CHAPTER I

INTRODUCTION AND REVIEW OF LITERATURE

1.1 INTRODUCTION

This chapter presents a brief history of options, its growth and its importance in global and Indian Financial, and Commodities Markets. Further it includes statement of the problem, need for the study, research objectives, the review of literature related to the basic formulation of the Black - Scholes (BS) model, and the findings of empirical verification done by other researchers.

Option is a financial instrument whose value depends upon the value of the underlying assets. Option itself has no value without underlying assets. Option gives the right to the buyer either to sell or to buy the specified underlying assets for a particular price (Exercise / Strike price) on or before a particular date (expiration date). If the right is to buy, it is known as “call option” and if the right is to sell, it is called as “put option”. The buyer of the option has the right but no obligation either to buy or to sell. The option buyer has to exercise the option on or before the expiration date, otherwise, the option expires automatically at the end of the expiration date. Hence, options are also known as contingent claims.

Such an instrument is extensively used in share markets, money markets, and commodity markets to hedge the investment risks and acts as financial leverage investment. Option is a kind of derivative instruments along with forwards, futures and swaps, which are used for managing risk of the investors. Though derivatives are theoretically risk management tools and leveraged investment tools, most use them as speculative tools.

Though the derivatives were very old as early as 1630s, the exchange-traded derivative market was introduced during 1970s. 1973 marked the
creation of both the Chicago Board Options Exchange and the publication of the most famous formula in finance, the option-pricing model of Fischer Black and Myron Scholes. These events revolutionized the investment world in ways no one could imagine at that time. The Black-Scholes model, as it came to be known, set up a mathematical framework that formed the basis for an explosive revolution in the use of derivatives. Chicago Board Options Exchange (CBOE) was founded as first United States of America (USA) options exchange and trading begins on standardized, listed options. April 26, the first day of trading sees 911 contracts traded on 16 underlying stocks. During 1975, computerized price reporting was introduced and Options Clearing Corporation was formed. The Black-Scholes model was adopted for pricing options in CBOE. In the year 2005, CBOE’s options contract volume was an all-time record of 468,249,301 contracts (up 30% over the previous year), and the notional value of this volume was more than US$1.2 trillion.

In 1983, the Chicago Board Options Exchange decided to create an option on an index of stocks. Though originally known as the CBOE 100 Index, it was soon turned over to Standard and Poor's and became known as the S&P 100, which remains the most actively traded exchange-listed option.

Options have the most peculiar property of capping the downside risk at the same time keeping the unlimited upside potential. Furthermore, the importance of the option trading and the requirement of its correct pricing are far more critical and useful in decision making, which are narrated below.

First, prices in an organized derivatives market reflect the perception of market participants about the future and lead the prices of underlying to the perceived future level. The prices of derivatives converge with the prices of the underlying at the expiration of the derivative contract. Thus derivatives help in discovery of future as well as current prices. Second, the derivatives market helps to transfer risks from those who have them but may not like them to those who have an appetite for them. Third, derivatives, due to their inherent nature,
are linked to the underlying cash markets. With the introduction of derivatives, the underlying market witness higher trading volumes, because more players participated who would not otherwise participate for lack of an arrangement to transfer risk. Fourth, the speculative trades shift to a more controlled environment of derivatives market. In the absence of an organized derivatives market, speculators trade in the underlying cash markets. Margining, monitoring and surveillance of the activities of various participants become extremely difficult in these kinds of mixed markets. Fifth, an important incidental benefit that flows from derivatives trading is that it acts as a catalyst for new entrepreneurial activity. The derivatives have a history of attracting many bright, creative, well-educated people with an entrepreneurial attitude. They often energize others to create new businesses, new products and new employment opportunities, the benefit of which are immense. Finally, derivatives markets help increase savings and investment in the long run. Transfer of risk enables market participants to expand their volume of activity.

In India, derivatives trading was introduced Index Futures Contracts from June 2000 and stock option trading in July 2001 grown very fast to reach an average daily turnover of derivatives at NSE, at Rs. 33,745 crores during May 2006 as against cash markets turnover of about Rs. 9202.15 crores (as on May 2006), which indicates the importance of the derivatives. Normally, the derivative turnover is three to four times the cash market turnover in India.

Option, being one of the derivatives is a unique type of hedging tool. Black – Scholes formula after mesmerize the western countries also entered into in Indian option market.
1.2 OPTION BASICS

1.2.1 OPTION BASICS

1.2.1.1 Options

An option is a contract to buy or sell a specific financial product officially known as the option's underlying instrument or underlying assets. For exchange-traded equity options, the underlying instruments are stocks of listed companies. The contract itself is very precise. It establishes a specific price, called the *strike price*, at which the contract may be exercised or acted on and it has an *expiration date*. When an option expires, it no longer has value and no longer exists. Option is known as security, or contingent claim, or contract, or derivative security or simply derivative. An option gives its holder the right to purchase (sell), a specified quantity (lot size) of an underlying asset for a specified price (exercise price or strike price) on or before some specified date called expiration date, but the holder has no obligation to purchase (sell).

1.2.1.2 Types of Options

Options come in two varieties, *calls and puts*, and you can buy or sell either type. You make those choices - whether to buy or sell and whether to choose a call or a put - based on what you want to achieve as an options investor. Call option gives its holder the right to purchase the underlying assets. Put option gives it holder the right to sell the underlying assets.

1.2.1.3 Option terminology

1. *Index options*: These options have the index as the underlying. Some options are European while others are American. Like index futures contracts, index options contracts are also cash settled.
2. *Stock options*: Stock options are options on individual stocks. Options currently trade on over 500 stocks in the USA. A contract gives the holder the right to buy or sell shares at the specified price.

3. *Buyer of an option*: The buyer of an option is the one who by paying the option premium buys the right but not the obligation to exercise his option on the seller/writer.

4. *Writer of an option*: The writer of a call/put option is the one who receives the option premium and is thereby obliged to sell/buy the asset if the buyer exercises on him.

5. *Call option*: A call option gives the holder the right but not the obligation to buy an asset on a certain date for a certain price.

6. *Put option*: A put option gives the holder the right but not the obligation to sell an asset on a certain date for a certain price.

7. *Option price*: Option price is the price which the option buyer pays to the option seller. It is also referred to as the option premium.

8. *Expiration date*: The date specified in the options contract is known as the expiration date / the exercise date / the strike date or the maturity.

9. *Strike price*: The price specified in the options contract is known as the strike price or the exercise price.

10. *American options*: American options are options that can be exercised at any time up to the expiration date. Most exchange-traded options are American.

11. *European options*: European options are options that can be exercised only on the expiration date itself. European options are easier to analyze than
American options, and properties of an American option are frequently deduced from those of its European counterpart.

12. **In-the-money option**: An in-the-money (ITM) option is an option that would lead to a positive cash flow to the holder if it were exercised immediately. A call option on the index is said to be in-the-money when the current index stands at a level higher than the strike price (i.e. spot price > strike price). If the index is much higher than the strike price, the call is said to be deep ITM. In the case of a put, the put is ITM if the index is below the strike price.

13. **At-the-money option**: An at-the-money (ATM) option is an option that would lead to zero cash flow if it were exercised immediately. An option on the index is at-the-money when the current index equals the strike price (i.e. spot price = strike price).

14. **Out-of-the-money option**: An out-of-the-money (OTM) option is an option that would lead to a negative cash flow if it were exercised immediately. A call option on the index is out-of-the-money when the current index stands at a level which is less than the strike price (i.e. spot price < strike price). If the index is much lower than the strike price, the call is said to be deep OTM. In the case of a put, the put is OTM if the index is above the strike price.

15. **Intrinsic value of an option**: The option premium can be broken down into two components – intrinsic value and time value. The intrinsic value of a call is the difference between stock price and the strike price, if it is ITM. If the call is OTM, its intrinsic value is zero. Putting it another way, the intrinsic value of a call is Max [0, S_t – X] which means the intrinsic value of a call is the greater of 0 or (S_t – X). Similarly, the intrinsic value of a put is Max [0, X - S_t], i.e. the greater of 0 or (X - S_t) where X is the strike price and S_t is the spot price.
16. **Time value of an option:** The time value of an option is the difference between its premium and its intrinsic value. Both calls and puts have time value. An option that is OTM or ATM has only time value. Usually, the maximum time value exists when the option is ATM. The longer the time to expiration, the greater is an option’s time value. At expiration, an option should have no time value.

### 1.2.1.4 Option Pricing:

The price of the option is determined by many methods like binomial method, Black Scholes option pricing formula, Volatility jump model etc. out of which the Black Scholes option pricing model is most popular and widely used throughout the world. It is based on the assumption that the stock prices as per **continuous – time, continuous – variable stochastic Markov process.** Markov process states that the future value of stock price depends only on the present value not on the history of the variable. The Markov property implies that the probability distribution of the stock prices at any particular future time is not dependent on the path followed by the price in the past. The Markov property of the stock prices is consistent with the weak form of market efficiency.

The variables and the parameters that determine the call option price are diagrammatically given in Figure 1.1.
Future is uncertain and must be expressed in terms of probability distributions. The probability distribution of the price at any particular future time is not dependent on the particular path followed by the price in the past. This states that the present price of a stock impounds all the information contained in a record of past prices. If the weak form of market efficiency were not true, technical analysts could make above-average returns by interpreting charts of the past history of stock prices. There is very little evidence that they are in fact able to get above-average returns.
It is competition in the marketplace that tends to ensure that weak-form market efficiency holds. There are many, many investors watching the stock market closely. Trying to make a profit from it, leads to a situation where a stock price, at any given time, reflects the information in past prices. Assume that it was discovered a particular pattern in stock prices, which always gave a 65% chance of subsequent steep price rises. Investors would attempt to buy a stock as soon as the pattern was observed, and demand for the stock would immediately rise. This would lead to an immediate rise in its price and the observed effect would be eliminated, as would any profitable trading opportunities.

1.2.2 OPTION AND THE STOCK MARKET

1.2.2.1 Market Efficiency

The derivatives make the stock market more efficient. The spot, future and option markets are inextricably linked. Since it is easier and cheaper to trade in derivatives, it is possible to exploit arbitrage opportunities quickly, and keep the prices in alignment. Hence these markets help ensure that prices of the underlying asset reflect true values.

Options can be used in a variety of ways to profit from a rise or fall in the underlying asset market. The most basic strategies employ put and call options as a low capital means of garnering a profit on market movements, known as leveraging. Option route enable one to control the shares of a specific company without tying up a large amount of capital in the trading account. A small portion of money say, 20% (margin) is sufficient to get the underlying asset worth 100 percentages. Options can also be used as insurance policies in a wide variety of trading scenarios. One, probably, has insurance on his / her car or house because it is the responsible act and safe thing to do. Options provide the same kind of safety net for trades and investments already committed, which is known as hedging.
The amazing versatility that an option offers in today’s highly volatile markets is welcome relief from the uncertainties of traditional investing practices. Options can be used to offer protection from a decline in the market price of available underlying stocks or an increase in the market price of uncovered underlying stock. Options can enable the investor to buy a stock at a lower price, sell a stock at a higher price, or create additional income against a long or short stock position. One can also uses option strategies to profit from a movement in the price of the underlying asset regardless of market direction.

There are three general market directions: market up, market down, and market sideways. It is important to assess potential market movement when you are placing a trade. If the market is going up, you can buy calls, sell puts or buy stocks. Does one have any other available choices? Yes, one can combine long and short options and underlying assets in a wide variety of strategies. These strategies limit your risk while taking advantage of market movement.

The following tables show the variety of options strategies that can be applied to profit on market movement:

<table>
<thead>
<tr>
<th>Bullish Limited Risk Strategies</th>
<th>Bullish Unlimited Risk Strategies</th>
<th>Bearish Limited Risk Strategies</th>
</tr>
</thead>
<tbody>
<tr>
<td>Buy Call</td>
<td>Buy Stock</td>
<td>Buy Put</td>
</tr>
<tr>
<td>Bull Call Spread</td>
<td>Sell Put</td>
<td>Bear Put Spread</td>
</tr>
<tr>
<td>Bull Put Spread</td>
<td>Covered Call</td>
<td>Bear Call Spread</td>
</tr>
<tr>
<td>Call Ratio Back spread</td>
<td>Call Ratio Spread</td>
<td>Put Ratio Back spread</td>
</tr>
</tbody>
</table>
It is of paramount importance to be creative with trading. Creativity is rare in the stock and options market. That's why it's such a winning tactic. It has the potential to beat the next person down the street. One has a chance to look at different scenarios that he does not have the knowledge to construct. All you need to do is take one step above the next guy for you to start making money. Luckily the next person, typically, does not know how to trade creatively.

Thus the risk managing ability, low cost and its act as sentiment indicator of option drives the market more efficient.

### 1.2.2.2 Leverage and Risk

Options can provide leverage. This means an option buyer can pay a relatively small premium for market exposure in relation to the contract value (usually 100 shares of underlying stock). An investor can see large percentage gains from comparatively small, favorable percentage moves in the underlying index. Leverage also has downside implications. If the underlying stock price does not rise or fall as anticipated during the lifetime of the option, leverage can magnify the investment’s percentage loss. Options offer their owners a predetermined, set risk. However, if the owner’s options expire with no value,
this loss can be the entire amount of the premium paid for the option. An uncovered option writer, on the other hand, may face unlimited risk.

1.2.3 RISK MANAGEMENT TOOL

The market price reduction of the share is called as downside risk of the investor. The profit from the increase in the share price is known as upside potential. Option strategies help the investors to cap the downside risk at the same time keep the upside potential unlimited. This is the most desired need of the investors. Buying a call option and selling a put option works well in the bull market, limiting the loss to the premium paid but the upside potential in unlimited as market price increases. Similarly, in a bearish situation, selling a call and buying a put are the strategies of capping the downside risk. Apart from the above plain vanilla strategies, bull – spread, bear – spread, calendar spreads, butterfly spreads, diagonal spreads, straddle, strangle, strips, and straps are some of the famous strategies to cap the downside risks in any level required by the investors. “How this can be achieved?” is not the scope of the study but are practiced by the investing community as on date, but the upside potential is slightly reduced by using these strategies, which are minimum compare to the advantage gained by the investors. This property makes the option a unique tool for risk management and a preferred one.

1.3 DERIVATION OF BLACK – SCHOLES FORMULA

1.3.1 CONTINUOUS-TIME STOCHASTIC PROCESSES

Consider a variable that follows a Markov stochastic process. Suppose that its current value is 1.0 and that the change in its value during one year is \( \Phi (0, \sqrt{1}) \), where \( \Phi (\mu, \sigma) \) denotes a probability distribution that is normally distributed with mean \( \mu \) and standard deviation \( \sigma \). What would be the probability distribution of the change in the value of the variable during two years? The
change in two years is the sum of two normal distributions, each of which has a mean of zero and standard deviation of 1.0. Because the variable is Markov, the two probability distributions are independent. When we add two independent normal distributions, the result is a normal distribution in which the mean is the sum of the means and the variance is the sum of the variances\(^1\). The mean of the change during two years in the variable we are considering is therefore zero, and the variance of this change is 2.0. The change in the variable over two years is therefore $\Phi(0, \sqrt{2})$. Considering the change in the variable during six months, the variance of the change in the value of the variable during one year equals the variance of the change during the first six months plus the variance of the change during the second six months. We assume these are the same. It follows that the variance of the change during a six month period must be $\sqrt{0.5}$. Equivalently, the standard deviation of the change is 0.5, so that the probability distribution for the change in the value of the variable during six months is $\Phi(0, \sqrt{0.5})$.

A similar argument shows that the change in the value of the variable during three months is $\Phi(0, \sqrt{0.25})$. More generally, the change during any time period of length $T$ is $\Phi(0, \sqrt{T})$. In particular, the change during a very short time period of length $\delta t$ is $\Phi(0, \delta t)$.

The square root signs in these results may seem strange. They arise because, when Markov processes are considered, the variance of the changes in successive time periods are additive. The standard deviations of the changes in successive time periods are not additive. The variance of the change in the variable in our example is 1.0 per year, so that the variance of the change in two years is 2.0 and the variance of the change in three years is 3.0. The standard deviation of the change in two and three years is $\sqrt{2}$ and $\sqrt{3}$, respectively. Strictly speaking, we should not refer to the standard deviation of

\(^1\)The variance of a probability distribution is the square of its standard deviation. The variance of a one-year change in the value of the variable we are considering is therefore 1.0.
the variable as 1.0 per year. It should be “1.0 per square root of years”. The results explain why uncertainty is often referred to as being proportional to the square root of time.

### 1.3.2 Weiner Processes

The process followed by the variable we have been considering is known as *Wiener process*. It is a particular type of Markov stochastic process with a mean change of zero and a variance rate of 1.0 per year. It has been used to describe the motion of a particle that is subject to a large number of small molecular shocks and is sometimes referred as *Brownian motion*.

Expressed formally, a variable $z$ follows a Weiner Process if it has the following two properties:

**Property 1:** The change $\delta z$ during a small period of time $\delta t$ is

$$\delta z = \varepsilon \sqrt{\delta t}$$  \hspace{1cm} (1.3.1)

where $\varepsilon$ is a random drawing from a standard normal distribution, $\Phi(0,1)$.

**Property 2:** The values of $\delta z$ for any two different short intervals of time $\delta t$ are independent.

It follows from the first property that $\delta z$ itself has a normal distribution with

- Mean of $\delta z = 0$
- Standard deviation of $\delta z = \sqrt{\delta t}$
- Variance of $\delta z = \delta t$

The second property implies that $z$ follows a Markov process.
Consider the increase in value of \( z \) during a relatively long period of time, \( T \). This can be denoted by \( z(T) - z(0) \). It can be regarded as the sum increases in \( z \) in \( N \) small time intervals of length \( \delta t \), where

\[
N = \frac{T}{\delta t}
\]

Thus,

\[
z(T) - z(0) = \sum_{i=1}^{N} \epsilon_i \sqrt{\delta t}
\]

(1.3.2)

where the \( \epsilon_i \) (\( i = 1, 2, \ldots, N \)) are random drawings from \( \Phi(0,1) \). From second property of Weiner Processes the \( \epsilon_i \)'s are independent of each other. It follows from the equation (1.3.2) that \( z(T) - z(0) \) is normally distributed with

Mean of \([z(T) - z(0)] = 0\)

Variance of \([z(T) - z(0)] = N \delta t = T\)

Standard deviation of \([z(T) - z(0)] = \sqrt{T}\).

This is consistent with our earlier logic.

### 1.3.3 GENERALIZED WIENER PROCESS

The basic Weiner Process, \( \delta z \), which has been developed so far, has a drift rate of zero and a variance rate of 1.0. The drift rate of zero means that the expected value of \( z \) at any future time is equal to its current value. The variance rate of 1.0 means that the variance of the change in \( z \) in a time interval of length \( T \) is equal to \( T \). A generalized Weiner Process for a variable \( x \) can be defined in terms of \( dz \) as follows:

\[
dx = a \, dt + b \, dz
\]

(1.3.3)

where \( a \) and \( b \) are constants.
To understand equation (1.3.3), it is useful to consider two components of right hand side separately. The $a \, dt$ term implies that $x$ has an expected drift rate of $a$ per unit of time. Without $b \, dz$ term the equation is

$$dx = a \, dt$$

which implies that

$$\frac{dx}{dt} = a$$

Integrating with respect to time, we get

$$x = x_0 + at$$

where $x_0$ is the value at the time zero. In a period of length $T$, the value of $x$ increases by an amount "at". The $b \, dz$ term on the right-hand side of equation (1.3.3) can be regarded as adding noise or variability of the path followed by $x$. The amount of noise or variability is $b$ times a Wiener Process has a standard deviation of 1.0. It follows that $b$ times a Wiener Process has a standard deviation of $b$. In a small time interval $\delta t$, the change $\delta x$ in the value of $x$ is given by equations (1.3.1) and (1.3.3) as

$$\delta x = a \, \delta t + b\varepsilon \sqrt{\delta t}$$

where, as before, $\varepsilon$ is a random drawing from a standardized normal distribution. Thus $\delta x$ has a normal distribution with

Mean of $\delta x = a \, \delta t$

Standard deviation of $\delta x = b\sqrt{\delta t}$

Variance of $\delta x = b^2 \, \delta t$
Similar arguments to those given for a Wiener Process show that the change in the value of $x$ in any time interval $T$ is normally distributed with

\[
\text{Mean of change in } x = a T
\]

\[
\text{Standard deviation of change in } x = b \sqrt{T}
\]

\[
\text{Variance of change in } x = b^2 T
\]

Thus, the generalized Wiener Process given in equation (1.3.3) has an expected drift rate (i.e., average drift per unit of time) of “$a$” and a variance rate (i.e., variance per unit of time) of $b^2$. It is illustrated in the Figure 1.2.

**FIGURE 1.2**

**WIENER AND GENERALIZED WIENER PROCESSES**
1.3.4 ITÔ PROCESS

A further type of stochastic process can be defined. This is known as an Itô process. This is a generalized Wiener Process in which the parameters $a$ and $b$ are functions of the value of the underlying variable $x$ and time $t$.

Algebraically, an Itô process can be written

$$dx = a(x,t)dt + b(x, t) dz \quad (1.3.4)$$

Both the expected drift rate and variance rate of an Itô process are liable to change over time. In a small time interval between $t$ and $t + \delta t$, and the variable changes from $x + \delta x$, where

$$\delta x = a(x,t) \delta t + b(x, t) \epsilon \sqrt{\delta t}$$

The relationship involves a small approximation. It assumes that the drift and variance rate of $x$ remains constant, equal to $a(x, t)$ and $b(x, t)^2$, respectively, during the time interval between $t$ and $t + \delta t$.

1.3.5 THE PROCESS OF STOCK PRICES

In this section it is dealt about the stochastic process for the price of non-dividend paying stock. It is tempting to suggest that a stock price follows a generalized Wiener Process, that is, that it has a constant drift rate and a constant variance rate. However, this model fails to capture a key aspect of stock prices. This is the expected percentage return required by the investors from a stock is independent of the stock price. If the investors require a 20% per annum expected return when the stock price is Rs. 1000, then ceteris paribus, they will also require a 20% per annum expected return when it is Rs.5000. Clearly, the constant expected drift-rate assumption is inappropriate and
needs to be replaced by the assumption that the expected return (i.e., expected drift divided by the stock price) is constant. If \( S \) is the stock price at time \( t \), the expected drift rate in \( S \) should be assumed to be \( \mu S \) for some constant parameter \( \mu \). This means that in a short interval of time, \( \delta t \), the expected increase in \( S \) is \( \mu S \delta t \). The parameter \( \mu \) is the expected rate of return on the stock, expressed in decimal form.

If the volatility of the stock price is always zero, this model implies that

\[
\delta S = \mu S \delta t
\]

In the limit as \( \delta t \to 0 \),

\[
\delta S = \mu S dt
\]

\[
\frac{\delta S}{S} = \mu dt
\]

Integrating between time zero and time \( T \), it becomes

\[
S_T = S_0 e^{\mu t}
\]  \hspace{1cm} (1.3.5)

where \( S_T \) and \( S_0 \) are stock prices at the time of \( T \) and at the time of zero. Equation (1.3.5) shows that, when the variance rate is zero, the stock price grows at a continuously compounded rate of \( \mu \) per unit.

In practice, the stock price does exhibit volatility. A reasonable assumption is that the variability of the percentage return in a short period of time, \( \delta t \), is the same regardless of the stock price. In other words, an investor is just uncertain of the percentage return when the stock price is Rs.5000 as when it is Rs.1000. This suggests that the standard deviation of the change in a short period of time \( \delta t \) should be proportional to the stock price and leads to the model

\[
dS = \mu S \delta t + \sigma S dz
\]

or
This equation is the most widely used model for the stock behaviour. The variable $\sigma$ is the volatility of the stock price and the variable $\mu$ is the expected rate of return.

For example, consider a stock that pays no dividends, has volatility of 30% per annum, and provides expected return of 15% per annum with continuous compounding. The process of stock price is

$$\frac{dS}{S} = \mu \delta t + \sigma dz$$

$$= 0.15 \delta t + 0.30 dz$$

If $S$ is the stock price at a particular time and $\delta S$ is the increase in the stock price in the next small interval of time, then

$$\frac{\delta S}{S} = a(x,t) \delta t + b(x, t) \epsilon \sqrt{\delta t}$$

$$= 0.15 \delta t + 0.30 \epsilon \sqrt{\delta t}$$

where $\epsilon$ is a random drawing from a standardized normal distribution. Consider a time interval of one week, or 0.0192 years, and suppose that the initial stock price is Rs.100. Then $\delta t = 0.0192$ and $S = 100$ and

$$\delta S = 100(0.00288 + 0.0416 \epsilon)$$

$$= 0.288 + 4.16 \epsilon$$

showing that the price increase is a random drawing from a normal distribution with mean Rs.0.288 and a standard deviation Rs.4.16. This process is known as Geometric Brownian motion.
1.3.6 THE PARAMETERS

The process of stock prices involves two parameters; \( \mu \) and \( \sigma \). The parameter \( \mu \) is the expected continuously compounded return earned by an investor per year. Most investors require higher expected returns to induce them to take higher risks. It follows that the value of \( \mu \) should depend on the risk of the return from the stock. It should also depend on the interest rate in the economy. The higher the level of interest rates, the higher the expected return required on any given stock.

Fortunately, BS formula is independent of \( \mu \) and hence the determination of \( \mu \) is not required. The parameter \( \sigma \), the stock price volatility, is, by contrast, critically important to the determination of the value of the most derivatives. The standard deviation of the proportional change in the stock price in a small interval of time \( \delta t \) is \( \sigma \sqrt{\delta t} \). As a rough approximation; the standard deviation of the proportional change in the stock price over a relatively long period of time \( T \) is \( \sigma \sqrt{T} \). This means that, as an approximation, volatility can be interpreted as standard deviation of the stock price in one year.

1.3.7 ITÔ’S LEMMA

The price of the stock option is a function of the underlying stock’s price and time. More generally, the price of any derivative is a function of stochastic variables underlying the derivative and time. An important result in the area of the behaviour of functions of stochastic variables was discovered by the mathematician Kiyosi Itô in 1951, which is known as Itô’s process, explained below.

Suppose the value of a variable \( x \) follows the Itô’s process

\[
dx = a(x,t)dt + b(x,t)dz
\]

(1.3.7)
where $dz$ is a Wiener process and $a$ and $b$ are functions of $x$ and $t$. The variable $x$ has a drift rate of $a$ and a variance rate of $b^2$. Itô's lemma shows that a function $G$ of $x$ and $t$ follows an Itô's process. It has a drift rate of

$$dG = \left( - \frac{\partial G}{\partial x} a + \frac{\partial G}{\partial t} \frac{\partial G}{\partial x} \frac{\partial^2 G}{\partial x^2} b^2 \right) dt + \frac{\partial G}{\partial x} b dz$$

where the $dz$ is the same Wiener process as in equation (1.3.7) above. Thus, $G$ also follows an Itô's process. It has a drift rate of

$$- \frac{\partial G}{\partial x} a + \frac{\partial G}{\partial t} \frac{\partial G}{\partial x} \frac{\partial^2 G}{\partial x^2} b^2$$

and variance rate of

$$\left( - \frac{\partial G}{\partial x} b \right)^2 b^2$$

Lemma can be viewed as an extension of well-known results in differential calculus.

Earlier we argued that

$$dS = \mu S dt + \sigma S dz$$

with $\mu$ and $\sigma$ constant, is a reasonable model of stock price movements. From Itô's lemma, it follows that the process followed by a function $G$ of $S$ and $t$ is

$$dG = \left( - \frac{\partial G}{\partial x} \mu S + \frac{\partial G}{\partial t} \frac{\partial^2 G}{\partial S^2} \sigma^2 S^2 \right) dt + \frac{\partial G}{\partial S} \sigma S dz$$
It is to be noted that both $S$ and $G$ are affected by the same underlying source of uncertainty, $dz$. This proves to be very important in the derivation of the Black-Scholes results.

### 1.3.8 APPLICATION TO FORWARD CONTRACTS

To illustrate Itô’s lemma, consider a forward contract on a non-dividend-paying stock. Assume that risk-free rate of interest is constant and equal to all maturities. For continuously compounded investment, the value of the future contract to be

$$F_0 = S_0 e^{rT}$$  \hspace{1cm} (1.3.11)

where $F_0$ is the forward contract price at time zero, $S_0$ is the spot price at time zero and $T$ is the time to maturity of the forward contract.

Let us study the process of forward price as time passes. Define $F$ as forward price and $S$ as spot price, respectively, at a general time $t$ with $t < T$. The relationship between $F$ and $S$ is

$$F = Se^{r(T-t)}$$  \hspace{1cm} (1.3.12)

Assuming that the process of $S$ is given by equation (1.3.8), we can use Itô’s lemma to determine the process for $F$. From equation (1.3.11),

$$\frac{\partial F}{\partial S} = e^{r(T-t)} \hspace{1cm} \frac{\partial^2 F}{\partial S^2} = 0 \hspace{1cm} \frac{\partial F}{\partial t} = -rS e^{r(T-t)}$$

From equation (1.7.9), the process for $F$ is given by

$$dF = [ e^{r(T-t)} \mu S - rS e^{r(T-t)} ] dt + e^{r(T-t)} \sigma S dz$$

By substituting the value of $F$ from equation (1.3.12)
\[ dF = (\mu - r) F \, dt + \sigma F \, dz \quad (1.3.13) \]

Like the stock price \( S \), the forward price \( F \) also follows *Geometric Brownian* motion. It has an expected growth rate of \((\mu - r)\) rather than \(\mu\). The growth rate in \( F \) is the excess return of \( S \) over risk-free rate of interest.

### 1.3.9 THE LOG-NORMAL PROPERTY

\( \text{Itô's lemma can be used to derive the process followed by} \ln S \text{ when } S \text{ follows the process in equation (1.3.9). Define} \)

\[ G = \ln S \]

Because

\[
\begin{align*}
\frac{\partial G}{\partial S} &= \frac{1}{S}, \\
\frac{\partial^2 G}{\partial S^2} &= \frac{1}{S^2}, \\
\frac{\partial G}{\partial t} &= 0
\end{align*}
\]

It follows from equation (1.7.9) that the process followed by \( G \) is

\[ dG = \left( \mu - \frac{\sigma^2}{2} \right) dt + \sigma dz \quad (1.3.14) \]

Because \( \mu \) and \( \sigma \) are constant, this equation indicated that \( G = \ln S \) follows a generalized Wiener process. It has a drift rate \((\mu - \sigma^2 / 2)\) and constant variance rate of \( \sigma^2 \). The change in \( \ln S \) between time zero and some future time, \( T \) is therefore normally distributed with mean

\[ \frac{\sigma^2}{2} \left( \mu - \frac{\sigma^2}{2} \right) T \]

and variance \( \sigma^2 T \). This means that

\[ \ln S_T - \ln S_0 \sim \Phi \left( \left( \mu - \frac{\sigma^2}{2} \right) T, \sigma \sqrt{T} \right) \quad (1.3.15) \]

Or
\begin{equation}
\ln S_T \sim \Phi \left( \ln S_0 + \left( \mu - \frac{\sigma^2}{2} \right) T, \sigma \sqrt{T} \right) \tag{1.3.16}
\end{equation}

where $S_T$ is the stock price at a future time $T$, $S_0$ is the stock price at time zero, and $\Phi \left( m, s \right)$ denotes a normal distribution with mean $m$ and standard deviation $s$.

Equation (1.3.15) shows that $\ln S_T$ is normally distributed. A variable has a lognormal distribution if the natural logarithm of the variable is normally distributed. The model of stock behaviour that was developed therefore implies that a stock price at a time $T$, given its price today, is lognormally distributed. The standard deviation of the logarithm of the stock price is $\sigma \sqrt{T}$. It is proportional to the square root of how far ahead we are looking.

From the equation (1.3.16) and the properties of lognormal distribution, it can be shown that the expected value, $E \left( S_T \right)$, of $S_T$ is given by

$$E \left( S_T \right) = S_0 e^{\mu T} \tag{1.3.17}$$

1.3.10 DERIVATION OF THE BLACK-SCHOLES DIFFERENTIAL EQUATION

Let us consider the stock price process

\begin{equation}
dS = \mu S \, dt + \sigma S \, dz \tag{1.3.18}
\end{equation}

Suppose that $f$ is the price of the call option or other derivative contingent on $S$. The Variable $f$ must be some function of $S$ and $t$. Hence, from the equation (1.3.10)

\begin{equation}
\begin{aligned}
\frac{\partial f}{\partial S} &+ \frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \, dt + \frac{\partial f}{\partial S} \, dz = 0
\end{aligned} \tag{1.3.19}
\end{equation}
The discrete versions of the above equations are as below

\[ \delta S = \mu S \delta t + \sigma S \delta z \quad (1.3.20) \]

\[ \frac{\partial f}{\partial S} \delta t + \frac{\partial f}{\partial S^2} \delta S + \frac{\partial f}{\partial t} \delta t + \frac{\partial^2 f}{\partial S \partial t} \delta S \delta t + \frac{\partial f}{\partial S} \delta S \delta z = \quad (1.3.21) \]

where, \( \delta S \) and \( \delta f \) are the changes in \( f \) and \( S \) in a small time interval \( \delta t \).

From Itô’s lemma it is understood that the Wiener processes underlying \( f \) and \( S \) are the same. In other words, the \( \delta z (=\epsilon \sqrt{\delta t}) \) in equation (1.3.20) and (1.3.21) are the same. It follows that, by choosing a portfolio of the stock and the derivative, the Wiener process can be eliminated.

The appropriate portfolio is as follows:

\[ \begin{align*}
- \frac{1}{\partial f} : & \text{derivatives} \\
+ \frac{1}{\partial S} : & \text{shares}
\end{align*} \]

The holder of the portfolio is short one derivative and long an amount \( \frac{\partial f}{\partial S} \) of shares. Define \( \pi \) as the value of the portfolio. By definition,

\[ \pi = -f + \frac{\partial f}{\partial S} S \quad (1.3.22) \]

The change in \( \delta \pi \) in the value of the portfolio in the time interval \( \delta t \) is given by

\[ \delta \pi = -\delta f + \frac{\partial f}{\partial S} \delta S \quad (1.3.23) \]

Substituting equations (1.3.20) and (1.3.21) into equations (1.3.22) yields

\[ \delta \pi = \left( -\frac{\partial f}{\partial t} - \frac{\partial^2 f}{\partial S \partial t} \right) \delta t + \frac{\partial f}{\partial S^2} \delta S \delta t \quad (1.3.24) \]
Because this equation does not involve $\delta z$, the portfolio must be riskless during time $\delta t$. The assumptions listed in preceding section imply that the portfolio must instantaneously earn the same rate of return as other short-term risk-free securities. If it earned more than this return, arbitrageurs could make a riskless profit by borrowing money to buy the portfolio; if it earned less, they could make a riskless portfolio by shorting the portfolio and buying risk-free securities. It follows that

$$\delta \pi = r \pi \delta t$$

where, $r$ is the risk-free interest rate. Substituting from equation (1.3.22) and (1.3.24), it becomes

$$\frac{\partial f}{\partial t} + \frac{1}{2} \frac{\partial^2 f}{\partial S^2} \sigma^2 S^2 \delta t = r (f - \frac{\partial f}{\partial S}) \delta t$$

so that

$$\frac{\partial f}{\partial t} + r S \frac{\partial f}{\partial S} + \frac{1}{2} \sigma^2 S^2 \frac{\partial^2 f}{\partial S^2} = r f$$

Equation above is the Black–Scholes differential equation. It has many solutions, corresponding to all the different derivatives that can be defined with $S$ as the underlying variable. The particular derivative that is obtained when the equation is solved depends on the boundary conditions that are used. These specify the values of the derivatives at the boundaries of possible values of $S$ and $t$. In the case of a European call option, the key boundary condition is

$$f = \max (S - X, 0) \quad \text{when } t = T$$

In the case of a European put option, it is
\[ f = \max (X - S, 0) \quad \text{when } t = T \]

One point that should be emphasized about the portfolio used in the derivation of equation (1.3.25) is that it is not permanently riskless. It is riskless only for an infinitesimally short period of time. As \( S \) and \( t \) change, \( \frac{\partial f}{\partial S} \) also changes. To keep the portfolio riskless, it is therefore necessary to frequently change the relative proportions of the derivative and the stock in the portfolio.

### 1.3.10.1 The prices of tradable derivatives

Any function \( f(S, t) \) that is a solution of the differential equation (1.3.25) is the theoretical price of a derivative that could be traded. If a derivative with that price existed, it would not create an arbitrage opportunities. Conversely, if a function \( f(S, t) \) does not satisfy the differential equation (1.3.25), it cannot be the price of the derivative without creating arbitrage opportunities for the traders.

To illustrate this point, consider the function \( e^S \). This does not satisfy the differential equation (1.3.25). It is therefore not a candidate for being the price of a derivative dependent on the stock price. If an instrument whose price was always \( e^S \) existed, there would be an arbitrage opportunity.

Also consider the function \( e^{(\sigma^2 - 2r)(T - t)} / S \). This does not satisfy the differential equation, and so is, in theory, the price of a tradable security.

### 1.3.11 RISK – NEUTRAL VALUATION

Risk – neutral valuation is the single most important tool for analysis of derivatives. It arises from one key property of the Black – Scholes differential equation. This property is that the equation does not involve any variable that is affected by the risk preference of the investors. The variables that do appear in the equation are the stock price, time, stock price volatility, and risk-free rate of interest. All are independent of risk preferences.
Black – Scholes differential equation would not be independent of risk preferences if it involved the expected return on the stock $\mu$. This is because the value of $\mu$ does depend on risk preferences. The higher the level of risk aversion by the investors, the higher $\mu$ will be for any given stock. It is fortunate that $\mu$ happens to drop out in the derivation of the differential equation.

Because the Black – Scholes differential equation is independent of risk preferences, an ingenious argument can be used. If risk preferences do not enter the equation, they cannot affect its solution. Any set of risk preferences can, therefore be used when evaluating $f$. In particular, the very simple assumption that all investors are risk neutral can be made.

In a world where investors can be risk neutral, the expected return on all securities is the risk-free rate of interest, $r$. The reason is that risk- neutral investors do not require a premium to induce them to take risks. It is also true that the present value of any cash flow in a risk neutral world would be obtained by discounting its expected value at risk-free rate of interest. The assumption that the world is risk neutral, therefore, considerably simplifies the analysis of derivatives.

Consider a derivative that provides a payoff at one particular time. It can be valued using risk - neutral valuation by using the following procedure:

1. Assume that the expected return from the underlying asset is the risk-free rate of interest, $r$ (i.e. assume $\mu = r$).
2. Calculate the expected payoff from the options at its maturity.
3. Discount the expected payoff at the risk-free rate of interest.

It is important to appreciate the risk - neutral valuation (or assume that all investors are risk - neutral) is merely an artificial device for obtaining solutions to the Black – Scholes differential equation. The solutions that are obtained are
valid in all worlds, not just those were investors are risk – neutral. When we move from risk - neutral world to a risk averse world, two things happen. They are the expected growth rate in the stock price changes and the discount rate that must be used for any payoffs from any derivative changes. It happens that these two changes always offset each other exactly.

1.3.11.1 Application to Forward Contracts on a Stock

Assume that the interest rates are constant and equal to r. This is somewhat restrictive. Consider a long forward contract that matures at time T with delivery price X. The value of the contract at maturity is

$$S_T - X$$

where $S_T$ is the stock price at time T. From the risk - neutral valuation argument, the value of the forward contract at time zero is its expected value at time T discounted at risk-free rate of interest. If we denote the value of the forward contract at time zero by f, this means that

$$f = e^{-rT} \hat{E}(S_T - X) \quad (1.3.26)$$

where $\hat{E}$ is the expected value in a risk - neutral world. Because $X$ is a constant, equation (1.3.26) becomes

$$f = e^{-rT} \hat{E}(S_T) - X e^{-rT} \quad (1.3.27)$$

The expected growth rate of the stock price, $\mu$, becomes r in a risk - neutral world, which, can be expressed as:

$$\hat{E}(S_T) = S_0 e^{rT} \quad (1.3.28)$$

Substituting equation (1.7.28) into equation (1.7.25) gives

$$f = S_0 - X e^{-rT} \quad (1.3.29)$$
1.3.12 BLACK – SCHOLES OPTION PRICING FORMULA

One way of deriving the Black - Scholes formula is by solving the differential equation (1.7.25) subject to the boundary conditions explained in the above section 1.7.9. Another approach is to use the risk - neutral valuation. Consider a European call option. The expected value of the option at any maturity in a risk - neutral world is

\[ \hat{E} \left[ \max (S_T - X, 0) \right] \]

where \( \hat{E} \) denotes the expected value in a risk - neutral world. From the risk - neutral valuation argument, the European call option, \( c \), is the expected value discounted at the risk-free rate of interest, that is,

\[ C_0 = e^{-rT} \hat{E} \left[ \max (S_T - X, 0) \right] \quad (1.3.30) \]

Let us consider a call option on a non-dividend-paying stock maturing at time \( T \). Under the stochastic process assumed by Black – Scholes, \( S_T \) is lognormal.

Also from the equation (1.3.16) and (1.3.17), \( \hat{E} (S_T) = S_0 e^{rT} \) and the standard deviation of \( \ln S \) is \( \sigma \sqrt{T} \).

**Key result:**

If \( V \) is lognormally distributed and the standard deviation of \( \ln V \) is \( s \), then

\[ E \left[ \max (V - X, 0) \right] = E (V) N (d_1) - X N (d_2) \quad (1.3.31) \]

where

\[ d_1 = \frac{\ln [E(V) / X] + s^2 / 2}{s} \quad (1.3.32) \]

\[ d_2 = \frac{\ln [E(V) / X] - s^2 / 2}{s} \quad (1.3.33) \]
E denotes the expected value.

From the key result just proved, the equation (1.7.31) implies that

\[ C_0 = e^{-rT} \left[ S_0 e^{rT} N(d_1) - X N(d_2) \right] \]

\[ = S_0 N(d_1) - X e^{-rT} N(d_2) \quad (1.3.34) \]

where

\[ d_1 = \frac{\ln \left( \frac{\hat{E}(S_T)}{X} + \sigma^2 T/2 \right)}{\sigma \sqrt{T}} \]

\[ d_2 = \frac{\ln \left( \frac{S_0}{X} + (r + \sigma^2 /2) T \right)}{\sigma \sqrt{T}} \quad (1.3.35) \]

\[ d_2 = \frac{\ln \left( \frac{\hat{E}(S_T)}{X} - \sigma^2 T/2 \right)}{\sigma \sqrt{T}} \]

\[ = \frac{\ln \left( \frac{S_0}{X} + (r - \sigma^2 /2) T \right)}{\sigma \sqrt{T}} \quad (1.3.36) \]

\[ = d_1 - \sigma \sqrt{T} \]

**1.4 STATEMENT OF THE PROBLEM**

Buying decision depends upon the intrinsic (correct) value of the asset to be purchased and the investor’s ability to estimate the same. If the market price is more than the estimated intrinsic price, one should sell the asset, as the market price will converge with the intrinsic value in due course. If the market price is less than the intrinsic value, then one should buy the asset before it rises as more and more investors will find that the asset is undervalued and buy
the same. There are many asset pricing models being practiced in capital market like Dividend Discount Model (DDM), Relative Valuation Models, Capital Asset Pricing Model (CAPM), Single factor model, Stephen Ross’s Arbitrage Pricing Theory (APT) etc. Likewise in last two decades, option pricing has witnessed an explosion of new models after the celebrated and Nobel Prize won Black – Scholes (1973) European formula. Examples include (i) the Stochastic – Interest – rate option models of Merton (1973) and Amin and Jarrow (1992); (ii) the Jump – Diffusion / Pure Jump models of Bates (1991), Madan and Chang (1996), and Merton (1976); (iii) the Constant - Elasticity – of - Variance model of Cox and Ross (1976); (iv) the Markovian models of Rubinstein (1994) and Aït – Sahalia and Lo (1996); (v) the Stochastic – Volatility models of Heston (1993), Hull and White (1987a), Melino and Turnbull (1990, 1995), Scott (1987), Stein and Stein (1991), and Wiggins (1987); (vi) the Stochastic – Volatility and Stochastic – Interest – rates models of Amin and Victor Ng (1993), Baily and Stulz (1989), Bakshi and Chen (1997a,b), and Scott (1997). But most of them are not as parsimonious as Black – Scholes model and have difficulties in practice. Till now, Black - Scholes is considered as benchmark for option pricing.

As seen early, the Black - Scholes (BS) option pricing formula, won the Nobel Prize for economics in 1997, revolutionized the capital market. So many empirical studies were conducted on the BS formula in developed nations, which interestingly revealed the strengths and weaknesses of it. As India introduced exchange - traded options only in 2001, an elaborate empirical study is required in the usage of BS formula in Indian stock option market. First, the sensitivity of the call option price on change in the variables / parameters is to be studied in detail. That is the relationships between the call option price and its determinants like price of the underlying asset, Strike Price, Risk-free Interest rate, volatility of the returns of the underlying asset, remaining life of the option etc. are to be analyzed and understood. Second, the research study is on the predictability of the model, and biases of the model towards the above
determinants are to be identified and taken care when use. Third, model's specification is to be tested for its correctness, using residual analysis. Fourth, it is to understand that the assumptions of the model are real and practically correct. Lastly, the empirical study can also focus on any weakness of the model and to improve the same to improve the predictability of the model.

1.5 NEED FOR THE STUDY

Once the investor decided to cap the downside risk of his investment, using options, the next question arises that what is the premium to be paid or the price to be given to enjoy such a risk management tool. This price shall be logical and correct to make a decision of buying or selling an option. The question of pricing the asset can be done in any number of ways explained in earlier paragraph 1.2. But the investor - friendly, parsimonious method of option pricing is the Black - Scholes option pricing model. Universally this model is used in all the leading option exchanges including the developed nations like USA, UK, Japan and emerging nations like India, China etc.

If we keenly observe, the volume and the range of options offered by NSE in India and the actual options that are traded, it is easily understood that still the Indian investors are not familiar with the options. At random, samples of 30 companies were taken which are given in Table no.1.1.
### TABLE 1.1

**DETAILS OF THE STOCK CALL OPTIONS OFFERED, TRADED AND NON-DIVIDEND PAYING STOCKS AT NSE FROM 1.1.02 TO 31.10.07**

<table>
<thead>
<tr>
<th>S. No.</th>
<th>Company</th>
<th>From</th>
<th>To</th>
<th>Offered</th>
<th>Traded</th>
<th>Non-Dividend Paying</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Tata Steel</td>
<td>01/01/02</td>
<td>31/10/07</td>
<td>59,912</td>
<td>18,462</td>
<td>16,100</td>
</tr>
<tr>
<td>2</td>
<td>Reliance Ind.</td>
<td>01/01/02</td>
<td>31/10/07</td>
<td>53,118</td>
<td>16,271</td>
<td>14,145</td>
</tr>
<tr>
<td>3</td>
<td>Infosys</td>
<td>31/01/03</td>
<td>31/10/07</td>
<td>60,653</td>
<td>18,046</td>
<td>12,559</td>
</tr>
<tr>
<td>4</td>
<td>ACC</td>
<td>01/01/02</td>
<td>31/10/07</td>
<td>56,006</td>
<td>11,577</td>
<td>9,334</td>
</tr>
<tr>
<td>5</td>
<td>MTNL</td>
<td>01/01/02</td>
<td>31/10/07</td>
<td>49,049</td>
<td>13,085</td>
<td>9,298</td>
</tr>
<tr>
<td>6</td>
<td>Satyam</td>
<td>01/01/02</td>
<td>31/10/07</td>
<td>53,376</td>
<td>16,122</td>
<td>8,673</td>
</tr>
<tr>
<td>7</td>
<td>HUL</td>
<td>01/01/02</td>
<td>31/10/07</td>
<td>49,742</td>
<td>12,444</td>
<td>7,776</td>
</tr>
<tr>
<td>8</td>
<td>Ranbaxy</td>
<td>01/01/02</td>
<td>31/10/07</td>
<td>57,502</td>
<td>9,975</td>
<td>7,481</td>
</tr>
<tr>
<td>9</td>
<td>ITC</td>
<td>01/01/02</td>
<td>31/10/07</td>
<td>50,349</td>
<td>8,864</td>
<td>7,264</td>
</tr>
<tr>
<td>10</td>
<td>M &amp; M</td>
<td>01/01/02</td>
<td>31/10/07</td>
<td>56,020</td>
<td>8,739</td>
<td>7,232</td>
</tr>
<tr>
<td>11</td>
<td>Maruti</td>
<td>09/07/03</td>
<td>31/10/07</td>
<td>46,591</td>
<td>8,599</td>
<td>7,157</td>
</tr>
<tr>
<td>12</td>
<td>Ambuja Cements</td>
<td>01/01/02</td>
<td>31/10/07</td>
<td>47,152</td>
<td>7,643</td>
<td>6,793</td>
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<tr>
<td>13</td>
<td>ICICI</td>
<td>31/01/03</td>
<td>31/10/07</td>
<td>47,754</td>
<td>7,989</td>
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<td>14</td>
<td>ONGC</td>
<td>31/01/03</td>
<td>31/10/07</td>
<td>48,223</td>
<td>9,567</td>
<td>5,978</td>
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<td>15</td>
<td>SCI</td>
<td>31/01/03</td>
<td>31/10/07</td>
<td>45,178</td>
<td>6,962</td>
<td>5,574</td>
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<td>16</td>
<td>Hindalco</td>
<td>01/01/02</td>
<td>31/10/07</td>
<td>56,464</td>
<td>6,114</td>
<td>5,353</td>
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<td>17</td>
<td>BPCL</td>
<td>01/01/02</td>
<td>31/10/07</td>
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<td>7,780</td>
<td>5,347</td>
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<td>18</td>
<td>Cipla</td>
<td>01/01/02</td>
<td>31/10/07</td>
<td>56,632</td>
<td>5,665</td>
<td>4,833</td>
</tr>
<tr>
<td>19</td>
<td>Dr. Reddy’S</td>
<td>01/01/02</td>
<td>31/10/07</td>
<td>55,490</td>
<td>5,805</td>
<td>4,721</td>
</tr>
<tr>
<td>20</td>
<td>Bank Of India</td>
<td>29/08/03</td>
<td>31/10/07</td>
<td>40,364</td>
<td>6,203</td>
<td>4,660</td>
</tr>
<tr>
<td>21</td>
<td>Andhra Bank</td>
<td>29/08/03</td>
<td>31/10/07</td>
<td>33,559</td>
<td>5,896</td>
<td>4,518</td>
</tr>
<tr>
<td>22</td>
<td>Wipro Ltd.</td>
<td>31/01/03</td>
<td>31/10/07</td>
<td>47,780</td>
<td>6,417</td>
<td>4,505</td>
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<tr>
<td>23</td>
<td>Syndicate Bank</td>
<td>26/09/03</td>
<td>31/10/07</td>
<td>32,941</td>
<td>5,759</td>
<td>4,389</td>
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<td>24</td>
<td>UBI</td>
<td>29/08/03</td>
<td>31/10/07</td>
<td>36,327</td>
<td>5,166</td>
<td>4,122</td>
</tr>
<tr>
<td>25</td>
<td>BHEL</td>
<td>01/01/02</td>
<td>31/10/07</td>
<td>65,471</td>
<td>6,051</td>
<td>4,083</td>
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<tr>
<td>26</td>
<td>PNB</td>
<td>29/08/03</td>
<td>31/10/07</td>
<td>49,229</td>
<td>4,661</td>
<td>3,870</td>
</tr>
<tr>
<td>27</td>
<td>Bank Of Baroda</td>
<td>29/08/03</td>
<td>31/10/07</td>
<td>49,764</td>
<td>4,457</td>
<td>3,589</td>
</tr>
<tr>
<td>28</td>
<td>Canara Bank</td>
<td>29/08/03</td>
<td>31/10/07</td>
<td>46,500</td>
<td>4,676</td>
<td>3,262</td>
</tr>
<tr>
<td>29</td>
<td>Bajaj Auto</td>
<td>01/01/02</td>
<td>31/10/07</td>
<td>63,292</td>
<td>2,331</td>
<td>1,790</td>
</tr>
<tr>
<td>30</td>
<td>Grasim</td>
<td>01/01/02</td>
<td>31/10/07</td>
<td>64,195</td>
<td>2,086</td>
<td>1,761</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td></td>
<td><strong>15,32,58</strong></td>
<td><strong>2,53,412</strong></td>
<td><strong>1,92,642</strong></td>
</tr>
</tbody>
</table>

Source: Column 1 to 6 from [www.nseindia.com](http://www.nseindia.com)

Note: The details for non-dividend paying stock are explained in paragraph 2.2.3.5 under research methodology.
From the table, it is observed that out of 15,32,587 call options offered by the NSE, only 2,53,412 were traded during the period related to the said 30 companies. Out of the options offered by NSE only about 16.43% of the offered options are traded.

This is the proof that Indian investors are not trading / using the most of the options offered by the NSE. The reasons may be many. They may not be interested in options as they do not understand them. Many books on derivatives may offer some theoretical knowledge. But, many Indians might be of the opinion that theories are useless in practice. Or the detailed aspects of the options are not available to them. The other reasons may be lack of awareness of correct asset pricing model and understanding it in all respects like its weaknesses and strengths.

Hence, these kinds of empirical analysis and research studies will induce the investors to understand the option pricing method, its strengths and weaknesses. By understanding the systematic errors (biases) of the model, the investors can adjust the prices accordingly and benefited. As confident goes higher, more people will participate in the options market. In turn, as the breadth and depth of the market will convert it as efficient market and benefit to the investors themselves.

1.6 SCOPE OF THE STUDY

This empirical study is made in Indian stock option market, which is only about seven years old. Index Futures Contracts were introduced in June 2000, Index Options, introduced in June 2001 and Stock Options, introduced in July 2001.

Initially, only the blue chip stocks with high trading volume and financial soundness were allowed to trade in the Futures & Options (F&O) (derivatives) markets. Only 31 company’s stocks were in option trading till 2003. Gradually, the rules were relaxed and more and more stock were allowed in the F&O
sections. In 2007, about 223 company’s stocks were included in option trading. This study is made on those stocks options which are traded at least from January 2002 to October 2007. Both National Stock Exchange and Bombay Stock Exchange are trading stock options, but, the volume of the National Stock Exchange is more than 98% of the total traded value and volume in India and hence the study is confined to the options traded at National Stock Exchange (NSE). More details are given in under research methodology, in the chapter II. The options are offered in Stock market Index such as Nifty, the stocks, etc. This study is restricted to the stock options that too call options only, as the BS model itself is basically designed for call options.

1.7 OBJECTIVES OF THE STUDY

1.7.1 TESTS FOR THE BLACK-SCHOLES MODEL

Assessments of a model's validity can be done in two ways. First, the model's predictions can be confronted with historical data to determine whether the predictions are accurate, within some statistical standard of confidence. Second, the assumptions made in developing the model can be assessed to determine if they are consistent with observed behavior or historical data.

A long tradition in economics focuses on the first type of tests, arguing that "the proof is in the pudding". It is argued that any theory requires assumptions that might be judged "unrealistic", and that if we focus on the assumptions, we can end up with no foundations for deriving the generalizations that make theories useful. The only proper test of a theory lies in its predictive ability: The theory that consistently predicts best is the best theory, regardless of the assumptions required to generate the theory.
Tests based on assumptions are justified by the principle of "garbage in-
garbage out." This approach argues that no theory derived from invalid assumptions can be valid. Even if it appears to have predictive abilities, those can slip away quickly when changes in the environment make the invalid assumptions more pivotal.

Our analysis takes an agnostic position on this methodological debate, looking at both predictions and assumptions of the Black-Scholes model.

The main objective of this research is to make an empirical study of Black - Scholes option pricing model in Indian stock call - option market and to find an improvement in the model for better prediction ability, if possible.

The sub objectives are

i) To measure the sensitivity of the model in respect of each factors of option pricing such as Stock Price, Strike Price, Time to Maturity, Volatility of stock returns, and Risk – Free Rate of Interest.

ii) To analyze the predictability and the biases of the model, if any, towards volatility, Time to maturity, Moneyness, risk free rate of interest etc.

iii) To verify the model’s specification by analyzing the residuals of the model, such as distribution of the residuals, mean, median, and momentum analysis, correlation of residuals with the factors of option price, etc.

iv) To analyze the validity of the model assumptions such as lognormal returns of stocks, random walk of the stock price etc.

v) To find an improvement in the theoretical or practical part of the model so that its prediction ability improves at least 5 to 10 percent.
1.8 REVIEW OF LITERATURE – RESEARCH STUDIES

There are many studies about the Black - Scholes model, the most important studies published in the Journal of Finance, Number one Finance journal in the world, are studied in depth and the findings are listed below.

The authors Black, Fisher, and Scholes, Myron,[24] themselves admitted some biases of the model in their research paper, “The Valuation of Option Contracts and a Test of Market Efficiency”, expressed as “Using the past data to estimate the variance caused the model to overprice options on high variance stocks and underprice options on low variance stocks. While the model tends to overestimate the value of an option on a high variance security, market tends to underestimate the value, and similarly while the model tends to underestimate the value of an option on a low variance security, market tends to overestimate the value”.

During 1979, Macbeth, James D., and Merville, Larry J.[92] in their research paper, “An Empirical Examination of the Black - Scholes Call Option pricing Model” revealed that B-S model predicted prices are on average less (greater) than market prices for in the money options (out of the Money) and also had biases over the life of the options also. This study has some coincidences and differences with the above findings which are also explained in this chapter.

LIU, JINLIN [86], while researching the topic, “An Empirical Investigation of Option Bounds Method”, opined that the BS formula worked better as a whole than the Option Bounds Method. This phenomenon is interesting because although one key assumption of BS’ is untrue, BS still works well for real data of options. One possible explanation is that too many participants in the market are using BS formula to price the options. Even when BS cannot work well in reality, they apply something as OAS to modify the results.
However, the modifiers are still are based on the BS method. Consequently, the BS method would fit the price in the market well.

Fortune, Peter, in his series of Federal Reserve Bank of Boston studies titled “Anomalies in option pricing: the Black-Scholes model revisited” published in New England Economic Review, March-April, 1996 [53], had concluded that “the combined results suggest a 10 to 100 percent error for calls”. He also added “In summary, the probability distribution of the change in the logarithm of the S&P 500 does not conform strictly to the normality assumption. Not only is the distribution thicker in the middle than the normal distribution, but it also shows more large changes (either up or down) than the normal distribution. Furthermore, the distribution seems to have shifted over time. After the Crash an increase in the kurtosis and a shift in skewness occurred”

Ball, Clifford A. and Torous, Walter N. [13] during their study, "On Jumps in Common Stock Prices and Their Impact on Call Option Pricing," compared between Merton’s Jump- diffusion model and the Black - Scholes model. They observed that there were no operationally significant differences in the models.

Empirical evidences confirm the systematic mispricing of the Black - Scholes call option pricing Model. These biases have been documented with respect to the call option’s exercise price, its time to expiration, and the underlying common stock’s volatility. Black [24] reports that the model over prices the deep in-the-money options, while it underprices deep out-the-money options. By contrast, Macbeth, James D. and Merville, Larry J. [92] state that deep in-the-money options have model prices that are lower than the market prices, whereas, deep out -the-money options have model prices that are higher. These conflicting results may perhaps be reconciled by the fact that the studies examined market prices at different point in time and these systematic errors vary with time (Rubinston [111]).
A number of explanations for the systematic price bias have been suggested (Geske and Roll [57]). Among these is the fact that Black - Scholes assumptions of lognormally distributed security price fails to systematically capture the important characteristics of the actual security price process.

1.9 LIMITATIONS OF THE STUDY

The study is limited to National Stock Exchange and limited to stock options, which are traded from January 2002 till October 2007 as the trades in BSE has been less than one percentage compare to NSE trade. The risk-free rates are obtained from the Mumbai Inter-Bank Offer rates (MIBOR) and Mumbai Inter-Bank Bid Rates (MIBID) as taken by the NSE itself. The foreign countries are using T - Bill rates as risk-free rates. But in India the T-bill market is not matured and deep and hence the MIBOR / MIBID are taken as a proxy for the risk – free rates. NSE is also using the same. The study is limited to call options only as the BS model is basically derived for call options.

1.10 CHAPTER PLAN

The first chapter represents a brief history of options, its growth and its importance in International and Indian derivative Markets. Further it includes statement of the problem, objectives, basics of options, factors influencing options, pricing methods, basics and derivation of Black – Scholes formula and its assumptions. An overview of earlier studies, the empirical studies in the foreign nations and its findings are included. A brief notes on the individual journal papers relevant to our study and their findings are narrated. It ends with the limitations of the research.
The second chapter deals with a brief research methodology applied for the empirical verification, including the statistical tools used / procedures. It also narrates the data collection, culling out the traded data from the offered options, method of removing the risk-free arbitrage opportunities, exclusion of data related to ex-dividend dates within the option life and finalization of data for analysis.

In chapter three, each of the five factors is taken separately and studied in depth about the sensitivity of the price of option by changing the variables and parameters step by step. The price of the option is more sensitive to some of the factors than others. Full examination details are given in this chapter.

Fourth chapter explains the model’s prediction ability and pattern of the option pricing calculated using BS formula, towards its determinants like volatility of returns of the stock, exercise price, moneyness of the option, risk – free rate, life of the option etc. The findings of the study are explained in depth. Partial study with the data up to 30.6.2004 on biases of the model had been published in the Book titled “Business Management Practices, Policies and Principles” by The Allied Publishers Private Limited, New Delhi, after editing by faculty of Indian Institute of Management, Indore [105] (Annexure I).

In the Fifth chapter, residual analysis being one of the important tools of modern econometrics is used to analyze the model adequacy; the interesting findings are explained in length and breadth in this chapter. This study “Residual Analysis of the Model” was presented in the National Conference on Business Research conducted by P.S.G. Institute of Management and won the Best Paper Award in finance session and the same was published in the Journal of Management Research Volatility.1, No.2, April - June 2006, [106] (Annexure II).

Sixth chapter deals with the BS model assumptions while developing the model and examines the validity of these seven important assumptions in
practice. The empirical verification of one of the main assumptions of the model that the stock price follows a random walk itself is vast and equals to a mini research in technical analysis. The fascinating empirical findings are narrated in depth in this chapter.

Chapter seven reveals the efforts made to improve the BS model's predictability and the logic behind the selection of the variables / parameter for improvement. The number of improvements effected and the percentage of improvement achieved due to the envisaged new method are enlightened in depth.

Chapter eight deals with the fulfillment with the objectives of the study considered. The findings emerged out of this empirical study is enumerated and suggestions of the researcher are given to the investors and academicians and the conclusion. The scopes of further research and recommendations for further research are added towards the end.