CHAPTER V
SHAPE ANALYSIS
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5.1 Introduction

Most natural tectonically deformed or even undeformed pebbles in conglomeratic rocks hardly even confirm to true ellipsoidal shapes, that is, they do not satisfy the equation for an ideal ellipsoid given by

\[
\frac{x^2}{A^2} + \frac{y^2}{B^2} + \frac{z^2}{C^2} = 1
\]

eq 5.1

Where A, B and C are semi-axes (in decreasing order of length) of the seemingly ellipsoidal mass or pebble, arranged parallel to the cartesian co-ordinates X, Y and Z respectively. This fact was first pointed out by Burns and Spry (1969) while analysing strain from deformed conglomeratic rocks of Goat Island, north of Tasmania. Gardner (1965) in a short note as a popular article suggested that to exactly define a seemingly ellipsoidal particulate shape, the exponent 2 in the equation for an ideal ellipsoid be replaced by p (here called n). Hence the equation takes the form,

\[
\frac{x^n}{A^n} + \frac{y^n}{B^n} + \frac{z^n}{C^n} = 1
\]

eq 5.2

If n < 2, the overall ellipsoidal shape may be described as subellipsoidal. If n > 2, the shape may be termed superellipsoidal.

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For elliptical shapes also, the ideal ellipse equation,

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1$$  

may be modified to

$$\frac{x^n}{A^n} + \frac{y^n}{B^n} = 1$$  

The elliptical particulate shapes may be described as subelliptical ($n < 2$) or superelliptical ($n > 2$).

At about the same time the sedimentologists were engaged in research on seemingly particulate shapes of sand particles. For example Ehrlich and Weinberg (1970) devised a technique based on Fourier analysis in terms of the peripheral points which are converted into polar co-ordinates. A computer programme was first written to find the exact centre of the grain shape and then compute the harmonics of each of the peripheral co-ordinates to define or characterize the exact shape. However, the analysis is valid for rather irregular shapes of sand particles such as those that are wind-borne.

5.2 **Analytical Background**

In the area under investigation, the chocolate tablet boudins, extracted from the outcrop, do not show a true
ellipsoidal shape but instead, the shapes are of various kinds. For example, the ellipsoidal shapes may be like that of a barrel, an asymmetrical barrel as a result of shearing, nearly lozenge shaped or like that of "diamond suite" in playing cards, tear-drop shapes, asymmetrical lozenzes, very long ribbons, ellipsoidal with tips turned due to modification under more ductile regime of deformation, dominated by shear couples.

In this chapter an attempt is made to exactly define the shape of a boudin in various sections and relate this shape to the rheology of the material, the viscosity contrast between the layer undergoing boudinage, and the matrix that immediately surrounds it. All shapes for an apparent elliptical sections are quantified in terms of the exponent. In the first account, any kind of shape is shown to be departing from the true elliptical shape in four different quadrants of the section being examined, generally by finding the value of n for each quadrant. For pinch and swell type of structures, the value of n becomes even less than unity. In other words, to define the shape of a boudin in cross section, the original basic true elliptical shape is taken into consideration and how this shape departs under different conditions.

In the second section, the shapes of some of the three dimensional chocolate tablet boudins are considered
by finding their \( n \) values in all three sections. The overall volume of these ellipsoids is calculated on the basis of the equation for a true ellipsoid first, given by

\[
V = \frac{4}{3} \pi r_1 r_2 r_3
\]

eq. 5.5

Where, \( r_1, r_2 \) and \( r_3 \) are the three radii of the apparent ellipsoid. The volume is then calculated assuming the ellipsoid of revolution concept (Spiegel 1968 pp 105 and 235). However, since the original shape was not spherical or spheroidal but simply planar bedding, this latter study is rather of subordinate interest. But, the study of sections and overall shapes is related to the deformation regime, that is, whether elastic or plastic, rheology contrasts and the role of shear during deformation and its degree or extent in each of the three principal planes of the finite deformation ellipsoid.

5.3 The Spectrum of Shapes based on \( n \) values

Fig. 5.1C schematically shows the outline of a true ellipse that satisfies the equation for the ellipse with exponent of the values of denominators and numerators being 2. But the first part of the same figure (Fig. 5.1A, see also 8.1B) shows a typical "diamond" or lozenge shape. For such a shape, the value of the exponent \( n \) in the equation for ellipse takes a value of unity. In other words, the equation is reduced to
FIG 5.1
\[
\frac{x}{A} + \frac{y}{B} = 1
\]  \hspace{1cm} \text{eq. 5.6}

Fig. 5.1B shows a subelliptical shape of \( n \) value equalling 1.5.

On the other side of the spectrum, we have values of exponent exceeding 2 so that, the overall elliptical shape is actually superelliptical. For example Figs. 5.1D and 5.1E (see also 5.1I) show superelliptical shapes with \( n \) values 3 and 4 respectively. Such boudins are typically barrel shaped, formed by elastic/brittle failure (Lloyd and Fergusan 1982, Lloyd et al. 1981), but, these barrel shaped boudins get their tips modified by flow of material under an advanced stage of plastic deformation. The exactly rectangular boudin shapes as would be found in quartz veins enclosed in pelitic schists, and in case of these, if the basic elliptical equation is followed, the value of \( n \) equals \( \infty \) (Fig. 5.1F).

In boudins that are formed under ductile deformation, it is usually found that their sides are curved inwards in some cases. A term "infraelliptical" is used here in such a case where the value of exponent is even less than unity, the value one being for ideal diamonds. Figs. 5.1G and 5.1H show such shapes (see also Fig. 5.1J) the values for the exponent are 0.75 and 0.50 respectively.
It is needless to point out that we cannot take ratios \( x/A \) and \( y/B \) and then obtain exponents for these ratios. Nor does there exist a relationship between exponent \( n \) and the sum obtained. That is, if in the equation,

\[
\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1
\]  

the value obtained is more than 1, the value of exponent must exceed 2 and if the value obtained is less than 1, the exponent must be less than 2. But if all values \( x, y, A \) and \( B \) are taken together with the value of the sum on the right hand side of the above equation, the value of exponent cannot be found out.

Based on this observation, a computer program was written in BASIC language to run on a PC/XT computer, to compute the value of exponent. As pointed out later, the program also calculates values of \( x_1 \) and \( y_1 \) at the \( i \)th point on the periphery, given the angle between \( x \) axis and the radius from centre that intersects the periphery in the \( i \)th point \( (x_i, y_i) \).

5.4 Analytical Procedure

The long axis \( 2A \) of a single boudin was drawn by joining the two end points and the short axis \( 2B \) at \( A \) (midpoint of long axis) irrespective of the fact whether this was the maximum short axis width of the boudin or not.
In each quadrant of the elliptical shape, from the centre, radii \( r_i \) were drawn at regular interval of 5° to meet the periphery of the boudin section in the \( i \)th point. The values of \( X_i \) and \( Y_i \) at the \( i \)th point were computed from the simple relationship:

\[
X_i = X \cos \theta_i \quad \text{and} \quad X_i = X \sin \theta_i
\]

\[
\text{eq. 5.8a} \quad \text{eq. 5.8b}
\]

where \( \theta_i \) is the angle between the \( i \)th radius from the \( X \) co-ordinate axis, \( X \) being equal to \( A \), the half of the long axis for the quadrant and \( Y_i \), the length of the short axis for the quadrant. Thus 18 values of \( X_i \) and \( Y_i \) at 18 \( i \) points were obtained and these were averaged for each quadrant \(( \bar{X}_i \text{ and } \bar{Y}_i \) from;

\[
\bar{X}_i = \frac{\sum_{i=1}^{n} X_i}{n} \quad \text{where } n = 18
\]

\[
\text{eq. 5.9a}
\]

and

\[
\bar{Y}_i = \frac{\sum_{i=1}^{n} Y_i}{n} \quad \text{where } n = 18
\]

\[
\text{eq. 5.9b}
\]

Thus for each quadrant \( \bar{X}_i \) and \( \bar{Y}_i \) values were obtained and these were used to compute the mean \( n \) value for each quadrant. However, since a large volume of data had to be handled, all these computations were done using the computer program given in Appendix A3.4. It may be pointed out here that the ellipticity of a section may approach a near circular pattern in some cases. Such shapes may show quadrants which
Fig. 5.2 - A - The shape of an ellipsoidal boudin. AB view. Note the horizontal lineation $L_1$.

Fig. 5.2 - B - Subelliptical AB outline of the boudin. SL - Stretching downdip lineation.
Fig. 5.3 - A - Five different shapes of the "Pebbles", all showing their AB outlines, AB being parallel to foliation plane.

Fig. 5.3 - B - View of a "pebble" as seen on the fabric plane.
Fig. 5.4 - A - Tear drop shapes (flattened) of boudin in horizontal plane.

Fig. 5.4 - B - Shapes of boudins in vertical plane (AC sections of boudins).
Fig. 5.5 - A - Two boudins with nearly elliptical outlines in horizontal section.

Fig. 5.5 - B - Tear drop and subelliptical shapes of boudins in horizontal sections
Fig. 5.6(i)- A - $S_2$ crenulation cleavage planes. Note folding of $S_1$ and early boudins. Locality. North of GirI dam.

Fig. 5.6 (i) - B - A chain of boudins in a vertical section perpendicular to fabric, showing clear stretching of a layer. Locality - South of Megardha - This is the $\lambda_1 \lambda_3$ vertical section since at the final stage of deformation, the $\lambda_2 \lambda_3$ vertical plane becomes one of the $\lambda_1 \lambda_3$ because of swapping between $\lambda_1$ and $\lambda_2$. Note that the quartzite boudins are sinistrally displaced which clearly suggests that this was an updip sinistral shearing along the foliation during late stages. Long boudin and part of exposed $\lambda_1 \lambda_2$ can be seen at extreme right.
Fig. 5.6(ii) - A - Views of the foliation plane from the locality near canal, 2 kms south of Giri, showing stretching of the boudins. This is particularly noticeable in the lower photograph (see arrow).
are subelliptical because, the ellipticity is a measure of the length of both $X$ and $Y$ axes or $A$ and $B$ axes parallel to $X$ and $Y$ co-ordinates respectively. The angle $\theta$ of the straight line joining tips of $X$ and $Y$ axes with $Y$ axis is given by

$$\theta = \cos^{-1} \frac{Y}{X}$$  \hspace{1cm} \text{eq. 5.10}

and the angle $\theta'$ with $X$ axis of this line will naturally be given by

$$\theta' = 90 - \theta \text{ or}$$

$$\theta' = \sin^{-1} \frac{Y}{X}$$  \hspace{1cm} \text{eq. 5.11}

In other words, shapes appearing to be closer to circular, may not necessarily indicate superelliptical shapes but could also indicate subelliptical shapes.

5.5 **Two-dimensional Shape Analysis**

Fig. 5.7F shows $\lambda_1 \lambda_3$ section of a boudin which shows tapering at both ends and has a near lozenge shape. A closer examination of this shape reveals that opposite quadrants are similar. But in one of the quadrants, there is slight inward "bowing" of the quadrant periphery but this is compensated by the slight outward "bowing" in the opposite quadrant (right top and left bottom respectively). In case of the other two opposite quadrants (left top and right bottom) the curvature is nearly the same. Thus the
values for these two quadrants are quite alike, being
given by exponents 1.20999 and 1.0999, the latter value
because of a slight inflexion in the otherwise smooth peri-
phery. The values in the former case obtained to be 1.1899
and 1.25. Thus the opposite quadrants do not exactly show
like values but one slightly higher than the other. This
is due to the fact that out of the two opposite quadrants
one has a tendency to slightly "bow in" and the other a
tendency to "bow out". This is due to the effect of shear
and since one tip is more sharp and rather indistinctly
curved along a ductile sinistral shear, the shear sense
along the foliation planes themselves may be dextral.

Fig. 590 is the shape of another boudin in $\lambda_1 \lambda_3$
plane. In this case also we have a similar situation, though
slightly different. Overall shape appears to be superellipti-
cal. Two quadrants on the left hand side have n values 1.58
and 1.52 while the quadrants on the right hand side have
superelliptical n values of 2.2099 and 2.029. This probably
suggests that originally, the boudin was formed as a
symmetrical barrel but then inhomogenous flattening within
the plane of fabric caused a flow that changed its shape as
a half barrel half lozenge type. This shape has evolved due
presumably to less viscosity contrast on side of the boudin
than on the other (between the boudin and the matrix that
immediately is contiguous with it). This reflects a gradational
composition change between the boudinaged quartzite layer and the surrounding semipelitic material, or the compositional difference between the boudin itself.

Fig. 5.8K shows a typical tear drop shape boudin with one end tapering and the other rounded. The tapering ends of the boudins clearly give n values of subellipticity, these being 1.21 (left) and 1.32 (right), while the lagging or bottom end shows superelliptical shapes. The values of n for the bulging end (bottom of a regular tear drop when it is falling down on the cheek) obtain to be 4.5 for the left hand quadrant and 2.459 for the right hand side quadrant.

Fig. 5.9T is also that of a tear drop shaped boudin in $\lambda_2 \lambda_3$ plane. One end is tapering than the other but the tear is an elongated one. The values of n which form the quadrants for the tapering end are $1.7699$ (left hand side) and $1.1199$ (right hand side). These shapes are therefore subelliptical. Since the tear drop shape is elongated, the lagging end or the bulging end also shows the higher values, one being nearly true elliptical (right hand side) with n value of 2.099 and the other (left hand side) 1.9999. Thus though the shape is a tear drop one the overall shape is subelliptical, but tapering end quadrants show higher subellipticity with lower values of n while bulging end show lower subellipticity with higher values of n.
Fig. 5.9R shows a boudin with bulging centre and tapering ends. The opposite quadrants right top and left bottom show lower values of \( n \), these being 1.13 and 0.969 (note slight "bowing in" of periphery). The other two quadrants show values of 1.29 (left top) and 1.279 (bottom right). Hence, the overall like values of \( n \) in opposite quadrants certainly indicate the role of shear in producing this very slight, perhaps, entirely imperceptible asymmetry of the lozenge.

Fig. 5.7E shows a perfectly symmetrical boudin. At a first glance it appears that it is truely elliptical. But, it has an overall subelliptical shape. It's opposite side quadrants show greater degree of subellipticity with \( n \) values of 1.33 and 1.52 while the right side shows a mixed shape, the right top quadrant being superelliptical with \( n \) values of 2.4199 and the lower left quadrant being slightly subelliptical with \( n \) value of 1.6199.

Fig. 5.7D is the asymmetrical foliation "fish" of Hanmer (1986) with large drawn out tips shown in photograph (Fig. 3.42A) suggestive of dextral shearing along the foliation planes. The long horns of these seeming barrel shaped boudins were ignored in the shape analysis. The overall shape is again subelliptical with opposite quadrants showing different degrees of subellipticity. The principally low subellipticity is due to the inward "bowing" of boudin
in opposite quadrants and the matrix material flowing at an acute angle into dextral shears in case of the two remaining opposite quadrants, which tend to straighten the periphery. At some places "infraelliptical" type of periphery is observed, even though only at few places along the periphery of a quadrant. Since the radii are equally spaced, the values of subellipticity get considerably reduced because $\bar{x}_1$ and $\bar{y}_1$ values are considered. Thus, for such peripheries the values of $n$ are low. But, asymmetry of the boudin is reflected in the $n$ values of 1.39 and 1.3 for the two opposite quadrants and 1.24 and 1.39 for the other quadrants. The shape is obviously due to low viscosity contrast, between boudins and matrix as is clear from the photograph (Fig. 3.42A) as well.

At a first glance, the Fig. 5.7C shows a typical broad barrel with much rounded ends and a superelliptical shape. But, the values for the sides (if ends are ignored) obtain to be subelliptical, with $n$ values of 1.45, 1.96, 1.58 and 1.54. But if one considers the broad blunt ends of these boudins, the values obtained are 2.1, 2.23, 2.4 and 2.05. Thus, were it not for the blunt ends, the overall shape of this boudin also is subelliptical but the data at the tapering ends gives it an overall superelliptical shape. Presumably, this boudin escaped flattening and deformation at its end, due to its high competence or high competence
contrast between the boudin and the immediately contiguous matrix.

Fig. 5.83 also has a tear drop shape with two quadrants more subelliptical than the other. The two quadrants which taper show low values of $n$, i.e. 1.36 and 1.37 (the symmetry of tapering end is so striking indeed). The bulging end quadrants show $n$ values of 1.7699 and 1.7399. These two values are also so much alike that they typically reflect the bilateral symmetry of the boudin about its long axis in the $\lambda_1 \lambda_2$ plane (actually intermediate axis when overall deformation is considered). This reflects homogeneity of composition of the boudin and also of the matrix adjacent to it on all sides.

Fig. 5.95 shows a very unusual shape in the $\lambda_1 \lambda_2$ section of the boudin. The overall boudin is quite bulging but it suddenly tapers at its end giving rise to a beak with "bowing" in of margins near the beak or local "infra-elliptical" shape. The tapering and beak like quadrants have therefore extremely low values of $n$ of 1.069 and 1.06 while the bulging end has both sub- and superelliptical values of 1.58 and 3.099. This shape clearly indicates that how initiation of boudinage after swapping of principal strains occur on the $\lambda_1 \lambda_2$ planes.

Fig. 5.7B shows a asymmetrical lozenge type structure in $\lambda_1 \lambda_3$ plane with clockwise turning of tips suggesting dextral shear couple operating along foliation planes. One
of the quadrants shows pronounced curving and "infraelliptical" shape and has the lowest mean value of \( n \) equalling 0.999. The quadrant opposite to this gives a value of \( n \) equal to 1.1399. The other two quadrants give quite identical values of \( n \) for this asymmetrical lozenge of 1.25 and 1.299.

Fig. 5.9P is the shape of a boudin on the \( \lambda_1 \lambda_2 \) fabric plane. The boudin is sheared and has the shape of a torpedo, as described by Heron (1953). Two opposite quadrants with slightly bulging peripheries show values of \( n \) of 1.619 and 1.899. The pronounced outwardly bowing periphery shows \( n \) value of 2.049 while its opposite quadrant shows \( n \) values of 1.279. Thus the effect of shearing within the fabric planes is noticeable on boudins on the \( \lambda_1 \lambda_2 \) planes as well.

Fig. 5.9Q at the first glance appears to have a superelliptical shape. This too is observed on the \( \lambda_1 \lambda_2 \) plane. But, again, this is due to very small ratio of long to short axes. The values of \( n \) for the relative more blunt end are 1.949 and 1.609. On the other hand, the less blunt end gives \( n \) values of 1.44 and 1.48. This shape perhaps shows high finite strain on XY planes because, in case the strains are low, usually long boudins or very long pinch and swell structures are observed. Basically, the high viscosity contrast between the pelitic matrix and psammite boudin causes further splitting of the boudin in the \( \lambda_1 \lambda_2 \) plane.
Fig. 5.10W shows another boudin on the $\lambda_1 \lambda_2$ plane with a typical shear sense that is anticlockwise or sinistral. Two opposite quadrants are typically superelliptical in shape with $n$ values 2.06 and 2.43 and the other two are strikingly or very distinctly sub- to infraelliptical with $n$ values of 1.11 and 1.24. The sinistral sense is typical of $\lambda_1 \lambda_2$ planes.

Another boudin on $\lambda_1 \lambda_2$ plane (Fig. 5.10X) again shows a bulging middle with pinching, which is pronounced on both sides but more pronounced in the updip direction of the fabric plane. Because of extreme bulging in the centre, the lower two quadrants which show less pronounced pinching (only in the form of a little beak) show one superelliptical outline (left hand top quadrant) with $n$ values of 2.33 and the other nearly elliptical (left hand bottom quadrant) with $n$ values 1.93. The more prominently pinched quadrants show typically low $n$ values of 1.28 and 1.92. But the values are not symmetrical suggestive of sinistral shearing.

Fig. 5.10C' is also shown in a photograph (Fig. 5.2B). It typically shows an overall subelliptical shape in $\lambda_1 \lambda_2$ plane of which the boudin depicts the view. The $n$ values for two quadrants suggest subelliptical shape i.e. 1.759 and 1.739, one quadrant is nearly elliptical with $n$ value of 1.919 and one quadrant slightly superelliptical having $n$ values of 2.309. If originally conglomerate pebbles were present, they would have got considerably flattened in the
plane of schistosity, with superelliptical outline in $\lambda_1 \lambda_2$ planes. The fragments are therefore, tectonically ruptured boudins and therefore show anomalous shapes.

Fig. 5.10A' shows again another boudin with $\lambda_1 \lambda_2$ plane view and traced from photograph (Fig. 5.2A). The shape is anomalous with angular edges at both ends. Its one side is nearly straight and the other side conspicuously convex. For the nearly straighter side, i.e. right hand side quadrants, the values of n are low, being 1.58 and 1.279. For the side convexity of two quadrants, the n values are 1.859 and 1.749. These shapes are rather strange as they show that the "pebble" or the boudin has got this shape owing to subhorizontal shear, parallel to fabric planes which is adduced by the subhorizontal mineral lineation on the boudins. This subhorizontal shear may have given rise to torpedo shapes, also observed more particularly near Giri, with ends of boudins dragged differentially to produce an overall fish like shape.

Fig. 5.10B' is also a $\lambda_1 \lambda_2$ view of a boudin traced from the photograph (Fig. 5.3B). The overall shape in this case is strikingly superelliptical with n values for four quadrants being 2.04, 1.99, 2.43 and 2.06. But again it is seen that, one quadrant is bulging than the rest. This in extreme cases may give rise to truncheon shapes or torpedo shapes with further shear which was perhaps superimposed during $F_1$ and $F_2$ folding.
Fig. 5.10Y shows a boudin in $\lambda_1 \lambda_2$ section. The upper half of the boudin is superelliptical with $n$ values of 2.51 and 2.67, and the lower half is subelliptical with $n$ values of 1.69 and 1.84. This feature with upward pinching is commonly seen on $\lambda_1 \lambda_2$ planes. The pinching of boudins in the downdip direction is rarely seen which suggests that there was sinistral thrusting or ductile shear, up the planes of fabric after $\lambda_1$ and $\lambda_2$ swapped during latter part of deformational process.

Fig. 5.10Z is very anomalous and if rounded quadrant had been present and the length greater, a typical cuttle bone type shape would have been produced. Again, left hand quadrants are distinctly flattened with nearly straight slightly convex side. These quadrants give $n$ values 1.38 and 1.66. On the other hand, the irregular quadrant gives $n$ values of 1.63 because of the bowing in of the boudin margin. The remaining quadrant gives a value of $n = 1.11$. Note that this is almost like a straight line.

Fig. 5.10D' shows a nearly elliptical shape in $\lambda_1 \lambda_2$ section with the four quadrants showing values of $n$ of 1.24, 1.91, 2.37 and 1.94.

Fig. 5.10E' is the commonly seen shape on the $\lambda_1 \lambda_2$ plane with a pronounced pinching into a large pinched portion in the updip direction of fabric and a broad middle and slight beak at the bottom, after which long pinched
neck of another boudin begins. The $n$ values for the long downward pinched out boudin neck has $n$ values of $0.939$ and $1.08$ while the upper bulging portion with little beak at the bottom, has $n$ values of $1.31$ and $1.07$. Thus, there is a perfect bilateral symmetry with regard to $n$ values along the long axis of the boudin.

Fig. 5.8L shows the outline of a boudin in the $\lambda_1 \lambda_3$ plane traced on a large scale from the photograph (Fig. 3.17A). The boudin has an overall subelliptical shape, the values of $n$ for the two quadrants on one side of the short axis are $1.4$ and $1.46$ and those for the other side of the short axis are $1.77$ and $1.51$. Alongside this boudin, also occurs another small boudin (Fig. 5.7I) traced from photograph (Fig. 3.17A) on large scale and this shows as in some of the previous cases, one end more tapering than the other. The pinched out portion of quadrants have $n$ values of $1.16$ and $1.07$. The quadrants on the other side of the short axis show $n$ values of $1.35$ and $2.02$. Hence bilateral symmetry either side of the long axis in this section is lacking. This may be due to the inhomogenous flow inside the boudin. One could describe this as a flattened tear drop shape.

Fig. 5.7G shows a boudin shape in $\lambda_1 \lambda_3$ plane and anticlockwise turning of tips suggests a sinistral shear sense. It is a typical feature of shear regime and an overall asymmetrical lozenge with slightly bulging peripheries of a
quadrant and slightly "infraelliptical" or "bowing in" shape of the opposite quadrant. Thus the values of the opposite quadrants are not alike. One would yield a shape of high \( n \) values and the other of lower \( n \) values. Thus the two sets of opposite quadrants, depicting more difference in their shapes, have \( n \) values of 1.5 and 1.19. While, the other two quadrants depicting less curvature difference have \( n \) values of 1.35 and 1.34, or are more or less identical. This is typical of some of the boudins or asymmetrical pull-aparts that have horn like ends.

Fig. 5.7H shows inhomogenous flattening, with more convexity on one side than the other, tapering end with sharp point and the other end being blunt. The outline is found on the horizontal \( \lambda_1 \lambda_3 \) plane. For the less convex and flat side of the boudin, the \( n \) values for the two quadrants are less than those on the relatively more convex side. For the flatter side quadrant, the \( n \) value of the tapering one is 1.05 and that for the blunt end is 1.23. The \( n \) values for the relatively more convex side, are 1.32 and 1.609 for tapering side quadrant and blunt side quadrant respectively. Thus this boudin lacks the bilateral symmetry.

Fig. 5.1A outline of the boudin is in \( \lambda_1 \lambda_3 \) plane. The shape is clearly superelliptical on one side of the short axis, the values of \( n \) for the two quadrants being 2.49 and 2.05. On the other side of the short axis of the boudin
outline, the n values for the two quadrants are 1.67 and 2.05, there being slight disparity in this case.

Fig. 5.9U shows a boudin outline on a horizontal $\lambda_1 \lambda_3$ plane. The overall shape is slightly superelliptical or symmetrical barrel with the shape slightly modified in case of one quadrant. For this anomalous quadrant, the value of n is 1.37, but the n values for the remaining three quadrants are in excess of 2, these being 2.44, 2.65 and 2.23.

Fig. 5.9N shows the outline of a boudin on the $\lambda_1 \lambda_2$ plane. The material seems to pinch at one end, but this was ignored while analysing the shape. One half of the boudin shows subelliptical shape and the other half nearly elliptical. The long axis is of bilateral symmetry. The subelliptical quadrants have n values of 1.23 and 1.65 while the nearly elliptical shapes have n values of 1.61 and 1.32.

Fig. 5.9M is also an outline on the XY or $\lambda_1 \lambda_2$ plane with upper prominently pinched portion and the lower prominently bulged one. The pinched portion quadrants have n values of 0.84 and 1.22 while the bulged quadrants have n values of 1.64 and 2.36. The asymmetry may be because of shear as well as inhomogenous flattening.

Fig. 5.11 is an outline, traced from the photograph (Fig. 3.45B) of the pinch and swell type of boudin seen near Giri on a plane at $5^\circ$ to the $\lambda_1 \lambda_2$ plane. The boudin shows a typical asymmetrical pull-apart structure with two opposite
FIG5-11
The boudin traced from photographs (Fig3.45B) on large scale and analysed for shape in terms of n values.
FIG 5.12
A typical "torpedo" shaped boudin near Giri village on $\lambda_1 \lambda_2$ plane (Traced from a 35 mm coloured transparency diapositive) scale half of natural.
quadrants having matching value of $n$ of 1.413 and 1.507, while the other two quadrants having $n$ values of 1.771 and 1.82. The asymmetrical pull-apart structure has a sinistral sense.

Fig. 5.12 is a typical torpedo shaped boudin (see Fig. 3.49B) found on the $\lambda_1 \lambda_2$ plane near Giri. The general outline has a near plane convex shape but not quite so. The four quadrants have $n$ values as shown of 1.652, 1.20, 1.51 and 1.85. These structures basically originate from sub-horizontal shear couple which tends to drag one end of the boudin pinching it and flattening other end with a slight drag, with development of subhorizontal lineations on the boudin.

On the basis of the above analysis, a classification of the different kinds of boudin shapes has been proposed. This is now described in the following account and a possible mechanism for their formation is suggested.

The classification is also summarised in Table 5.1.

1. $n$ values of all four quadrants exceed 2 - Generally symmetrical barrel shaped boudins formed by elastic deformation followed by plastic deformation and rupture. Matrix would generally flow into the gaps between boudins. But this may not necessarily occur if the fabric develops late during deformation history. On fabric planes this shape would suggest flattening deformation.
Table 5.1. Classification of boudin sectional shapes on the basis of $n$ values. Quadrants are named 1, 2, 3 and 4 clockwise and designated $n_1$, $n_2$, $n_3$ and $n_4$.

<table>
<thead>
<tr>
<th>S.No.</th>
<th>$n$ values</th>
<th>Shape</th>
<th>Mechanism</th>
<th>Fig</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$n_1$ to $n_4$, all greater than 2 barrels</td>
<td>Symmetrical</td>
<td>Initial elastic deformation followed by plastic deformation and rupture.</td>
<td></td>
</tr>
<tr>
<td>2.</td>
<td>$n_1$-$n_4$, all exceed 2 but $n_1$ and $n_3$ and $n_2$ and $n_4$ both have close but different values</td>
<td>Asymmetrical barrels</td>
<td>Foliation parallel shear or breaking along tensile fractures not perpendicular to 1 (Ludar bands ?).</td>
<td></td>
</tr>
<tr>
<td>3.</td>
<td>$n_1$ to $n_4$ equal 1</td>
<td>Ideal lozenge</td>
<td>Shear failure of isotropic rocks under brittle regime.</td>
<td></td>
</tr>
<tr>
<td>4.</td>
<td>$n_1$ to $n_4$ between 1 and 2</td>
<td>Lozenge with curved boundaries pinch flow and swell structures</td>
<td>Ductile shearing or necking and distribution of stresses on two sides of the boudins.</td>
<td></td>
</tr>
<tr>
<td>5.</td>
<td>$n_1$ to $n_4$ less than 2 but $n_1$, $n_3$ and $n_2$, $n_4$ pairs, each have closely matching but different values</td>
<td>Asymmetrical lozenzes</td>
<td>Boudin formed by unequal distribution of stresses on two sides of the boudins.</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.1 (Contd.)

<table>
<thead>
<tr>
<th>S.No.</th>
<th>n values</th>
<th>Shape</th>
<th>Mechanism</th>
<th>Fig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td>(n_1) and (n_4) less than 2 and nearly close, (n_2) and (n_3) also nearly close but exceed (n_1) and (n_4) or even exceed 2.</td>
<td>Typical tear drop shapes</td>
<td>Ductile flow and rupture at initial perturbations where stresses inhomogeneous.</td>
<td></td>
</tr>
<tr>
<td>7.</td>
<td>(n_1) and (n_3) less than 1, (n_2) and (n_4) between 1 and 2</td>
<td>Typical shear related boudins with curving tips</td>
<td>Foliation parallel shear or drag along ductile shears opposed to the sense along foliation plane.</td>
<td></td>
</tr>
<tr>
<td>8.</td>
<td>(n_1) and (n_4) less than 1, (n_2) and (n_3) greater than 2</td>
<td>Typical tear nip or onion cross-section shapes.</td>
<td>Flattening and stretching under ductile regime.</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>(n_1) and (n_4) less than 2 but matching, (n_2) and (n_3) also less than 2 but greater than those of (n_1) and (n_4)</td>
<td>Flattened tear drop shapes or batons.</td>
<td>Flow and flattening under ductile regime.</td>
<td></td>
</tr>
<tr>
<td>10.</td>
<td>(n_1) and (n_2) less than 2 - (n_3) and (n_4) also less than 2 but much less than those for (n_1) and (n_2).</td>
<td>Planoconvex shape or torpedo shaped.</td>
<td>Ductile shear (HSS) parallel to short axis of the boudin in the plane of shear.</td>
<td></td>
</tr>
</tbody>
</table>
(ii) \( n \) values of all four quadrants greater than 2 but those of opposite quadrants have matching or close values, the other two also have much similar values but the values for one set of opposite quadrants are less than those for another set. Asymmetrical barrel shaped boudins, generally formed by development of shear fractures or oblique tensile fractures (tensile fracture is not perpendicular to \( \lambda_1 \) but oblique to it) like ladder bands. In some cases, role of foliation parallel shear cannot also be ruled out.

(iii) \( n \) values of all four quadrants equal to 1 - typical lozenzes or diamond shaped masses, need not be related to a typical boudinage formation but by compression on isotropic rocks.

(iv) \( n \) values of all four quadrants less than 2 - lozenze like boudins or pinch and swell structures formed by ductile stretching or by shear.

(v) \( n \) values of all four quadrants less than 2 but those for the set of two opposite quadrants are identical or close, but less or greater than for the other set - typical boudins formed by shear, under ductile deformation regime - Asymmetrical lozenzes.

(vi) \( n \) values of two adjacent quadrants identical and less than 2. On the other side of the short axis, \( n \) values of adjacent quadrants are generally identical and may
or may not be greater than 2 - typical tear drop shaped boudins showing combination of elastic - plastic initial rupture followed by stretching, or representing viscosity contrast variability that leads to the tapering forward ends and bulging lagging portions.

(vii) n values of two opposite quadrants less than 1 and of the other set of opposite quadrants greater than 1 but less than 2 - typical foliation parallel shear related boudins with both tips curved with the same sense.

(viii) n values of two adjacent quadrants on one side of the short axis less than 1, while, the n values for the two quadrants on the other side of the short axis greater than 2 - typical onion or turnip cross - sectional type boudins flattening together with stretching of layer in $\lambda_{\text{max}}$ direction predominant.

(ix) n values of all quadrants less than 2 but n values of two adjacent quadrants on one side of the short axis have matching values and greater or less than the other pair of quadrants - flattened tear drop shaped boudins formed by ductile extension.

(x) n values for two quadrants on one side of the long axis less than those on the other side - planoconvex
shaped boudins formed by HSS parallel to the shorter axis of the boudin. Slight variation of this gives the torpedo shaped boudinage in $\lambda_1 \lambda_2$ planes.

5.6 Three-dimensional Shape Analysis

The three dimensional shapes of boudins were also computed by choosing twenty boudins and computing the $n$ values for three sections of each boudin, viz. the AC section (containing longest and shortest axes), AB section (containing longest and intermediate axes) and BC section (containing intermediate and short axes). Thus, a dozen $n$ values for each boudin were worked out. From these values, the mean $n$ values were computed for the entire boudin. The data is tabulated in Table 5.2. The shapes of the three half the natural size boudins are shown in all three sections in Figs. 5.13 to 5.22. From the table 5.2, it is clear that all have an overall subellipsoidal shape, one of the pebbles is overall nearly true ellipsoidal with a mean $n$ value of 1.944 and one has a superellipsoidal with $n$ value of 2.89.

Fig. 5.13 (Top) shows the three sections AB, BC and AC of the boudin. The AB section (comparable to $\lambda_1 \lambda_2$ section as in case of the 2D analysis) shows that one of the quadrants (1) has a superelliptical shape, the other three quadrants being subelliptical and the quadrant 4 being nearly truely elliptical, giving higher values than
Table 5.2. Quadrantwise values of n for three dimensional shape analysis

<table>
<thead>
<tr>
<th>Boudin No.</th>
<th>AB Section</th>
<th>BC Section</th>
<th>AC Section</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>n for Q₁</td>
<td>n for Q₂</td>
<td>n for Q₃</td>
</tr>
<tr>
<td>1</td>
<td>1.459</td>
<td>1.759</td>
<td>2.209</td>
</tr>
<tr>
<td>2</td>
<td>1.779</td>
<td>1.369</td>
<td>1.429</td>
</tr>
<tr>
<td>3</td>
<td>1.829</td>
<td>1.469</td>
<td>1.509</td>
</tr>
<tr>
<td>4</td>
<td>1.529</td>
<td>1.609</td>
<td>1.549</td>
</tr>
<tr>
<td>5</td>
<td>1.489</td>
<td>1.199</td>
<td>1.419</td>
</tr>
<tr>
<td>6</td>
<td>1.329</td>
<td>1.539</td>
<td>1.489</td>
</tr>
<tr>
<td>7</td>
<td>1.189</td>
<td>1.289</td>
<td>1.339</td>
</tr>
<tr>
<td>8</td>
<td>1.429</td>
<td>1.449</td>
<td>1.589</td>
</tr>
<tr>
<td>9</td>
<td>1.439</td>
<td>1.429</td>
<td>1.289</td>
</tr>
<tr>
<td>10</td>
<td>1.239</td>
<td>1.649</td>
<td>1.459</td>
</tr>
</tbody>
</table>
Table 5.2 (Contd.)

<table>
<thead>
<tr>
<th>Boudin No.</th>
<th>AB Section n</th>
<th>BC Section n</th>
<th>AC Section n</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$n$ for $Q_1$</td>
<td>$n$ for $Q_2$</td>
<td>$n$ for $Q_3$</td>
</tr>
<tr>
<td>11</td>
<td>1.459</td>
<td>1.849</td>
<td>1.659</td>
</tr>
<tr>
<td>12</td>
<td>1.339</td>
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</tr>
<tr>
<td>13</td>
<td>1.399</td>
<td>1.489</td>
<td>1.419</td>
</tr>
<tr>
<td>14</td>
<td>1.769</td>
<td>1.669</td>
<td>1.899</td>
</tr>
<tr>
<td>15</td>
<td>1.769</td>
<td>1.409</td>
<td>1.299</td>
</tr>
<tr>
<td>16</td>
<td>1.359</td>
<td>1.709</td>
<td>1.369</td>
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<tr>
<td>17</td>
<td>1.239</td>
<td>1.589</td>
<td>1.659</td>
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<td>18</td>
<td>1.349</td>
<td>1.199</td>
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<tr>
<td>19</td>
<td>0.969</td>
<td>1.139</td>
<td>1.229</td>
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<tr>
<td>20</td>
<td>1.609</td>
<td>1.459</td>
<td>1.369</td>
</tr>
</tbody>
</table>
for the quadrants 2 and 3. The mean shape in AB plane may be subelliptical but the shape is reflective of a flattening deformation in the $\lambda_1 \lambda_2$ plane.

The BC section is typically asymmetrical lozenge with n value of opposite quadrants almost matching but different. The shape in the AC section is highly sub-elliptical. The mean n values for BC and AC sections are more or less identical but the small n values are comparable for large ones for two quadrants in BC section, while in AC section, three of the quadrants have typically sub-elliptical shapes.

Fig. 5.13 (bottom) shows the three sections for the boudin No. 2. The overall shape of AB section is subelliptical but uniformity of curvatures of peripheries of each of the quadrants is reflecting again of the flattening in $\lambda_1 \lambda_2$ plane. The BC section is a typical tear drop shape while AC section is flattened tear drop shape. The anomalies may be matched for the ten types of shapes given in the Table 5.1 of the classification based on n values of the four quadrant.

Fig. 5.14 (Top) shows a subelliptical shape but AB section is again suggestive of flattening deformation in the $\lambda_1 \lambda_2$ plane. The BC section has a "torpedo" or near "plano-convex" shape as given in Table 5.1 and AC section is typical subellipsoid, the overall shape being an asymmetrical lozenge.
In case of boudin 4 (Fig. 5.14, bottom) all three sections are overall subelliptical but the subellipticity goes on becoming more and more pronounced from the AB to BC section through AC. In all three sections, there is a more or less bilateral symmetry about the boudin axis (i.e. either A or B). The mean values of $n$ thus reduce progressively from 1.51 for AB to 1.487 to AC, to 1.45 for BC. Thus, the subellipticity is higher in the $\lambda_1\lambda_3$ plane of the finite deformation ellipsoid. This is probably reflective of a higher degree of pure shear and/or flattening component subsequent the formation of original barrel shaped boudins.

Boudin No. 5 (Fig. 5.15 top) also shows overall sub-ellipticity in all sections and a more or less bilateral symmetry about the longer axis $\lambda$ with case of boudin 4, the interpretation for this is therefore similar to that for boudin No. 4.

Fig. 5.15 bottom shows the AB, AC and BC sections of boudin No. 6. It is clear that the AB section suggests a shape similar to boudin No. 6 in Table 5.1 with broader outlines. For BC section it is only slightly subelliptical with bilateral symmetry about the larger B axis; in AC section, the shape is characteristically that of a lozenge with curved boundaries, identifying with the shape under category 4 given earlier.
The form of the boudin No. 7 (Fig. 5.16 top) in AB section is squarish, therefore, distinctly subelliptical but cannot fall into that of asymmetrical lozenge. But a square with its sides curved is characteristic feature of asymmetrical lozenge, as far as the n values are concerned. However, the shape is typically of flattening result, in $\lambda_1 \lambda_2$ plane.

In the horizontal plane, the shape is typically tear drop with the values of n for the quadrants with tapering ends exceeding 1 by only slight amounts, while for the other two quadrants with a broaden end, the n values are exactly identical. There is a bilateral symmetry about the long B axis.

The AC section is a typical symmetrical lozenge with sides curved, similar to the AB section. The only difference being that the shape is lozenge shaped and not squarish.

The AB section of boudin 8 (Fig. 5.16 bottom) is also typically subelliptical with the outline very slightly resembling a torpedo, or closer to a planoconvex shape in a very broad or general way. However, the horizontal HSS component was not so strong as to drag the tips to give the exact shape. The AC section is a flattened tear drop (category 9, Table 5.1) while the BC section is nearly perfect subelliptical with almost identical n values for all four quadrants.
Fig. 5.17 top shows the outline in AB, AC and BC sections for the boudin No. 9. The AB section is sub-elliptical but showing again a flattening component, the values of n being of higher order than BC section. The AC section shows a flattened tear drop shape and a bilateral symmetry about the A axis, which is also supported by one tapering end in AB plane. The BC section is a nearly symmetrical lozenge with value of n showing greater than unity, except for one quadrant which is suggestive of asymmetrical lozenzes.

The boudin No. 10 (Fig. 5.17 bottom) is with a flattening type of shape of AB section, slightly asymmetrical lozenge in the AC section, suggestive of sinistral shearing and the horizontal or BC section with higher values for one side of the longer B axis than the other but, the shape is rather not like a torpedo. The shape may be because of the heterogeneous ductile deformation. The viscosity of the matrix material being different on two sides of the long axis of the boudin.

Fig. 5.18 top shows the three sections of boudin No. 11. The overall shape, through subelliptical approaches that of a true ellipse in AB section. Even then the two broad sides (quadrants on side of shorter axis) have higher n values than the other. The BC section is slightly that of a tear drop but more complex with superellipticity of
one quadrant, due to a kink like structure in the periphery of this quadrant. The AC section is more like a flattened tear drop but again the typical bilateral symmetry about the longer axis is wanting.

The boudin 12 has AB section (Fig. 5.18 bottom) that shows flattening in $\lambda_1 \lambda_2$ plane but because of again an irregular to squarish outline, it has a shape that appears like a true cross-section of a balloon. One of the quadrant is superelliptic. The AC section has a slightly torpedo like shape while the shape in the BC section is much variable, from a high superellipticity for one quadrant ($n = 3$) to one side nearly straight ($n = 1.05$).

The shapes of the boudin (Fig. 5.19 top) as far as the degree of ellipticity is concerned, in case of boudin 13, are more or less alike. In all three sections, the mean $n$ values for the four quadrants are alike though the sizes of the axes $A$, $B$ and $C$ are variable.

The AB section of boudin 14 (Fig. 5.19 bottom) is overall subelliptical but again suggestive of an initial pinch and swell structure, further modified by flattening in $\lambda_1 \lambda_2$ plane. The AC section is typically a flattened tear drop shape while the BC section is an overall symmetrical lozenge with curved boundaries.

Boudin 15 (Fig. 5.20 top) is interesting in the sense that it shows tear drop shape in all three sections
though overall subelliptical. The AB section is a symmetrical tear drop while AC section is an asymmetric tear drop. BC section is also an asymmetrical tear drop but the degree of asymmetry is less than that of the one observed in AC section.

The No. 16 boudin shows a subelliptical shape in AB section (Fig. 5.20 bottom), an asymmetrical lozenge with highly curved sides and a sinistral shear sense in the BC section; and a pencil kind of highly subelliptical shape in the AC section, suggesting more or less like, continuous layering with slight "swelling" in the central portion. The section BC is interesting because it shows identical values of opposite quadrants, typical of asymmetrical lozenge and indicative of strong but an overall regular shape, with the degree of shearing constant at all places.

The horizontal BC section (Fig. 5.21 top) of boudin 17 is a typical symmetrical lozenge with curved sides, the AC section a modified tear drop shape, again suggestive of shearing in the $\lambda_2$ $\lambda_3$ plane; and the AB section though subelliptical suggests of a flattening component.

The boudin 18 is interesting in the sense that (Fig. 5.21 bottom) in all three sections AB, BC and AC it shows an overall symmetrical lozenge shape with curved side. BC is the most typical and uniform shape, AC is slightly modified to take a look of a flattened tear drop (but not to any great extent). The AB section has one side of the long
axis more bulging than the other, a double convex shape in
the process of being modified to a planoconvex shape.

The boudin 19 is a highly flattened structure with a
'banana' shape in the AB section (Fig. 5.22 top) a pencil
shape in the AC section and a typical near lozenge with
curved sides in BC section. This is the only boudin that
gives overall superelliptical shape with $n > 2$ in AB and AC
sections and highly subelliptical in BC section.

The last of the analysed pebbles shows (boudin 20,
Fig. 5.22 bottom) a very uniform subellipticity in AB
section, almost that of an ideal subellipse of $n = 1.5$.

In BC section, however, it has the shape of the
asymmetrical lozenge with a clockwise or dextral shear
sense. In the AC section, it is a highly flattened tear
drop with superelliptical shape for the broad end on one
side of the short axis, the value close to 1 on the tapering
side of the short axis of the ellipse.

The data on $n$ values for each of the quadrants, for
the 20 boudins shown in Figs. 5.13 to 5.22 are given quadrant-
wise and mean in Table 5.2.

The above analysis points out the fact that generally
in the AB planes, a boudin shows a flattening form and even
though subelliptical, the values of $n$ are close to 2. In
the BC sections the pebbles shows either symmetrical/
asymmetrical lozenge or flattened tear drop shapes. In the
AC sections, the shapes are like long pencils. The torpedo type shape is sometimes seen in AB sections.

5.7 Volume Ratio based on Concept of Ellipsoid of Revolution along axis A in AC section

The volume of a spheroid or sphere is generally given by the gamma function of Spiegel (1968) where,

\[ r = 4/3 \pi \]  

for a sphere of unit radius.

The volume of an ellipsoid \( V_e \), derived from a sphere is generally given by

\[ V_e = 4/3 \pi r_1 r_2 r_3 \]  \hspace{1cm} \text{eq. 5.13} \]

where \( r_1, r_2 \) and \( r_3 \) are the three radii of the ellipse, which gives the volume of the ellipsoid derived from a sphere. Usually, however, most of the ellipsoidal shapes are generally derived from previously existing ellipsoids or from the ductile deformation of three dimensional chocolate tablet boudins. These shapes are either superellipsoids or subellipsoids. Hence, the volume obtained by the above equation in case of an overall subellipsoid would be overestimation of the true ellipsoid volume. On the other hand, if the ellipsoidal mass or particle is a superellipsoidal, the \( V_e \) obtained using the above equation would be an underestimation of the true volume of a true ellipsoid.
This is principally due to the fact that the superellipticity and subellipticity would be defined by whether the surfaces of the ellipsoidal particle are curved, and to what degree. How much any point \((X, Y, Z)\) on the surface of an approximately ellipsoidal particle would depart from that of the \((X, Y, Z)\) values for a true ellipsoidal particle given by the equation:

\[
\frac{X^2}{A^2} + \frac{Y^2}{B^2} + \frac{Z^2}{C^2} = 1
\]

(5.14)

The volume of each boudin was calculated on the basis of half of the length of its true axes, multiplied by \(4/3\pi\) and is given by 20 'pebbles' in Table 5.3.

The \(V_e\) thus calculated is only the apparent volume of the superellipsoid and subellipsoid, not the real volume of the super- or subellipsoid. To compute the real volume of such an ellipsoid, the concept of ellipsoid of revolution of Spiegel (1968) using the gamma function is applied. The ellipsoid is rotated along one of its axis, usually the longer axis \(A\) and the volume of superellipsoid of revolution or subellipsoid of revolution is calculated. The \(Rv\) or volume ratio is given by

\[
Rv = \frac{V_e}{V_r}
\]

(5.15)

For superellipsoids, the ratio \(Rv\) is greater than 1 and for subellipsoids, the ratio \(Rv\) is less than 1. The
Table 5.3. The volume of each boudin calculated on the basis of half of the length of its true axes, multiplied by \(\frac{4}{3}\pi\) and given by 20 'pebbles'.

<table>
<thead>
<tr>
<th>Boudin No.</th>
<th>(\frac{1}{2} A) in cm</th>
<th>(\frac{1}{2} B) in cm</th>
<th>(\frac{1}{2} C) in cm</th>
<th>(V_e) cc</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.75</td>
<td>5.20</td>
<td>2.25</td>
<td>379.32</td>
</tr>
<tr>
<td>2</td>
<td>7.35</td>
<td>5.00</td>
<td>1.65</td>
<td>253.99</td>
</tr>
<tr>
<td>3</td>
<td>17.40</td>
<td>8.00</td>
<td>1.325</td>
<td>772.58</td>
</tr>
<tr>
<td>4</td>
<td>8.20</td>
<td>6.10</td>
<td>2.45</td>
<td>513.315</td>
</tr>
<tr>
<td>5</td>
<td>11.10</td>
<td>6.25</td>
<td>1.90</td>
<td>552.135</td>
</tr>
<tr>
<td>6</td>
<td>8.10</td>
<td>6.20</td>
<td>1.85</td>
<td>326.399</td>
</tr>
<tr>
<td>7</td>
<td>6.10</td>
<td>4.75</td>
<td>2.35</td>
<td>285.220</td>
</tr>
<tr>
<td>8</td>
<td>8.40</td>
<td>3.70</td>
<td>1.075</td>
<td>139.950</td>
</tr>
<tr>
<td>9</td>
<td>6.20</td>
<td>3.15</td>
<td>1.05</td>
<td>35.89</td>
</tr>
<tr>
<td>10</td>
<td>6.55</td>
<td>3.425</td>
<td>1.225</td>
<td>115.110</td>
</tr>
<tr>
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<td>6.85</td>
<td>3.375</td>
<td>0.900</td>
<td>37.1536</td>
</tr>
<tr>
<td>12</td>
<td>5.20</td>
<td>3.85</td>
<td>1.50</td>
<td>125.739</td>
</tr>
<tr>
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<td>3.65</td>
<td>1.50</td>
<td>137.601</td>
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<tr>
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<td>1.525</td>
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<tr>
<td>15</td>
<td>10.50</td>
<td>5.00</td>
<td>1.50</td>
<td>329.867</td>
</tr>
<tr>
<td>16</td>
<td>9.60</td>
<td>4.725</td>
<td>0.85</td>
<td>161.563</td>
</tr>
<tr>
<td>17</td>
<td>12.60</td>
<td>4.90</td>
<td>1.75</td>
<td>452.5778</td>
</tr>
<tr>
<td>18</td>
<td>8.30</td>
<td>3.75</td>
<td>1.75</td>
<td>228.123</td>
</tr>
<tr>
<td>19</td>
<td>18.50</td>
<td>4.625</td>
<td>1.00</td>
<td>358.405</td>
</tr>
<tr>
<td>20</td>
<td>22.00</td>
<td>6.75</td>
<td>1.25</td>
<td>777.500</td>
</tr>
</tbody>
</table>
basic theory is outlined in detail in Spiegel (1968, pp 105 and 235). For a superellipsoid of revolution, the $Rv$ is given by.

$$Rv = \frac{1.5 \Gamma(1 + 1/n) \Gamma(1 + 2/n)}{(1 + 3/n)} \quad \text{eq. 5.16}$$

Where $\Gamma$ (gamma) is a function of revolution and $n$ the exponent in AC section of the ellipsoid. Out of the 20 apparently ellipsoidal boudins analysed, only three have superelliptical shapes in AC sections. These are the boudin numbered 17, 19 and 20. The $Rv$s for these were calculated from the above equation by replacing the values of $n_{AC}$, i.e. exponent in AC section and the $Rv$ was calculated for the superellipsoid of revolution. All other seventeen boudins have subelliptical AC sections with the value of exponent greater than 2. The $Rv$ for these was calculated using the modified form of the equation given above, i.e.

$$Rv = 1 - \left[ \frac{(1 - \frac{1}{n})(1 - \frac{2}{n})}{(1 - \frac{3}{n})} \right] \quad \text{eq. 5.17}$$

The $Rv$ values for 20 boudins are plotted against respective $n$ values in AC sections in Fig. 5.23 and the $Rv$ values are also tabulated in Table 5.4.

Thus, it is apparent that the $Rv$ decreases with decrease in exponent $n$ in case of a subellipsoid of revolution and increases in case of a superellipsoid of revolution as $n$ increases.
FIG 5.23

SUPERELLIPSOIDS OF REVOLUTION

\[ R_v = \frac{V}{V_e} \]
Table 5.4. Rv values for boudins using n values in AC sections

<table>
<thead>
<tr>
<th>Boudin No.</th>
<th>Rv</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.66</td>
</tr>
<tr>
<td>2</td>
<td>0.83</td>
</tr>
<tr>
<td>3</td>
<td>1.125</td>
</tr>
<tr>
<td>4</td>
<td>0.87</td>
</tr>
<tr>
<td>5</td>
<td>0.85</td>
</tr>
<tr>
<td>6</td>
<td>0.825</td>
</tr>
<tr>
<td>7</td>
<td>0.81</td>
</tr>
<tr>
<td>8</td>
<td>0.85</td>
</tr>
<tr>
<td>9</td>
<td>0.83</td>
</tr>
<tr>
<td>10</td>
<td>0.844</td>
</tr>
<tr>
<td>11</td>
<td>0.76</td>
</tr>
<tr>
<td>12</td>
<td>0.84</td>
</tr>
<tr>
<td>13</td>
<td>0.725</td>
</tr>
<tr>
<td>14</td>
<td>0.91</td>
</tr>
<tr>
<td>15</td>
<td>0.86</td>
</tr>
<tr>
<td>16</td>
<td>0.665</td>
</tr>
<tr>
<td>17</td>
<td>1.08</td>
</tr>
<tr>
<td>18</td>
<td>0.825</td>
</tr>
<tr>
<td>19</td>
<td>1.225</td>
</tr>
<tr>
<td>20</td>
<td>0.81</td>
</tr>
</tbody>
</table>
A graphical plot to show the relationship between $n_{AB}$ and bulk strain in terms of $k$

\[ k = \frac{a - 1}{b - 1} \]

\[ a = \left( \frac{\lambda_1}{\lambda_2} \right)^2, \quad b = \left( \frac{\lambda_2}{\lambda_3} \right)^2 \]

FIG 5.24
In case of a large number of boudins, the n value is superelliptical on the average because of initial barrel shape of boudins which later gets modified. Another important point is that, as n values in $\lambda_1 \lambda_2$ or AB planes increases; the finite strain also increases and oblateness increases, k values generally decrease. Thus k value are inversely related to n values in $\lambda_1 \lambda_2$ planes and this is graphically recorded in Fig. 5.24 for 30 shapes where the strain data was known. However, the relationship is not straightforward everywhere, especially where the long boudins break on $\lambda_1 \lambda_2$ planes and undergo only little flattening with distance between individual boudins gradually increasing with further deformation.

5.8 Construction procedure for Superellipses and Subellipses

In this section, for an ellipse whose larger axis is double than its shorter axis, it has been shown that by increasing the distance of a given radius $r_i$ at $i^{th}$ point on the periphery, a number of shapes can be produced as 'ideal' shapes which have definite values of the exponent for the given X/Y ratio of ellipse. During this excercise for a large number of radii at regular intervals $\theta_i$, the total angle at each stage being the angle $\theta$ between the long axis of the ellipse and the $i^{th}$ radius.
The relationship is given by the equation.

\[ r_j = \left[ \frac{(A/2)^n \left\{ (E/2)^n - (r_i^n \sin \theta_i) \right\} }{(E/Z)^n} \right] \quad \text{eq. 5.18} \]

The equation would hold good for any X/Y ratio to produce a series of ideal shapes of different n values by simply changing the value \( \pm 0.5 \), in the above equation, depending on the actual length of X and Y axes. For example, if X has a length of 7 cm and Y, a length of 3 cm, then the equation would take form

\[ r_{v_j} = \left[ \frac{(A/2)^n \left\{ (E/2)^n - (r_i^n \sin \theta_i) \right\} }{(E/Z)^n} \right] \pm 0.4 \quad \text{eq. 5.19} \]

Thus in the above equation, the value of 0.4 will have to be subtracted from all radii making an angle of less than 23.57° (Sin⁻¹ 0.4) with X axis, that is greater than 23.57.

In the previous equation, the radii length will have to be decreased by 0.5 in case of all radii making an angle of less than 30° with X axis. All radii length will have to be increased by 0.5, when they make angle greater than 30° with X co-ordinate axis (see Fig. 5.25). Using the former equation where X doubles Y axis, some ideal shapes have been constructed using the method described above and shown diagrammatically in Fig. 5.25. Fig. 5.26 shows 20 ideal
shapes for different values of \( n \) produced by using such a method. Where \( X/Y \) is approximately 2, these shapes, produced on a perspex sheet can be matched with natural shapes to quickly compute the \( n \) values of a given quadrant of the ellipse, then the shapes are classified according to 4 \( n \) values obtained from four quadrants which would fall into one or two of the ten shapes categorised before, on the basis of 4 \( n \) values for each ellipse and given in Table 5.2.

5.9 Conclusions

To conclude, based on the \( n \) values of any shape that departs from an ideal ellipse, the shapes of boudins have been classified into ten principal types. Also, the mechanism for their formation have been suggested. A description of large number of elliptical shapes analysed in this manner are given. For finding the \( n \) values quickly, a computer program was written. The relative volumes of 20 three dimensional boudins have been computed. It has been shown that from the basic theory, one can produce ideal shapes of equal \( n \) values for visual inspection through a perspex sheet for visual estimation of \( n \). The procedure consists in increasing the length of several radii drawn within a lozenge by a given amount and then joining them by a smooth curve. This method would help to compute rapidly the shapes of apparently elliptical objects in terms of \( n \) values, of four quadrants of the ellipse and then classify the entire shape into one of the ten categories and a possible mechanism of their generation can be suggested.