0.5 respectively. Whether, this optimum renders
maximum overall heat transfer, will be identified later in Chapter-5. The maximum
enhancement of optimum local Nusselt number value is around 5-7% as compared to no
clearance spacing for fin-spacing $S^* = 0.3$ and $S^* = 0.5$. In addition, it is observed that local
Nusselt number of fully developed mixed convection is always higher than that of forced
convection for the same inlet velocity. In general, drop in pressure defect for forced
convection, induced velocity for mixed convection and overall Nusselt number for mixed
convection are correlated well with governing parameters of the considered problem.