SUMMARY

Stability analysis of fluid flows is essential for understanding the nature of fluid flows and their applications in various fields. Many physical phenomena are solved successfully with the help of this topic. In order to understand the behavior of a fluid flow, we need to know how it would react to various forces acting on it. The basic equation of continuity, momentum and energy are all non-linear partial differential equations and such as their analytic solutions are generally not possible. Therefore, to analyze any fluid flow theoretically we have to take some approximations depending upon the geometry of the flow and other considerations. In experimental investigations of any fluid flow, there are always present some disturbances. Therefore, to study any fluid flow, we must analyze the effect of these disturbances on the theoretically obtained solutions.

The present work is devoted to the investigation of stability analysis of couple-stress fluid and some visco-elastic fluids in porous medium. Macroscopic approach is adopted throughout the work and the normal mode technique is used to determine the stability or instability of the flows under consideration within the framework of linear analysis. By normal mode technique, the stability of a hydrodynamical system, the linearized perturbation equations are set up first in a single perturbation variable by eliminating the remaining variables from the linear equations derived from the equation of conservation of mass, momentum and energy, retaining only the linear terms in perturbed quantities. These equations are then solved either analytically of with the help of variational procedure or through an integral equation under a set of appropriate boundary conditions which lead to the dispersion relation in the parameter determining the stability of a system. Therefore, to determine the effect of a particular physical parameter on the growth rates, we analyze the change of varying
that parameter while keeping the other parameters fixed. An increase in growth rate implies the destabilizing influence of that particular parameter and a decrease in growth rate shows stabilizing influence of the parameter. For the investigation in any stability analysis to be complete, it is assumed that the perturbations can be resolved into dynamically independent wave like components, each component satisfying the linearized equations of motion and the boundary conditions separately. Disturbances are the essential point here, in all the cases, must be expended in all possible forms of time function constituting the time behavior of the quantities in the system.

Thus, if $A(x, y, z, t)$ is a typical wave component describing a disturbance, the expansion of this wave component if given by

$$A(z) \exp \left( ik_x x + ik_y y + nt \right),$$

where $k_x$ and $k_y$ are the wave numbers in $x$ and $y$ directions, respectively and $k = \sqrt{k_x^2 + k_y^2}$ is the real wave number of the disturbance and $n$ is a constant to be determined and in general, is a complex constant. It is to be remembered that the real parts are to be taken to get physical quantities, this being permissible for the linear problems. Further, since the perturbation equations are linear, the reaction of the system to a general disturbance can be determined if we know the reaction of the system to disturbances of all assigned wave numbers. In particular, the stability of the system will depend on its stability to disturbances of all wave numbers and instability will follow from the instability with respect to even one wave number. The research work in the present thesis is accomplished by dividing the work in following seven chapters:

**Chapter-1** deals with the introduction of the present thesis. In this chapter, we have discussed the basic principles and fundamental concepts of the topics relevant to the thesis. The thermal instability, thermosolutal instability and Rayleigh-Taylor instability problems have been described and effects of various factors like dust particles, uniform rotation, magnetic field and porous medium have been discussed.
Chapter-2 deals with the thermal instability of couple-stress fluid in the presence of variable gravity field and rotation. This chapter has been divided into two sections:

In section I, we formulate a mathematical problem of thermal instability of couple-stress fluid in the presence of variable gravity field and rotation.

In section II, we formulate a mathematical problem of thermal instability of couple-stress rotating fluid in the presence of variable gravity field and solute gradient.

In section I, effect of rotation and variable gravity field on couple-stress fluid heated from below is considered. After linearizing the relevant hydrodynamic equations, the perturbed quantities are analyzed in terms of normal modes. Dispersion relation governing the effects of rotation, couple-stress and variable gravity field is derived. The main results from the analysis of this section are as follows:

(i) Couple-stress has stabilizing/destabilizing effects on the system under the conditions

\[ T_A < [F_1(1 + x) + 1]^2(1 + x)^3 \quad \text{or} \quad T_A > [F_1(1 + x) + 1]^2(1 + x)^3. \]

But for the permissible range of values of various parameters, the said effect is stabilizing if

\[ T_A < [F_1(1 + x) + 1]^2(1 + x)^3. \]

In the absence of rotation, couple-stress is found to have a stabilizing effect on the thermal convection when gravity increases upwards from its value $g$. In this result, the effect of increasing the magnitude of the rotation parameter is to destabilize the system.

(ii) For the case of stationary convection, the rotation has a stabilizing effect on the system.

(iii) The oscillatory modes are introduced due to the presence of rotation whereas in their absence principle of exchange of stabilities (PES) is satisfied in the system.
(iv) The condition $k_T < v \left( 1 + \frac{H^T}{\rho_0 d^2} (1 + x) \right)$ is the sufficient condition for the non-existence of overstability.

In section II, the effect of solute gradient and variable gravity field on couple-stress rotating fluid heated from below is considered. After linearizing the relevant hydrodynamic equations, the perturbed quantities are analyzed in terms of normal modes. Dispersion relation governing the effects of stable solute gradient, rotation and couple-stress is derived. The main results from the analysis of this section are as follows:

(i) For the case of stationary convection, the stable solute gradient has a stabilizing effect on the system as can be seen from equation (II.3a.2) and graphically, from Fig. 2.1 and Fig. 2.2.

(ii) Couple-stress is found to have stabilizing/destabilizing effect on the thermosolutal instability of couple-stress rotating fluid under the conditions

$$T_{A_l} < (1 + x)^3 [F_l(1 + x) + 1]^2 .$$

But for the permissible values of various parameters, the said effect is stabilizing only if

$$T_{A_l} < \frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} + 1 \right)^2$$

as can be seen graphically, from Fig. 2.3 and Fig. 2.4. In the absence of rotation, couple-stress clearly has a stabilizing effect on the thermosolutal instability of couple-stress rotating fluid when gravity increases upward from its value $g_0$ as can be seen from equation (II.3a.4).

(iii) For the case of stationary convection, the rotation has a stabilizing effect on the system as can be seen from equation (II.3a.5) and graphically, from Fig. 2.5.

(iv) The oscillatory modes are introduced due to the presence of stable solute gradient and rotation whereas in their absence principle of exchange of stabilities (PES) is satisfied in the system.
(v) The conditions $k_s > k_T$ and $k_T < v\sqrt{\frac{\mu'}{\rho_o d^2 v}} (1 + x) + 1$ are the sufficient conditions for the non-existence of overstability.

Chapter-3 deals with the thermosolutal instability of couple-stress rotating fluid in the presence of magnetic field.

In this chapter, effect of magnetic field on couple-stress rotating fluid heated and soluted from below is considered. After linearizing the relevant hydrodynamic equations, the perturbed quantities are analyzed in terms of normal modes. Dispersion relation governing the effects of magnetic field, stable solute gradient, couple-stress and rotation is derived. The main results from the analysis of the chapter are as follows:

(i) For the case of stationary convection, the stable solute gradient has a stabilizing effect on the system as can be seen from equation (3.5a.2) and graphically, from Fig. 3.1.

(ii) Couple-stress has stabilizing/destabilizing effects on the system under the conditions

$$ T_A (1 + x) < \text{or} > \left[ F (1 + x) + 1 \right] (1 + x)^2 + Q_o^2. $$

But for the permissible values of various parameters, the said effect is stabilizing only if

$$ T_A (1 + x) < \left[ - \text{or} > \right] 2 $$

as can be seen from Fig. 3.2, Fig. 3.3 and Fig. 3.4. In the absence of rotation, couple-stress is found to have stabilizing effect on thermosolutal instability of couple-stress rotating fluid when gravity increases upward from its value $g_o$.

(iii) For the case of stationary convection, the rotation has a stabilizing effect on the system as can be seen from equation (3.5a.6) and graphically, from Fig. 3.5 and Fig. 3.6.
(iv) Magnetic field has stabilizing/destabilizing effect on the system under the conditions

\[ T_A(1+x) < or > \left[ F_1(1+x) + I_1^2(1+x)^2 \right]^{-2} \]

when gravity increases upwards from its value \( g_0 \) as can be seen graphically from Fig. 3.7. But for the permissible values of various parameters, the said effect is stabilizing only if

\[ T_A(1+x) < \left[ F_1(1+x) + I_1^2(1+x)^2 \right]^{-2} \]

In the absence of rotation, magnetic field is found to have stabilizing effect on thermosolutal instability of couple-stress rotating fluid in the presence of magnetic field when gravity increases upward from its value \( g_0 \).

(v) The oscillatory modes are introduced due to the presence of stable solute gradient, magnetic field and rotation whereas in their absence principle of exchange of stabilities (PES) is satisfied in the system. A discussion has been made that in the absence of stable solute gradient, the system is stable if

\[ \frac{\lambda g_0 \Omega k_T}{\sqrt{\beta}} \leq \frac{27\pi^4}{4} \quad \text{and unstable if} \quad \frac{\lambda g_0 \Omega k_T}{\sqrt{\beta}} > \frac{27\pi^4}{4}. \]

Chapter-4 deals with the problems on a couple-stress rotating fluid heated from below in hydromagnetics. This chapter has been divided into following two sections:

In section I, we have investigated the effect of dust particles on couple-stress rotating fluid heated from below in hydromagnetics.

In section II, we have investigated the effect of solute gradient on a couple-stress rotating dusty fluid heated from below in hydromagnetics in porous medium.

In section I, effect of dust particles on a couple-stress rotating fluid heated from below in hydromagnetics is considered. After linearizing the relevant hydrodynamic equations, the perturbed quantities are analyzed in terms of normal modes. Dispersion
relation governing the effects of dust particles, rotation, magnetic field and couple-stress is derived. The main results from the analysis from this section are as follows:

(i) For the case of stationary convection, the dust particles are found to have a destabilizing effect on the system as can be seen from equation (I.3a.2) and graphically, from Fig. 4.1.

(ii) Couple-stress is found to have stabilizing/destabilizing effect on the thermal instability of couple-stress rotating dusty fluid in hydromagnetics under the conditions

\[ T_A (1 + x) < or > \left[ \frac{F_i (1 + x) - 1}{(1 + x)^2} - Q_i \right]^2. \]

In the absence of rotation, couple-stress is found to have stabilizing effect on thermal instability of couple-stress rotating dusty fluid in hydromagnetics.

(iii) Rotation is found to have a stabilizing effect on thermal instability of couple-stress rotating dusty fluid in hydromagnetics as can be seen from equation (I.3a.5) and graphically, from Fig. 4.2 and Fig. 4.3.

(iv) The magnetic field is found to have stabilizing/destabilizing effect on thermal instability of couple-stress rotating dusty fluid in hydromagnetics under the conditions

\[ T_A (1 + x) < or > \left[ \frac{F_i (1 + x) - 1}{(1 + x)^2} - Q_i \right]^2 \]

as can be seen from graphically from Fig. 4.4. In the absence of rotation, magnetic field is found is found to have stabilizing effect on thermal instability of couple-stress rotating dusty fluid in hydromagnetics as can be seen from equation (I.3a.7).

(v) The oscillatory modes are introduced due to the presence of stable solute gradient, magnetic field and rotation whereas in their absence principle of exchange of stabilities (PES) is satisfied in the system.
In section II, we have investigated the effect of solute gradient on couple-stress rotating dusty fluid heated from below in hydromagnetics in porous medium. Dispersion relation governing the effects of stable solute gradient, medium permeability, dust particles, couple-stress, rotation and magnetic field is derived. The main results from the analysis from this section are as follows:

(i) For the case of stationary convection, stable solute gradient has a stabilizing effect on the system as can be seen from equation (II.3a.2) and graphically, from Fig. 4.5.

(ii) Medium permeability has stabilizing/destabilizing effects on the system under the conditions

\[ T_A (1 + x) > \varepsilon^2 \left[ \frac{(1+x)}{P} \left( 1 + F_1(1 + x)^2 \right) \right] \frac{Q_1}{\varepsilon} \]

But for the permissible values of various parameters, the medium permeability has a stabilizing effect contrary to its general destabilizing influence if

\[ T_A (1 + x) > \varepsilon^2 \left[ \frac{(1+x)}{P} \left( 1 + F_1(1 + x)^2 \right) \right] \frac{Q_1}{\varepsilon} \]

as can be seen graphically from Fig. 4.6, Fig. 4.7 and Fig. 4.8. In the absence of magnetic field and rotation, medium permeability clearly has a destabilizing effect on the system as can be seen from equation (II.3a.4).

(iii) For the case of stationary convection, dust particles have a destabilizing effect on the system as can be seen from equation (II.3a.5).

(iv) Couple-stress has stabilizing/destabilizing effects on the system under the conditions

\[ T_A (1 + x) < \varepsilon^2 \left[ \frac{(1+x)}{P} \left( 1 + F_1(1 + x)^2 \right) \right] \frac{Q_1}{\varepsilon} \]

and in the absence of rotation, couple-stress is found to have a stabilizing effect on the thermosolutal instability of couple-stress rotating dusty fluid in hydromagnetics through porous medium. In the absence of rotation and magnetic
field, couple-stress clearly has a stabilizing effect on the system as can be seen from equation (II.3a.7).

(v) For the case of stationary convection, the rotation has a stabilizing effect on the system as can be seen from equation (II.3a.8) and graphically, from Fig. 4.9.

(vi) Magnetic field has stabilizing/destabilizing effect on the system under the conditions

\[ T_A (1+x) < \sqrt{\frac{\lambda (1+x)}{\mu} \left\{ 1 + F_1 (1+x) \right\} + \frac{Q}{e}}. \]

But for the permissible values of various parameters, the said effect is stabilizing only if

\[ T_A (1+x) < \sqrt{\frac{\lambda (1+x)}{\mu} \left\{ 1 + F_1 (1+x) \right\} + \frac{Q}{e}}. \]

and in the absence of rotation, magnetic field has a stabilizing effect on the thermosolutal instability of couple-stress rotating dusty fluid in porous medium as can be seen from equation (II.3a.10) and graphically from Fig. 4.10.

(vii) The oscillatory modes are introduced due to the presence of stable solute gradient, magnetic field and rotation whereas in their absence principle of exchange of stabilities (PES) is satisfied in the system.

**Chapter-5** deals with the problem of Rayleigh-Taylor instability of Walters’ (Model B’) visco-elastic dusty plasma in presence of variable magnetic field through porous medium. After linearizing the relevant hydrodynamic equations, the perturbed quantities are analyzed in terms of normal modes. Dispersion relation governing the effect of magnetic field, dust particles and medium permeability is derived. In this chapter, we have discussed the following two cases:

(a) The case of two uniform fluids separated by a horizontal boundary.

(b) The case of exponentially varying density, viscosity, visco-elasticity, magnetic field and particles number density.
In case (a), we have considered two uniform fluids separated by a horizontal boundary. The main results are as follows:

(i) The system is stable under the conditions $k_1 > \alpha_2 \nu_2 + \alpha_1 \nu_1$ and $\alpha_1 > \alpha_2$.

(ii) The system is stable under the conditions $k_1 > \alpha_2 \nu_2 + \alpha_1 \nu_1$ and $\alpha_1 < \alpha_2$, provided $2k_x^2 \nu^2 > g(k_2 - \alpha_1)$.

(iii) The system is unstable under the conditions $k_1 > \alpha_2 \nu_2 + \alpha_1 \nu_1$ and $\alpha_1 < \alpha_2$, provided that $2k_x^2 \nu^2 < g(k_2 - \alpha_1)$.

(iv) If $k_1 < \alpha_2 \nu_2 + \alpha_1 \nu_1$ and $\alpha_1 < \alpha_2$, then system is stable under the condition $2k_x^2 \nu^2 > g(k_2 - \alpha_1)$.

In case (b), we have considered the case of exponentially varying density, viscosity, visco-elasticity, magnetic field and particles number density. The main results are as follows:

(i) For $\beta < 0$ and $F < 1/\varepsilon$, the system is always stable.

(ii) For $\beta > 0$ and $F < 1/\varepsilon$, the system is stable/unstable according as $k_x^2 \nu^2 > \frac{\varepsilon \beta k^2}{L}$ or $\frac{\varepsilon \beta k^2}{L}$.

(iii) If $\beta < 0$ and $F > 1/\varepsilon$, then the system is unstable.

(iv) If $\beta > 0$ and $F > 1/\varepsilon$, then the system is unstable even if $k_x^2 \nu^2 > \frac{\varepsilon \beta k^2}{L}$.

Case (b) has been divided into two sections:

In section-1, we have discussed the oscillatory modes in the system. The main results are as follows:

(i) For $\beta < 0$ and $F < 1/\varepsilon$, the estimate of $n$ for the growth rate of oscillatory stable modes are given by $|n| > \frac{D}{B}$.

(ii) For $\beta < 0$ and $F < 1/\varepsilon$, the estimate of $n$ for the growth rate of oscillatory unstable modes are given by $|n| < \frac{D}{B}$.

$B$ and $D$ are defined in equation (5.5b.7).
For $\beta > 0$, $F < 1/\varepsilon$ and $k_A^2 V_A^2 > \frac{g|\beta| k^2}{L}$, the estimate on $\eta$ for the growth rate of oscillatory stable or unstable modes are respectively given by $|\eta|^2 > \frac{D}{B}$ or $|\eta|^2 < \frac{D}{B}$.

In section-2, we have discussed the non-oscillatory modes in the system. The main results are as follows:

(i) For $\beta < 0$ and $F < 1/\varepsilon$, the non-oscillatory modes are always stable.

(ii) For $\beta > 0$, $F < 1/\varepsilon$, the non-oscillatory modes are stable if $k_A^2 V_A^2 > \frac{g|\beta| k^2}{L}$ or

\[
\varepsilon \frac{v_m K}{k_i m} > \frac{g|\beta| k^2}{L} \quad \text{and} \quad k_A^2 V_A^2 > \frac{g|\beta| k^2}{L}.
\]

(iii) For $\beta > 0$, $F < 1/\varepsilon$, then non-oscillatory modes are unstable if $k_A^2 V_A^2 < \frac{g|\beta| k^2}{L}$.

(iv) For $\beta > 0$, $F < 1/\varepsilon$ and $k_A^2 V_A^2 < \frac{g|\beta| k^2}{L}$, there are wave propagating for a given wave number: Two damped and one amplified.

Chapter-6 deals with the problem of thermal convection in a Walters’ (Model B’) elastico-viscous dusty fluid in hydromagnetics with the effect of compressibility and rotation.

In this chapter, effect of rotation on thermal convection in a Walters’ (Model B’) elastico-viscous dusty fluid in hydromagnetics with the effect of compressibility is considered. After linearizing the relevant hydrodynamic equations, the perturbed quantities are analyzed in terms of normal modes. Dispersion relation governing the effect of visco-elastic parameter, rotation, dust particles, magnetic field and compressibility is derived. The main results from the analysis of this chapter are as follows:

(i) For the case of stationary convection, a Walters’ (Model B’) elastico-viscous fluid behaves like an ordinary Newtonian fluid.
The compressibility has a stabilizing effect on the thermal convection in a compressible (Walters’ B’) dusty fluid for the case of stationary convection.

For the case of stationary convection, the rotation has a stabilizing effect on the system as can be seen from equation (6.5a.2) and graphically, from Fig. 6.1 and Fig. 6.2.

Magnetic field has both stabilizing/destabilizing effect on thermal convection in a Walters’ (Model B’) elastic-viscous dusty fluid under the conditions

\[ T_A (1 + x) < \left( 1 + x \right)^2 + Q_1 \]

But for the permissible values of various parameters, the said effect is stabilizing only if

\[ T_A (1 + x) \left[ (1 + x)^2 + Q_1 \right] \]

as can be seen graphically from Fig. 6.3. In the absence of rotation, magnetic field has stabilizing effect on thermal convection in a compressible Walter’s (model B’) elastic-viscous dusty fluid as can be seen from equation (6.5a.3) along with the Fig. 6.4. Thus, the effect of increasing the magnitude of rotation parameter is to destabilize the system.

Dust particles have a stabilizing effect on the system as can be seen from equation (6.5a.5), and graphically, from Fig. 6.5.

The oscillatory modes are introduced due to the presence of viscoelasticity, compressibility, magnetic field and rotation whereas in their absence principle of exchange of stabilities (PES) is satisfied in the system.

Chapter-7 deals with the problem of stability of two superposed Rivlin-Ericksen visco-elastic dusty fluids in the presence of magnetic field.

In this chapter, effect of magnetic field on stability of two superposed Rivlin-Ericksen visco-elastic fluids in the presence of dust particles is considered. Dispersion relation governing the effect of magnetic field and dust particles is derived. In this
chapter, we have discussed the two cases:

In case (a), we have considered two uniform viscoelastic (Walters’ B’) fluids separated by a horizontal boundary. The main results are as follows:

(i) For potentially stable arrangement \((\rho_1 > \rho_2)\), the system is always stable.

(ii) For potentially unstable arrangement \((\rho_1 < \rho_2)\), the system is stable provided 
\[2k^2V^2 > gk(\rho_1 - \rho_2).\]

(iii) For potentially unstable arrangement \((\rho_1 < \rho_2)\), the system is unstable provided 
\[2k^2V^2 < gk(\rho_1 - \rho_2).\]

Thus, for potentially stable configuration, the system is found to be stable for disturbances of all wave numbers. The magnetic field succeeds in stabilizing certain wave-number range, for the potentially unstable configuration.

In case (b), we have considered the case of exponentially varying density, viscosity, visco-elasticity, magnetic field and particles number density. The main results are as follows:

(i) For stable density stratification \((\beta = 0)\), the system is always stable.

(ii) For \(\beta > 0\), the system is stable or unstable provided 
\[k^2V^2 > \frac{g\beta k^2}{L} \quad \text{or} \quad \frac{g\beta k^2}{L}.\]

Thus, for stable density stratification, the system is found to be stable for disturbances of all wave numbers. The magnetic field succeeds in stabilizing the potentially unstable stratifications for a certain wave-number range which were unstable in the absence of the magnetic field.

Case (b) has been divided into two sections:

In section-1, we have discussed the oscillatory modes in the system. The main results are as follow:

(i) For \(\beta < 0\), the estimate of \(n\) for the growth rate of oscillatory stable modes is given by 
\[|n|^2 > \frac{D}{B}.\]

\(B\) and \(D\) are defined in equation (7.5b.6).
(ii) For $\beta < 0$, the estimate of $n$ for the growth rate of oscillatory unstable modes is given by
\[ |n|^2 < \frac{D}{B}. \]

(iii) For $\beta > 0$ and $k_x^2 \mathcal{V}_A^2 > \frac{g|\beta|k^2}{L}$, the estimate of $n$ for the growth rate of oscillatory stable or unstable modes are respectively given by $|n|^2 > \frac{D}{B}$ or $|n|^2 < \frac{D}{B}$.

In section-2, we have discussed the non-oscillatory modes in the system. The main results are as follows:

(i) For $0$, the non-oscillatory modes are always stable.

(ii) For $\beta > 0$, the non-oscillatory modes are stable provided $k_x^2 \mathcal{V}_A^2 > \frac{g|\beta|k^2}{L}$.

(iii) For $\beta > 0$, the non-oscillatory modes are unstable provided $k_x^2 \mathcal{V}_A^2 < \frac{g|\beta|k^2}{L}$.

(iv) For $\beta > 0$ and $k_x^2 \mathcal{V}_A^2 \leq \frac{g|\beta|k^2}{L}$, there are wave propagating for a given wave number in which two waves of propagation are damped and one is amplified for a given wave number.