INTRODUCTION

1.1. Background of Thesis

This thesis attempts to develop some optimal allocation designs in three areas. The first problem is in the area of balanced allocation of incoming cases to judges. Secondly, we find allocation of jobs to a single machine. And the third problem is to find the allocation of weights among individuals in multidimensional scaling. In these problems of balanced allocation, it is necessary to maintain uniformity of workloads assigned to judges, uniformity in completion times of jobs and to ensure that weights allotted to individuals are such that the theoretical model comes closer to the observed data. For this purpose, we need to have appropriate criterion.

It is usual practice to select the measure of performance as linear function of certain quantities. However, it is known that in statistical decision theory, variance is an important criterion to assess uniformity of observations. Recently, many research workers have found important applications of criterion functions based on variance. Thus in this thesis, we consider criterion functions based on variance type measures for balancing allocation in all the three problems.
1.2. Problems Considered

As mentioned above, this thesis contains three allocation problems, (1) balanced allocation models, (2) allocation of jobs to a single machine, (3) allocation of weights in individual multidimensional scaling.

In balanced allocation models, we consider the allocation of incoming cases to judges. The cases come sequentially and are to be allotted immediately. If the total number of cases which will arrive during the specific time period is known, then the problem is to allot these cases among \( n \) judges so that imbalance between the number of pending cases is minimum. As discussed before, for balancing allocation, we consider the variance type measurement of imbalance between the number of pending cases. Let \( a_{i0} \) be the initial number of pending cases with judge \( i \). Let \( a_{i1} \) be the number of pending cases with judge \( i \) after allocation of \( x_i \) cases.

Imbalance between the number of pending cases is given by

\[
\sum_{i=1}^{n} \left( a_{i1} - \bar{a}_i \right)^2
\]

where

\[
\bar{a}_i = \frac{1}{n} \sum_{i=1}^{n} \frac{a_{i1}}{n}
\]

and \( n \) is the number of judges.
Thus problem considered here is to find the allocation of \( K \) cases so that \( I \) is minimum. This problem is considered for both the situations, random and deterministic. In the case of random situation, allocation is to be done so that expected imbalance is minimum.

The second problem considered in the thesis is to allocate \( n \) jobs to a single machine. Given the processing times of the jobs, the jobs are to be allotted or scheduled so that variance of completion times is minimum. Let \( P_i \) be the processing time of job at position \( i \) in a sequence. \( C_i \) be the completion time of job at \( i \)th position. Then variance of completion times is given by

\[
V = \frac{1}{n} \sum_{i=1}^{n} (C_i - \overline{C})^2
\]

where
\[
\overline{C} = \left( \frac{1}{n} \sum_{i=1}^{n} C_i \right) / n
\]

This problem is also considered under both the situations, (i) when processing times are known deterministically, (ii) when processing times are random variables with known distributions. In the second situation, jobs are scheduled so that expected variance of completion times is minimum.
Next we consider the problem of finding the weights of individuals in multidimensional scaling technique. Given the interpoint distances for each individual, our problem is to find the common configuration as well as difference among the individuals. Let $D_i : n \times n \ (i = 1, \ldots, N)$ be the matrix of squared interpoint distances for individual $i$. $D_i$ for each individual is transformed into pseudo product matrix $P_i$ given as

$$P_i = (I - \frac{1}{n} JJ') (-\frac{1}{2} D_i) (I - \frac{1}{n} JJ')$$

where $J = (1, \ldots, 1)_{1 \times n}$ and $I$ is identity matrix.

Thus problem is reduced to, given $P_i$, $(i = 1, \ldots, N)$ matrices, find matrix $X : n \times t$ and diagonal matrices, $W_i = $ diagonal $(W_{i1}, \ldots, W_{it})$ such that

$$\text{STRAIN} = \sum_{i=1}^{N} \text{tr} \left( P_i - X W_i X' \right)^2$$

is minimum subject to the conditions.

$$\frac{1}{N} \sum_{i=1}^{N} W_i = I_{t \times t} \text{ and }$$

$$W_{ij} \geq 0 \ \text{for } i = 1, \ldots, N, \ j = 1, \ldots, t.$$
Here \( t_r (A) \) indicates the trace of matrix \( A \). The matrix \( X \) presents \( t \)-dimensional common configuration to all individuals and \( W_1 \) presents the weights attached to dimensions for individual \( i \). This is actually a problem of finding relative position of products and to scale the individual differences.

1.3. Present Work

In this thesis, an explicit optimal solution is obtained for allotting \( K \geq 1 \) cases to \( n \) judges when efficiency of judges is not considered. In earlier allocation models in literature, the efficiency of judges is not taken into account. In this thesis, efficiency factor is introduced and efficiency of judges is measured by the rate of disposal of the cases. Sequential and non-sequential allocation rules are obtained for allotting \( K \geq 1 \), cases when efficiency of judges is known. A few optimization criteria are examined for obtaining the allocation rules. Deterministic case is extended to random case when efficiency as well as the caseloads are random variables with known probability density functions. Allocation rules are obtained so that expected imbalance after allotting, the cases is minimum. Particular situations like uniform and geometric distributions are examined in detail.
In the problem of scheduling n jobs to a single machine, optimal sequence for $n \leq 5$ jobs was obtained. Here these results are extended to determine the optimal sequence for $n = 6$ and $n = 7$ jobs. Using the formula of partitioning variance of completion times, it is shown that for $n \leq 18$, the job with third largest processing time will always be at second position if the job with second largest processing time is at the last position. A heuristic procedure for obtaining a sequence is also proposed for general $n$. When processing times are random variables with known probability density function, characteristics of optimal sequence are examined under general assumptions. The results are also examined for the following particular distributions. (1) Exponential, (2) Gamma, (3) Uniform, (4) Erlang. For exponential distribution, the complete optimal sequence is obtained for $n \leq 7$ jobs.

For solving individual scaling problem as mentioned before, a new iterative procedure is proposed to estimate configuration and weights attached to dimensions for each individual. This procedure has two phases. In the first phase, for given configuration $X$, weights $W_{ij}$, $i = 1, \ldots, N$, $j = 1, \ldots, n$ are obtained by standard method available for
solving quadratic programming problem. In the second phase, given the weights, the configuration $X$ is obtained by numerical analysis methods. A few approaches are examined with numerical examples. These two phases are iterated till the results upto a desired accuracy are obtained.

1.4. **Organization of Thesis:**

The main work of the thesis is written in three parts. Part I deals with balanced allocation models. Chapter II discusses the allocation models when initial number of pending cases as well as efficiency of judges are deterministically known. At the end of the chapter, two alternative criteria are also examined. In Chapter III, allocation rules are obtained when initial number of pending cases and/or efficiency of judges are random variables with known probability density function. In particular, allocation rules are given for uniform and geometric distribution.

Part II is concerned with allocation of jobs to a single machine. In Chapter IV, we discuss scheduling of jobs when processing times are known in Sections 4.1 to 4.8. In Section 4.9 to 4.11 we discuss the case when processing times are random variables.
Part III deals with allocation of weights in individual scaling. In Chapter V, iterative procedure for obtaining configuration and weights for each individual is given. At the end of the chapter, the numerical examples which explain the proposed procedure are also given. Listing of the computer programmes for (i) Obtaining $p_i$, $i = 1, \ldots, N$ matrices from observations, and (ii) Newton-Raphson method for solving non-linear equations are given in Appendix 1 and Appendix 2 respectively.