Chapter 1

PREREQUISITES

1.1 Introduction

Topology is an indispensable object of study in Mathematics with open sets as well as closed sets as the most fundamental concepts in topological spaces. Open sets and closed sets have been generalized by several Mathematicians. This thesis is an elaborate study of a new type of generalized closed sets in ideal topological spaces called $I_w$-closed sets, their respective continuous maps, closed maps, open maps, homeomorphisms, connectedness, contra continuous maps, somewhat $b$-$I$-continuous, regular semi local functions and their extension to ideal bitopological spaces.

In this chapter, the recent developments of topology and an ideal topology contributed by various authors are mentioned and definitions cited by them are presented. Section 2 begins with the definitions of basic concepts of open sets and closed sets in topological spaces. Section 3 deals with the basic concepts of open sets and
closed sets in ideal topological spaces. Section 4 gives the notions of some stronger and weaker forms of continuous maps, irresolute maps, open maps and closed maps in topological and ideal topological spaces. In Section 5, we describe the structure of bitopological spaces.

Throughout the thesis $X$, $Y$ and $Z$ denote ideal topological spaces $(X, \tau, I)$, $(Y, \sigma, J)$ and $(Z, \eta, K)$ respectively on which no separation axioms are assumed unless otherwise explicitly mentioned. For any subset $A$ of a space $(X, \tau)$, the closure of $A$, interior of $A$, semi-interior of $A$, semi-closure of $A$, $rw$-interior of $A$, $rw$-closure of $A$, $\alpha g$-interior of $A$, $\alpha g$-closure of $A$, the complement of $A$ and $\text{int}^*(A)$ are denoted by $\text{cl}(A)$ or $\tau\text{-cl}(A)$, $\text{int}(A)$ or $\tau\text{-int}(A)$, $\text{sint}(A)$, $\text{scl}(A)$, $\text{rw-int}(A)$, $\text{rw-cl}(A)$, $\alpha g\text{-int}(A)$, $\alpha g\text{-cl}(A)$, $A^C$ or $X - A$ and interior of $A$ in $(X, \tau^*)$ respectively.

1.2 Basic Concepts of Open Sets and Closed Sets in Topological spaces

M. H. Stone [63], O. Njastad [51] and N. Levine [37] have introduced and investigated stronger forms of open sets called regular open sets, $\alpha$-open sets and semiopen sets respectively. D. E. Cameron [7], introduced regular semiopen sets weaker than regular open set and regular closed set. The complement of a regular semiopen set is a regular semiclosed set. N. Levine [38], H. Maki, R. Devi and K. Balachandran [39] and R. S. Wali [67] respectively introduced $g$-closed sets, $\alpha g$-closed sets and $rw$-closed sets which are some weaker forms of closed sets. The complement of the various types of open (closed) sets are called the some types of closed (open) sets. The complement of a regular open set is called a regular closed set and the comple-
ment of a generalized closed set is called a generalized open set and so on. We recall
the following definitions, which are prerequisites for present study.

**Definition 1.2.1** A subset $A$ of a topological space $(X, \tau)$ is called

(i) **regular open set** [63] if $A = \text{int}(\text{cl}(A))$ and **regular closed set** [63] if
$A = \text{cl}(\text{int}(A))$.

(ii) **pre-open set** [40] if $A \subseteq \text{int}(\text{cl}(A))$ and **pre-closed set** [40] if
$\text{cl}(\text{int}(A)) \subseteq A$.

(iii) **$\alpha$-open set** [51] if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$ and **$\alpha$-closed set** [51] if
$\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$.

(iv) **semiopen set** [37] if $A \subseteq \text{cl}(\text{int}(A))$ and **semiclosed set** [37] if
$\text{int}(\text{cl}(A)) \subseteq A$.

The family of all regular open and semiopen (resp. regular closed and semiclosed)
subsets of a space $(X, \tau)$ is denoted by $RO(X)$ and $SO(X)$ (resp. $RC(X)$ and
$SC(X)$).

The intersection of all semiclosed (resp. semiopen) subsets of $(X, \tau)$ containing $A$
is called the semi-closure (resp. semi-kernel) of $A$ and is denoted by $\text{scl}(A)$ (resp.
$\text{sker}(A)$). Also the intersection of all pre-closed (resp. regular closed) subsets of
$(X, \tau)$ containing $A$ is called pre-closure (resp. regular closure) of $A$ and is denoted
by $\text{pcl}(A)$ (resp. $\text{spcl}(A)$).

The family of all $\alpha$-open sets in a space $(X, \tau)$, denoted by $\tau^\alpha$, is a topology
on $X$ finer than $\tau$. The closure of a subset of $A$ in $(X, \tau^\alpha)$ is denoted by
$\alpha-\text{cl}(A)$ or $\text{cl}_\alpha(A)$.
**Definition 1.2.2** [68] Let \( X \) be a topological space. The finite union of regular open sets in \( X \) is said to be \( \pi \)-**open**. The complement of a \( \pi \)-open set is said to be \( \pi \)-**closed**.

**Definition 1.2.3** [7] A subset \( A \) of a space \((X, \tau)\) is said to be **regular semiopen** if there is a regular open set \( U \) such that \( U \subseteq A \subseteq cl(U) \). The complement of regular semiopen set is called regular semiclosed.

The family of all regular semiopen (resp. regular semiclosed) sets of \( X \) is denoted by \( RSO(X) \) (resp. \( RSC(X) \)).

The regular semiclosure of \( A \) in \((X, \tau)\) is denoted by the intersection of all regular semiclosed sets containing \( A \) and is denoted by \( rsccl(A) \).

**Definition 1.2.4** A subset \( A \) of a topological space \((X, \tau)\) is called

(i) **generalized closed set** (briefly, \( g \)-**closed**) [38] if \( cl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is open in \( X \).

(ii) \( \alpha \)-**generalized closed set** (briefly, \( \alpha g \)-**closed**) [39] if \( cl_{\alpha}(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is open in \( X \).

(iii) **regular weakly closed set** (briefly, \( rw \)-**closed**) [67] if \( cl(A) \subseteq U \) whenever \( A \subseteq U \) and \( U \) is regular semiopen.

The complement of the above mentioned closed sets are their respective open sets.

**Definition 1.2.5** [62] A topological space \( X \) is said to be **hyperconnected** if every pair of nonempty open sets of \( X \) has nonempty intersection.
Definition 1.2.6 [68] A space $(X, \tau)$ is said to be **quasi-normal** if for disjoint \( \pi \)-closed sets \( F_1 \) and \( F_2 \) there exist disjoint open sets \( U_1, U_2 \) such that \( F_1 \subseteq U_1 \) and \( F_2 \subseteq U_2 \).

Definition 1.2.7 If \( X \) is a set and \( \tau \) and \( \sigma \) are topologies for \( X \), then \( \tau \) is said to be **weakly equivalent to** \( \sigma \) provided if \( U \in \tau \) and \( U \neq \phi \), then there is an open set \( V \) in \( (X, \tau) \) such that \( V \neq \phi \) and \( V \subseteq U \) and if \( U \in \sigma \) and \( U \neq \phi \), there is an open set \( V \) in \( (X, \tau) \) such that \( V \neq \phi \) and \( V \subseteq U \).

### 1.3 Basic Concepts of Open Sets and Closed Sets in Ideal Topological spaces

The subject of an ideals in topological spaces has been studied by Kuratowski [36] and Vaidyanathaswamy [65]. In [65], using the concept of an ideal in a topological space, local function is defined and it is established that the local function generates a topology which is finer than the given topology. In 1990, Jankovic and Hamlett [31] consolidated all the results established by various authors in ideal topological spaces, deduce results established in topological spaces from the results established in ideal topological spaces and establish more new results. Every topological space is an ideal topological space and all the results of ideal topological spaces are generalizations of the results established in topological spaces.

Before entering into our work we recall the following definitions and results from various authors which are useful in the sequel of the thesis.
Definition 1.3.1  [36, 65] A nonempty collection $I$ of subsets of a set $X$ is said to be an **ideal** on $X$, if it satisfies the following two conditions:

(i) $A \in I$ and $B \subseteq A \implies B \in I$ (heredity)

(ii) $A \in I$ and $B \in I \implies A \cup B \in I$ (finite additivity).

If $I$ is an ideal on $X$, then $(X, \tau, I)$ is called an **ideal topological space** or simply an **ideal space**.

Definition 1.3.2  Given a topological space $(X, \tau)$ with an ideal $I$ on $X$ and if $\varphi(X)$ is the set of all subsets of $X$, a set operator $(.)^* : \varphi(X) \rightarrow \varphi(X)$, called a **local function** [36] of $A$ with respect to $\tau$ and $I$ is defined as follows:

for $A \subseteq X$, $A^*(I, \tau) = \{x \in X | U \cap A \notin I \text{ for every } U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau | x \in U\}$.

Definition 1.3.3  Let $(X, \tau, I)$ be an ideal topological space and $A$ be a subset of $X$. Then $A^*(I, \tau) = \{x \in X | A \cap U \notin I \text{ for every } U \in SO(X, x)\}$ is called the **semi-local function** [35] of $A$ with respect to $I$ and $\tau$, where $SO(X, x) = \{U \in SO(X) | x \in U\}$.

A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(I, \tau)$, called the $*$-topology, finer than $\tau$ is defined by $cl^*(A) = A \cup A^*(I, \tau)$ [66] and $int^*(A)$ will denote the interior of $A$ in $(X, \tau^*)$. Clearly, if $I = \{\phi\}$, then $cl^*(A) = cl(A)$ for every subset $A$ of $X$. For every ideal topological space $(X, \tau, I)$, there exists a topology $\tau^*(I)$, finer than $\tau$, generated by the base $\beta(I, \tau) = \{U - J | U \in \tau \text{ and } J \in I\}$. It is known in [31] that $\beta(I, \tau)$ is not always a topology. When there is no chance for confusion, we will simply write $A^*$ for $A^*(I, \tau)$ and $\tau^*$ for $\tau^*(I, \tau)$. 
Definition 1.3.4 Let \((X, \tau, I)\) be an ideal topological space. If \(A\) is a subset of \(X\), then \((A, \tau_A, I_A)\), where \(\tau_A\) is the relative topology on \(A\) and \(I_A = \{A \cap J : J \in I\}\) is an ideal topological space.

Definition 1.3.5 A subset \(A\) of an ideal space \((X, \tau, I)\) is said to be

1. **I-open** [1] if \(A \subseteq int(A^*)\).
2. **semi-I-open** [25] if \(A \subseteq cl^*(int(A))\).
3. **b-I-open** [6] if \(A \subseteq int(cl^*(A)) \cup cl^*(int(A))\).

The complement of I-open (resp. semi-I-open and b-I-open) set is called I-closed (resp. semi-I-closed and b-I-closed). If the set \(A\) is I-open and I-closed (resp. semi-I-open and semi-I-closed, b-I-open and b-I-closed), then it is called I-clopen (resp. semi-I-clopen, b-I-clopen).

Definition 1.3.6 [31] A subset \(A\) of an ideal space \((X, \tau, I)\) is said to be \(*\)-closed (or \(\tau^*\)-closed) if \(A^* \subseteq A\). The complement of \(*\)-closed set is said to be \(*\)-open.

Definition 1.3.7 [15] A subset \(A\) of an ideal space \((X, \tau, I)\) is said to be \(*\)-dense if \(cl^*(A) = X\).

Definition 1.3.8 [28] A subset \(A\) of an ideal space \((X, \tau, I)\) is said to be \(*\)-dense in itself if \(A \subseteq A^*\).

Definition 1.3.9 [17] An ideal space \((X, \tau, I)\) is said to be \(*\)-hyperconnected if \(A\) is \(*\)-dense for every open subset \(A \neq \phi\) of \(X\).
Definition 1.3.10 [28] A subset $A$ of an ideal space $(X, \tau, I)$ is said to be $*$-perfect if $A = A^*$. 

Definition 1.3.11 [54] A space $(X, \tau, I)$ is said to be $*$-normal if for any two disjoint closed sets $A$ and $B$ in $(X, \tau)$, there exist disjoint $*$-open sets $U$, $V$ such that $A \subseteq U$ and $B \subseteq V$. 

Definition 1.3.12 [14] A subset $A$ of an ideal space $(X, \tau, I)$ is said to be $I_g$-closed if $A^* \subseteq U$ whenever $U$ is open and $A \subseteq U$. The complement of a $I_g$-closed set is said to be $I_g$-open. 

Definition 1.3.13 [14] An ideal space $(X, \tau, I)$ is said to be a $T_I$-space if every $I_g$-closed set is $*$-closed. 

Definition 1.3.14 [31, 66] An ideal $I$ is said to be codense or a boundary ideal if $\tau \cap I = \{\phi\}$. 

Definition 1.3.15 [15] An ideal $I$ is said to be completely codense if $PO(X) \cap I = \{\phi\}$, where $PO(X)$ is the family of all pre-open sets in $(X, \tau)$. $\mathcal{N}$ denotes the ideal of all nowhere dense subset in $(X, \tau)$. 

The following lemmas will be useful in the sequel. 

Lemma 1.3.16 [[59], Lemma 1.2] Let $(X, \tau, I)$ be an ideal space and $A \subseteq X$. If $A \subseteq A^*$, then $A^* = cl(A^*) = cl(A) = cl^*(A)$. 

Lemma 1.3.17 [[59], Lemma 1.1] Let $(X, \tau, I)$ be an ideal space. Then $I$ is codense if and only if $A \subseteq A^*$ for every semiopen set $A$ in $X$. 

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Lemma 1.3.18  [58] If $(X, \tau, I)$ be an ideal space and $I$ is completely codense, then $\tau^* \subseteq \tau^a$.

Lemma 1.3.19  [32] Let $(X, \tau, I)$ be an ideal topological space and $B \subseteq A \subseteq X$. Then $B^*(\tau_A, I_A) = B^*(\tau, I) \cap A$.

Lemma 1.3.20  [27] Let $(X, \tau, I)$ be an ideal topological space and $B \subseteq A \subseteq X$. Then $Cl^*_A(B) = Cl^*(B) \cap A$.

Lemma 1.3.21  [15] Let $(X, \tau, I)$ be an ideal space. For each $U \in \tau^*$, $\tau_U^* = (\tau_U)^*$.

1.4 Continuous Maps, Irresolute Maps, Open Maps and Closed Maps in Topological and Ideal Topological Spaces

This section deals with the stronger and weaker forms of continuous maps and irresolute maps contributed by various topologists. After that, several Mathematicians like S. P. Arya, R. Gupta [5], D. Caranahan [8], M. Rajamani, V. Indhumathi and S. Krishnaprakash [56] and R. S. Wali [67] have respectively introduced and studied some stronger and weaker forms of continuous functions namely, completely continuous, $R$-map, $*$-continuous, $I_g$-continuous and $rw$-continuous functions. E. Hatir and T. Noiri [26] introduced and investigated $I$-irresolute maps which are stronger than semi-$I$-continuous but are independent of $I$-continuous maps.

We give definitions of some stronger and weaker forms of continuous, irresolute, open and closed maps which are used in our present study.
**Definition 1.4.1** A map $f : (X, \tau) \to (Y, \sigma)$ is said to be

(i) **completely continuous** [5] if $f^{-1}(V)$ is regular closed in $(X, \tau)$ for every closed set $V$ of $Y$.

(ii) **$rw$-continuous** [67] if $f^{-1}(V)$ is $rw$-closed in $(X, \tau)$ for every closed set $V$ of $Y$.

(iii) **regular semicontinuous** if $f^{-1}(V)$ is regular semi closed in $(X, \tau)$ for every closed set $V$ of $Y$.

(iv) **$R$-map** [8] if $f^{-1}(V)$ is regular closed in $(X, \tau)$ for every regular closed set $V$ of $Y$.

**Definition 1.4.2** A function $f : (X, \tau, I) \to (Y, \sigma)$ is said to be

(i) **$*$-continuous** [56] if $f^{-1}(V)$ is $*$-closed in $X$ for every closed set $V$ of $Y$.

(ii) **$I_g$-continuous** [56] if $f^{-1}(V)$ is $I_g$-closed in $X$ for every closed set $V$ of $Y$.

(iii) **contra $I$-continuous** if $f^{-1}(V)$ is $I$-closed in $X$ for every open set $V$ of $Y$.

**Definition 1.4.3** A function $f : (X, \tau, I) \to (Y, \sigma, J)$ is said to be

(i) **$I$-irresolute** [26] if $f^{-1}(V)$ is semi-$I$-open in $X$ for each semi-$J$-open set $V$ of $Y$.

(ii) **contra $I$-irresolute** if $f^{-1}(V)$ is semi-$I$-open in $X$ for each semi-$J$-closed set $V$ of $Y$. 

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Definition 1.4.4 A function \( f : (X, \tau, I) \to (Y, \sigma) \) is said to be **somewhat-I-continuous function** if for \( U \in \sigma \) and \( f^{-1}(U) \neq \emptyset \) there exists \( I \)-open set \( V \) in \( X \) such that \( V \neq \emptyset \) and \( V \subseteq f^{-1}(U) \).

Definition 1.4.5 A function \( f : (X, \tau, I) \to (Y, \sigma) \) is said to be **somewhat semi-I-continuous** if for \( U \in \sigma \) and \( f^{-1}(U) \neq \emptyset \) there exists an semi-\( I \)-open set \( V \) in \( X \) such that \( V \neq \emptyset \) and \( V \subseteq f^{-1}(U) \).

Definition 1.4.6 A function \( f : (X, \tau, I) \to (Y, \sigma) \) is said to be **\( I \)-continuous** [1], the inverse image of each open set is \( I \)-open.

Definition 1.4.7 A function \( f : (X, \tau, I) \to (Y, \sigma) \) is said to be **semi-\( I \)-continuous** [25], the inverse image of each open set is semi-\( I \)-open.

Definition 1.4.8 A function \( f : (X, \tau) \to (Y, \sigma, I) \) is said to be **\( I \)-open** (resp. **semi-\( I \)-open**) function if the image of open set \( U \) in \( (X, \tau) \) is \( I \)-open (resp. semi-\( I \)-open) in \( (Y, \sigma, I) \).

Definition 1.4.9 A function \( f : (X, \tau) \to (Y, \sigma, I) \) is said to be **somewhat-I-open function** provided that for \( U \in \tau \) and \( U \neq \emptyset \), there exists \( I \)-open set \( V \) in \( Y \) such that \( V \neq \emptyset \) and \( V \subseteq f(U) \).

Definition 1.4.10 A function \( f : (X, \tau) \to (Y, \sigma, I) \) is said to be **somewhat semi-I-open function** provided that for \( U \in \tau \) and \( U \neq \emptyset \), there exists a semi-\( I \)-open set \( V \) in \( Y \) such that \( V \neq \emptyset \) and \( V \subseteq f(U) \).
1.5 Ideal Bitopological Spaces

The concept of bitopological space was introduced by J. C. Kelly [34]. Following the work of Kelly on bitopological spaces, various authors like T. Noiri and N. Rajesh [53], K. Kannan, D. Narasimhan, K. Chandrasekhara Rao and M. S. Srinivasan [33] and R. S. Wali [67] have turned their attention to the various concepts of topology by considering bitopological spaces and ideal topological spaces. Let \((X, \tau_1, \tau_2, I)\) or simply \(X\) denote an ideal bitopological space. The intersection (resp. union) of all \(\tau_i\)-semi closed sets containing \(A\) (resp. \(\tau_i\)-semi open sets contained in \(A\)) is called the \(\tau_i\)-semi closure (resp. \(\tau_i\)-semi interior) of \(A\), denoted by \(\tau_i\text{-scl}(A)\) (resp. \(\tau_i\text{-sint}(A)\)). For any subset \(A \subseteq X\), \(\tau_i\text{-int}(A)\) and \(\tau_i\text{-cl}(A)\) denote the interior and closure of a set \(A\) with respect to the topology \(\tau_i\) respectively. The closure and interior of \(B\) relative to \(A\) with respect to the topology \(\tau_i\) are written as \(\tau_i\text{-cl}_A(B)\) and \(\tau_i\text{-int}_B(A)\) respectively. The set of all \(\tau_i\)-regular closed sets in \(X\) is denoted by \(\tau_i\text{-RC}(X, \tau_1, \tau_2)\). The set of all \(\tau_j\)-regular open sets in \(X\) is denoted by \(\tau_j\text{-RO}(X, \tau_1, \tau_2)\). \(A^C\) denotes the complement of \(A\) in \(X\) unless explicitly stated. Here we present some of the definition, which are used in our study.

**Definition 1.5.1** [67] Let \(i, j \in \{1, 2\}\) be fixed integers. In a bitopological space \((X, \tau_1, \tau_2)\), a subset \(A\) of \(X\) is said to \((i, j)\text{-rw-closed}\) if \(\tau_j\text{-cl}(A) \subseteq U\) whenever \(A \subseteq U\) and \(U\) is regular semiopen in \(\tau_i\).

The complement of \((i, j)\text{-rw-closed}\) set is called \((i, j)\text{-rw-open set} \).

**Definition 1.5.2** Let \(i, j \in \{1, 2\}\) be fixed integers. A subset \(A\) of a bitopological space \((X, \tau_1, \tau_2)\) is called
(i) \( \tau_i \tau_j \)-open [29] if \( A \in \tau_i \cup \tau_j \).

(ii) \( \tau_i \tau_j \)-closed [29] if \( A^C \in \tau_i \cup \tau_j \).

(iii) \( \tau_i \tau_j \)-regular open [9] if \( \tau_i \cdot \text{int}[\tau_j \cdot \text{cl}(A)] = A \).

(iv) \( \tau_i \tau_j \)-regular closed [9] if \( \tau_i \cdot \text{cl}[\tau_j \cdot \text{int}(A)] = A \).

**Definition 1.5.3** Let \( i, j \in \{1, 2\} \) be fixed integers. In an ideal bitopological space \((X, \tau_1, \tau_2)\), a subset \( A \) of \( X \) is called

(i) \( (i, j) \)-generalized closed with respect to an ideal \{\( (i, j) \)-\( I_g \) closed\} [53] in \( X \) if and only if \( \tau_j \cdot \text{cl}(A) - U \in I \) whenever \( A \subseteq U \) and \( U \) is \( \tau_i \)-open in \( X \).

(ii) \( (i, j) \)-regular generalized star closed with respect to an ideal \{\( (i, j) \)-\( I_{rg} \) closed\} [33] in \( X \) if and only if \( \tau_j \cdot \text{rcl}(A) - U \in I \) whenever \( A \subseteq U \) and \( U \) is \( \tau_i \tau_j \)-regular open in \( X \).

**Definition 1.5.4** In a bitopological space \((X, \tau_1, \tau_2)\) a set \( A \) of \( X \) is said to be quasi open [11] if it is a union of a \( \tau_1 \)-open set and a \( \tau_2 \)-open set.

The complement of a quasi open set is termed as a quasi closed.

Every \( \tau_1 \)-open (resp. \( \tau_2 \)-open) set is quasi open but the converse may not be true. Any union of quasi open sets of \( X \) is quasi open in \( X \). The intersection of all quasi closed sets which contains \( A \) is called quasi closure of \( A \). It is denoted by \( qcl(A) \). The union of all quasi open sets contained in \( A \) is called quasi interior of \( A \). It is denoted by \( qint(A) \).

**Definition 1.5.5** A set \( A \) in a bitopological space \((X, \tau_1, \tau_2)\) is called quasi
**regular semiopen** if it is a union of a $\tau_1$-regular semiopen set and a $\tau_2$-regular semiopen set.

Complement of a quasi regular semiopen set is called a quasi regular semiclosed.

Every $\tau_1$-regular semiopen ($\tau_2$-regular semiopen, quasi open) set is quasi regular semiopen but the converse may not be true. Any union of quasi regular semiopen sets of $X$ is a quasi regular semiopen set in $X$. The intersection of all quasi regular semiclosed sets which contains $A$ is called quasi regular semi closure of $A$. It is denoted by $qrscl(A)$. The union of all quasi regular semiopen sets contained in $A$ is called quasi regular semi interior of $A$. It is denoted by $qrsint(A)$.

**Definition 1.5.6** Given an ideal bitopological space $(X, \tau_1, \tau_2, I)$ the **quasi local function** [30] of $A$ with respect to $\tau_1, \tau_2$ and $I$ denoted by $(\tau_1, \tau_2, I)$ (in short $A_q^*$) is defined as follows:

$$A_q^*(\tau_1, \tau_2, I) = \{x \in X | U \cap A \notin I, \forall \text{ quasi open set } U \text{ containing } x\}.$$

**Definition 1.5.7** A subset $A$ of an ideal bitopological space $(X, \tau_1, \tau_2)$ is said to be **$qI$-open** [30] if $A \subseteq qintA_q^*$.

**Definition 1.5.8** A mapping $f : (X, \tau_1, \tau_2, I) \rightarrow (Y, \sigma_1, \sigma_2)$ is called **$qI$-continuous** [30] if $f^{-1}(V)$ is $qI$-open in $X$ for every quasi open set $V$ of $Y$. 

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