CHAPTER IV

GOAL PROGRAMMING TECHNIQUES FOR PRODUCTION PLANNING PROBLEM IN INDUSTRY

4.1 INTRODUCTION

The objective of production planning is either to maximize profit or minimize cost and is formulated to a single-objective function in linear programming. Recently, many researchers and practitioners are increasingly aware of presence of multiple objectives in real-life problems (Evans [1993]), (Vincke [1992]). Decision-makers always want to develop a model that can consider real-life situations with multiple objectives. To achieve this, in this paper, a lexicographic (preemptive) goal programming and weighted goal programming techniques are used to determine optimal production plans.

As opposed to linear programming, which directly optimizes objectives, preemptive goal programming is used to manage a set of conflicting objectives by minimizing deviations between the target values and the realized results (Rifai [1994]). The original objectives are re-formulated as a set of constraints with target values and two auxiliary variables. Two auxiliary variables are called positive deviation $d^+$ and negative deviation $d^-$, which represent the distance from this target value. The objective of preemptive goal programming is to minimize the deviations hierarchically so that the goals of primary importance receive first-priority attention;
those of secondary importance receive second-priority attention, and so on and so forth (Lee [1972]). Then, the goals of first-priority are minimized in the first phase. Using the obtained feasible solution result in the phase, the goals of second priority are minimized, and so on. With fast computational growth (Ignizio [1983]), both linear and non-linear goal programming can be solved using well-developed software such as Linear Interactive and Discrete Optimization (LINDO [2011]) or meta-heuristics such as simulated annealing, genetic algorithms, tabu search and so on (Jones and Tamiz [2010]).

To solve multi dimensional planning problems, a flexible and practical methodology, known as goal programming (GP), was developed by Charnes and Cooper [1961]. Leung and Chen [2007] developed a preemptive goal programming model to maximize profit, minimize repairing cost and maximize machine utilization of the Chinese production plant hierarchically. Oliveira et al. [2003] used the multiobjective (weighted) goal programming in planning the farm problem for manage the timber, harvesting of erva-mate leaves, pasture, and tourism. Lee [1972] applied the goal programming approach to production planning and then to aggregate production planning. Ghosh et al. [2005] presents a goal programming technique for nutrient management by determining the optimum fertilizer combination for rice production. Tamiz et al. [1998] has studies the modeling approach of goal programming does not attempt to maximize or minimize the objective function directly as in the case of conventional linear programming. Instead of that the goal
programming (GP) model seeks to minimize the deviations between the desired goals and the actual results to be obtained according to the assigned priorities.

4.2 LEXICOGRAPHIC GOAL PROGRAMMING MODEL

The vast majority of the early goal programming formulations (Lee [1972] and (Romero [1991]) used the lexicographic goal programming (LGP) variant. This is also sometimes termed ‘preemptive’ goal programming in place of lexicographic goal programming. The distinguishing feature of lexicographic goal programming is the existence of a number of priority levels. Each priority level contains a number of unwanted deviations to be minimized.

To formulate a generic lexicographic goal programming algebraically we define the number of priority levels as L with corresponding index \( l = 1, 2, \ldots, L \). Each priority level is now a function of a subset of unwanted deviational variables which we define as \( h_l (\eta, \Gamma) \). This leads to the following formulation:

\[
\text{LEX Min } \alpha = \left[ h_1 (\eta, \Gamma), h_2 (\eta, \Gamma), \ldots, h_L (\eta, \Gamma) \right]
\]  

Subject to:

\[
f_q (x) + n_q - p_q - b_q \quad q = 1, 2, \ldots, Q
\]  

\[
x \in F
\]  

where \( n_q, p_q \geq 0 \)  

\[
q = 1, 2, \ldots, Q
\]
\[ n_q \times p_q = 0 \]  

(5)

where \( F \) is the feasible region made up of points in decision space that satisfy all of the constraints and sign restrictions. Each \( h_l(p, n) \) contains a number of unwanted deviational variables. The exact nature of \( h_l(p, n) \) depends on the nature of the goal programming to be formulated, but if we assume that it is linear and separable then it will assume the form

\[
h_l(p, n) = \sum_{q=1}^{Q} \left( \frac{u^l_q n_q}{k_q} + \frac{v^l_q p_q}{k_q} \right)
\]

(6)

where \( u^l_q \) is the preferential weight associated with the minimization of \( n_q \) in the \( l^{th} \) priority level. \( v^l_q \) is the preferential weight associated with the minimization of \( p_q \) in the \( l^{th} \) priority level. The preferential weights are used to model the relative importance of the minimization of the associated deviational variable to the decision maker. If a deviational variable is not included in a particular priority level then its preferential weight for that priority level is set equal to zero. Deviational variables whose minimization is considered unimportant to the decision maker(s) (e.g. positive deviation from a profit goal) are assigned a preferential weight of zero in every priority level. \( \tilde{n}_q \) is the normalization constant associated with the \( q^{th} \) goal. These constants are necessary in order to scale all the goals onto the same units of measurement. The meaning of the lexicographic minimization of the achievement function is that the minimization of deviational variables placed in a higher priority
level is regarded as infinitely more important than that of deviational variables placed in a lower priority level.

### 4.3 WEIGHTED GOAL PROGRAMMING MODEL

The weighted goal programming variant allows for direct trade-offs between all unwanted deviational variables by placing them in a weighted, normalized single achievement function. Weighted goal programming (WGP) is sometimes termed "non-preemptive" goal programming. If we assume linearity of the achievement function then we can represent the linear weighted goal programming by the following formulation:

\[
\text{Min:} a = \sum_{q=1}^{Q} \left( u_q n_q + v_q p_q \right)
\]

Subject to:

\[
f_q(\bar{x}) + n_q - p_q = b_q \quad q = 1, 2, \ldots, Q
\]

\[\bar{x} \in F\]

where

\[
n_q, p_q \geq 0 \quad q = 1, 2, \ldots, Q
\]

and

\[
n_q \times p_q = 0
\]

with the variable definitions identical to those introduced for the lexicographic goal programming variant, except that the preference weights \(u_q\) and \(v_q\) are no longer indexed by priority level.
4.4 AN ILLUSTRATIVE EXAMPLE OF PRODUCTION PLANNING PROBLEM

The problem proposed by Jones and Tamiz [2010] in which a company produces three types of cups, termed grade A, grade B and grade C. Each grade A cup requires 2 hr of furnace time and 3 hr of finishing labor. Each grade B cup requires 3 hr of furnace time and 5 hr of finishing labor. Each grade C cup requires 4 hr of furnace time and 10 hr of finishing labor. A grade A cup yields a profit of £1.00, a grade B cup a profit of £1.50, and a grade C cup a profit of £2.20. The company currently has 1000 hr of furnace time and 2000 hr of finishing labor per day. They have a high level of demand and therefore they have a strategic aim of increasing production to 1000 cups per day by increasing the level of furnace and finishing hours available.

(i) Formulate a lexicographic goal programming with the following priority structure:

Priority 1: Achieve the strategic aim of 1000 cups per day.

Priority 2: Achieve a profit of at least £1250.

Priority 3: Minimize the extra finishing and furnace hours needed.

Priority 4: Ensure that at least 300 of each type of cup is manufactured.

(ii) Formulate a normalized weighted goal programming with the preference weights given below in Table 4.1:
<table>
<thead>
<tr>
<th>S. No.</th>
<th>Goal</th>
<th>Preference Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Achieve the strategic aim of 1000 cups per day</td>
<td>5.0</td>
</tr>
<tr>
<td>2.</td>
<td>Achieve a profit of at least £1250</td>
<td>3.0</td>
</tr>
<tr>
<td>3.</td>
<td>Minimize the extra finishing and furnace hours needed</td>
<td>2.0</td>
</tr>
<tr>
<td>4.</td>
<td>Ensure that at least 300 of each type of cup is manufactured</td>
<td>1.0</td>
</tr>
</tbody>
</table>

### 4.5 FORMULATION OF THE PROBLEM IN GOAL PROGRAMMING FORMAT

Let

\[ x_1 = \text{Number of cups of grade A produced per day} \]

\[ x_2 = \text{Number of cups of grade B produced per day} \]

\[ x_3 = \text{Number of cups of grade C produced per day} \]

\[ n_1 = \text{Underachievement of profit of at least £1250} \]

\[ p_1 = \text{Overachievement of profit of at least £1250} \]

\[ n_2 = \text{Underachievement of furnace time} \]

\[ p_2 = \text{Overachievement of furnace time} \]

\[ n_3 = \text{Underachievement of finishing time} \]
\[ p_3 = \text{Overachievement of finishing time} \]

\[ n_4 = \text{Underachievement of strategic aim of manufacturing 1000 cups / day} \]

\[ p_4 = \text{Overachievement of strategic aim of manufacturing 1000 cups / day} \]

\[ n_5 = \text{Underachievement of manufacturing grade A cup} \]

\[ p_5 = \text{Overachievement of manufacturing grade A cup} \]

\[ n_6 = \text{Underachievement of manufacturing grade B cup} \]

\[ p_6 = \text{Overachievement of manufacturing grade B cup} \]

\[ n_7 = \text{Underachievement of manufacturing grade C cup} \]

\[ p_7 = \text{Overachievement of manufacturing grade C cup} \]

Let us now formulate the above production planning problem as a goal programming model.

(i) **Strategic aim of the company:**

The company has the strategic aim of achieving 1000 cups per day.

\[ x_1 + x_2 + x_3 + n_4 \quad p_4 = 1000 \quad (12) \]

(ii) **Profit goal:**

The company desires to achieve a profit of at least £1250

\[ x_1 + 1.5 x_2 + 2.2 x_3 + n_1 - p_1 = 1250 \quad (13) \]

(iii) **Limit on finishing and furnace hours needed:**
The company wants to limit the finishing and furnace hours to 1000 hours of furnace time and 2000 hours of finishing labor per day.

\[
2 x_1 + 3 x_2 + 4 x_3 + n_2 \quad p_2 = 1000 
\] (14)

\[
3 x_1 + 5 x_2 + 10 x_3 + n_3 \quad p_3 = 2000 
\] (15)

(iv) **Cups manufactured:**

Company desires to manufacture at least 300 of each type of cup.

\[
x_1 + n_5 \quad p_5 = 300 
\] (16)

\[
x_2 + n_6 \quad p_6 = 300 
\] (17)

\[
x_3 + n_7 \quad p_7 = 300 
\] (18)

Lexicographic goal programming problem is formulated as:

\[
LexMin: \quad a = 5P_1n_4 + 5P_1p_4 + 3P_2n_1 + 2P_3p_2 + 2P_3p_3 + P_4 (n_5 + n_6 + n_7) 
\] (19)

Subject to:

\[
x_1 + x_2 + x_3 + n_4 \quad p_4 = 1000 
\] (20)

\[
x_1 + 1.5 x_2 + 2.2 x_3 + n_1 \quad p_1 = 1250 
\] (21)

\[
2 x_1 + 3 x_2 + 4 x_3 + n_2 \quad p_2 = 1000 
\] (22)

\[
3 x_1 + 5 x_2 + 10 x_3 + n_3 \quad p_3 = 2000 
\] (23)

\[
x_1 + n_5 \quad p_5 = 300 
\] (24)
\[ x_2 + n_6 \quad p_6 = 300 \]  \hspace{1cm} (25) \\
\[ x_3 + n_7 - p_7 = 300 \]  \hspace{1cm} (26) \\
\[ x_i, n_j, p_j \geq 0 \quad (i = 1, 2, 3 ; j = 1, 2,...,7) \]  \hspace{1cm} (27) 

By using normalization, objective function becomes:

\[ \text{LexMin: } a = \frac{5P_1 n_1}{1000} + \frac{5P_2 n_2}{1000} + \frac{3P_3 n_3}{1250} + \frac{2P_4 n_4}{2000} + \frac{P_5 n_5 + n_6 + n_7}{300} \]  \hspace{1cm} (28) 

Subject to:

\[ x_1 + x_2 + x_3 + n_4 \quad p_4 = 1000 \]  \hspace{1cm} (29) \\
\[ x_1 + 1.5 x_2 + 2.2 x_3 + n_1 - p_1 = 1250 \]  \hspace{1cm} (30) \\
\[ 2 x_1 + 3 x_2 + 4 x_3 + n_2 \quad p_2 = 1000 \]  \hspace{1cm} (31) \\
\[ 3 x_1 + 5 x_2 + 10 x_3 + n_3 - p_3 = 2000 \]  \hspace{1cm} (32) \\
\[ x_1 + n_5 \quad p_5 = 300 \]  \hspace{1cm} (33) \\
\[ x_2 + n_6 - p_6 = 300 \]  \hspace{1cm} (34) \\
\[ x_3 + n_7 - p_7 = 300 \]  \hspace{1cm} (35) \\
\[ x_i, n_j, p_j \geq 0 \quad (i = 1, 2, 3 ; j = 1, 2,...,7) \]  \hspace{1cm} (36) 

4.6 **COMPUTATIONAL RESULTS OF LGP MODEL**

Using the data presented, the proposed lexicographic goal programming model is tested using LINDO [2011] software package with priority \( P_1, P_2, P_3, P_4 \) and the results are shown in Table 4.2:
\[ x_1 = 500, x_2 = 500, x_3 = 0 \]  
\[ n_1 = 0, p_1 = 0 \]  
\[ n_2 = 0, p_2 = 1500 \]  
\[ n_3 = 0, p_3 = 2000 \]  
\[ n_4 = 0, p_4 = 0 \]  
\[ n_5 = 0, p_5 = 200 \]  
\[ n_6 = 0, p_6 = 200 \]  
\[ n_7 = 300, p_7 = 0. \]  

Table 4.2: Interpretation of the Results

<table>
<thead>
<tr>
<th>Goal</th>
<th>Description</th>
<th>Target Level</th>
<th>Satisfied</th>
<th>Achieved Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Strategic aim</td>
<td>1000</td>
<td>Yes</td>
<td>1000 ((x_1 + x_2 + x_3))</td>
</tr>
<tr>
<td>2</td>
<td>Profit</td>
<td>1250</td>
<td>Yes</td>
<td>1250 ((n_1 = p_1 = 0))</td>
</tr>
</tbody>
</table>
| 3    | Minimize extra finishing and furnace hours | (i) 1000  
(ii) 2000 | No        | 2500  
4000 |
| 4    | No. of cups manufactured | A 300  
B 300 | No        | 500  
500  
0 |
Weighted goal programming problem is formulated as:

\[ \text{Min: } a = 3 n_1 + 2 p_2 + 2 p_3 + 5 n_4 + 5 p_4 + (n_5 + n_6 + n_7) \]  

(45)

Subject to:

\[ x_1 + x_2 + x_3 + n_4 \quad p_4 = 1000 \]  

(46)

\[ x_1 + 1.5 x_2 + 2.2 x_3 + n_1 \quad p_1 = 1250 \]  

(47)

\[ 2 x_1 + 3 x_2 + 4 x_3 + n_2 \quad p_2 = 1000 \]  

(48)

\[ 3 x_1 + 5 x_2 + 10 x_3 + n_3 \quad p_3 = 2000 \]  

(49)

\[ x_1 + n_5 \quad p_5 = 300 \]  

(50)

\[ x_2 + n_6 \quad p_6 = 300 \]  

(51)

\[ x_3 + n_7 \quad p_7 = 300 \]  

(52)

\[ x_i, n_j, p_j \geq 0 \quad (i = 1, 2, 3 ; j = 1, 2, ..., 7) \]  

(53)

By using normalization, objective function becomes:

\[ \text{Min: } a = \frac{3 n_1}{1250} + \frac{2 p_2}{1000} + \frac{2 p_3}{2000} + \frac{5 n_4}{1000} + \frac{5 p_4}{1000} + \frac{(n_5 + n_6 + n_7)}{300} \]  

(54)

Subject to:

\[ x_1 + x_2 + x_3 + n_4 \quad p_4 = 1000 \]  

(55)

\[ x_1 + 1.5 x_2 + 2.2 x_3 + n_1 \quad p_1 = 1250 \]  

(56)

\[ 2 x_1 + 3 x_2 + 4 x_3 + n_2 \quad p_2 = 1000 \]  

(57)

\[ 3 x_1 + 5 x_2 + 10 x_3 + n_3 \quad p_3 = 2000 \]  

(58)
\[ x_1 + n_5 - p_5 = 300 \]  \hspace{1cm} (59)

\[ x_2 + n_6 - p_6 = 300 \]  \hspace{1cm} (60)

\[ x_3 + n_7 - p_7 = 300 \]  \hspace{1cm} (61)

\[ x_i, n_j, p_j \geq 0 \quad (i = 1, 2, 3 ; j = 1, 2,\ldots, 7) \]  \hspace{1cm} (62)

### 4.7 COMPUTATIONAL RESULTS OF WGP MODEL

Using the data presented, the proposed weighted goal programming model is tested using LINDO [2011] software package with priority and the results are shown in Table 4.3:

\[ x_1 = 700, \ x_2 = 300, \ x_3 = 0 \]  \hspace{1cm} (63)

\[ n_1 = 100, \ p_1 = 0 \]  \hspace{1cm} (64)

\[ n_2 = 0, \ p_2 = 1300 \]  \hspace{1cm} (65)

\[ n_3 = 0, \ p_3 = 1600 \]  \hspace{1cm} (66)

\[ n_4 = 0, \ p_4 = 0 \]  \hspace{1cm} (67)

\[ n_5 = 0, \ p_5 = 400 \]  \hspace{1cm} (68)

\[ n_6 = 0, \ p_6 = 0 \]  \hspace{1cm} (69)

\[ n_7 = 300, \ p_7 = 0. \]  \hspace{1cm} (70)
Table 4.3: Interpretation of the Results

<table>
<thead>
<tr>
<th>Goal</th>
<th>Description</th>
<th>Target Level</th>
<th>Satisfied</th>
<th>Achieved Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Strategic aim</td>
<td>1000</td>
<td>Yes</td>
<td>1000 (x_1 + x_2 + x_3)</td>
</tr>
<tr>
<td>2</td>
<td>Profit</td>
<td>1250</td>
<td>No</td>
<td>1150 (n_1 = 100)</td>
</tr>
<tr>
<td>3</td>
<td>Minimize extra finishing and furnace hours</td>
<td>(i) 1000</td>
<td>No</td>
<td>2300 (p_2 = 1300)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(ii) 2000</td>
<td>No</td>
<td>3600 (p_3 = 1600)</td>
</tr>
<tr>
<td>4</td>
<td>No. of cups manufactured</td>
<td>A 300</td>
<td>No</td>
<td>700 (p_5 = 400)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B – 300</td>
<td>Yes</td>
<td>300</td>
</tr>
<tr>
<td></td>
<td></td>
<td>C – 300</td>
<td>No</td>
<td>0 (n_7 = 300)</td>
</tr>
</tbody>
</table>

4.8 CONCLUSION

In this chapter, we have solved the production planning problem through the lexicographic goal programming (LGP) and weighted goal programming (WGP) technique. Two major objectives with target values are optimized through lexicographic goal programming and weighted goal programming. Two objectives are achieved the strategic aim of 1000 cups per day and a profit of at least £1250 by lexicographic goal programming technique. Two objectives are achieved the strategic aim of 1000 cups per day and 300 cup per day of grade B is manufactured by weighted
goal programming technique. The goals are relatively balanced by different goal programming variants. The similar technique could be used for short-term and intermediate production planning in other continuous process industries.