CHAPTER 7

Optimal design of a TM-mode gyro-TWT amplifier

Amplification is one of the most basic and prevalent microwave circuit functions in modern r.f. and microwave systems. Amplifier is an electronic circuit that strengthens a weak input signal. Amplifiers require an injected microwave signal which is then amplified, usually by several orders of magnitude in a single pass. A practical amplifier should be capable of giving a high gain over a broad range of frequencies around the design frequency.

In the design of any millimeter wave amplifier, there are several issues to consider like, the desired operating frequency, the required magnetic field, the best operating mode, beam current etc.

We will now illustrate the small-signal theory of TE and TM -mode interaction developed in Chapters 5 and 6 in the subsequent sections of this Chapter. We have already discussed in Chapter 5 that, it is not feasible to make gyro-TWT work as an amplifier by enforcing the cyclotron resonance condition over the entire frequency range. In case of TM-mode interaction, when the cyclotron-resonance condition is enforced over the entire frequency range by allowing the gyro-radius and the relativistic factor to be frequency-dependent the initial growth rate of the interacting TM wave is seen to exhibit two maxima on either side of the minimum, a sharp but large maximum to the left and a broad but a smaller maximum to the right. This double humped curve gives us the flexibility to design an amplifier.

7.1. TE mode discussion

On maintaining the cyclotron resonance condition, it can be observed from the plots in Figures 5.11-5.20, that on fixing the optimum values of normalized gyro-radius given in Table 5.2, the plots for variation of initial growth rate exhibit a sharp but large maximum to the left of chosen normalized frequency $\tilde{\omega}_M=2$. There is no flat maximum around $\tilde{\omega}_M=2$, which clearly indicates that gyro-TWT cannot be used as an amplifier in TE modes. It should be emphasized that such growth rate versus frequency curves are obtained by varying $\gamma_0(\tilde{\omega})$ and
\(X(\tilde{\omega})\) with respect to \(\tilde{\omega}\) in such a way as to maintain cyclotron resonance throughout the frequency range \([\tilde{\omega}_l, \tilde{\omega}_u]\). However, it is impractical to allow any such variation in an amplifier. Thus, it is mandatory to keep the values of the relativistic factor and the normalized gyro-radius constant at

\[
\gamma_0(\tilde{\omega}_M) = \frac{s_0 \tilde{\Omega}_e}{(\tilde{\omega}_M - \tilde{\beta}_{mn}(\tilde{\omega}_M)) \tilde{v}_{oz}}
\]

(7.1)

and

\[
X(\tilde{\omega}_M) = \left[ s_0^2 \tilde{\Omega}_e^2 \left( 1 - \tilde{v}_{oz}^2 \right) / \left( (\tilde{\omega}_M - \tilde{\beta}_{mn}(\tilde{\omega}_M) \tilde{v}_{oz})^2 - 1 \right) \right]^{1/2} / \tilde{\Omega}_e
\]

(7.2)

With \(\gamma_0(\tilde{\omega})\) and \(X(\tilde{\omega})\) frozen at \(\gamma_0(\tilde{\omega}_M)\) and \(X(\tilde{\omega}_M)\), the expressions (5.40) for the coefficients of the biquadratic (5.39) become

\[
a_{M2}(\tilde{\omega}) = \frac{\Lambda_o \sigma(X)}{\tilde{\beta}_{mn}^2(\tilde{\omega}) \tilde{\omega}_M \tilde{v}_{oz}} \left[ \frac{H_{smn}(X)}{s^2 \tilde{\omega}_M^2 \tilde{v}_{oz}^2} + Q_{smn}(X) \right]
\]

(7.3a)

\[
a_{M1}(\tilde{\omega}) = \frac{\Lambda_o \sigma(X)}{\tilde{\beta}_{mn}^2(\tilde{\omega}) \tilde{\omega}_M \tilde{v}_{oz}} \left[ \frac{2H_{smn}(X)}{s^2 \tilde{\omega}_M^2 \tilde{v}_{oz}^2} - \left( \frac{\tilde{\omega}}{\tilde{\omega}_{oz}} - \tilde{\beta}_{mn}(\tilde{\omega}) \right) Q_{smn}(X) \right]
\]

(7.3b)

\[
a_{M0}(\tilde{\omega}) = \frac{\Lambda_o \sigma(X)}{\tilde{\beta}_{mn}^4(\tilde{\omega}) s^2 \tilde{\omega}_M^3 \tilde{v}_{oz}^3} H_{smn}(X)
\]

(7.3c)

We will begin by computing and plotting in Figures 7.1 –7.5, the normalized growth rate corresponding to the expressions (7.3) for the biquadratic against \(\tilde{\omega}\) for a fixed frequency range.
Figure 7.1: Variation of the initial growth rate around the design frequency on fixing the parameters for $\text{TE}_{01}$ mode.

Figure 7.2: Variation of the initial growth rate around the design frequency on fixing the parameters for $\text{TE}_{02}$ mode.
Figure 7.3: Variation of the initial growth rate around the design frequency on fixing the parameters for TE$_{03}$ mode

Figure 7.4: Variation of the initial growth rate around the design frequency on fixing the parameters for TE$_{13}$ mode
It can be observed from Figures 7.1-7.5 that there is no flat maximum around $\tilde{\omega}_M = 2$. Once the amplifier parameters are fixed, the growth-rate curve no longer exhibits a peak around the chosen normalized frequency but rather a monontonic variation with respect to normalized frequency leading to severe frequency distortion of any signal with a frequency band centered around $\tilde{\omega}_M$. This behavior persists irrespective of the value of $\tilde{\omega}_M$. Even a decrease in the value of the normalized gyro-radius $X_M$ from $X_{M0}$ does not affect this behavior of the growth-rate curve. Thus it may be concluded that a gyro-TWT operating in a TE mode is incapable of functioning as a small-signal amplifier.

### 7.2. Designing a gyro-TWT amplifier in TM-mode interaction

We now illustrate the small-signal theory of TM-mode interaction developed in Chapter 6 by indicating the steps involved in the optimal design of a gyro-TWT amplifier for small-signal interaction with an axi-symmetric TM mode [35]. The design specifications are collected together in Table 7.1.
Table 7.1 Design specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam current</td>
<td>10 A</td>
</tr>
<tr>
<td>Wave-guide radius</td>
<td>a=0.54cm</td>
</tr>
<tr>
<td>Operating (center) frequency</td>
<td>f_{do}=94GHz</td>
</tr>
<tr>
<td>Annular beam width</td>
<td>b=a/2</td>
</tr>
<tr>
<td>Beam upper boundary</td>
<td>b_u=3a/4</td>
</tr>
<tr>
<td>Beam lower boundary</td>
<td>b_l=a/4</td>
</tr>
<tr>
<td>Operating mode</td>
<td>TM_{02}</td>
</tr>
<tr>
<td>s-number</td>
<td>2</td>
</tr>
</tbody>
</table>

We begin by computing and plotting in Figure 7.6, the normalized initial growth rate $|y_M|$ of the exponentially growing wave for the choice of $\tilde{\omega}_M = 1.53184$ against $X_M \in [0, X_u]$ using the amplifier data of Table 7.1.

![Figure 7.6: Variation of the initial growth rate against $X_M$](image)

Let $X_{smn}$ be the value of $X_M$ at which the growth rate peaks. This optimum value $X_{smn}$ of $X_M$ turns out to be independent of the value of $\tilde{\omega}_M$ and remains constant at $X_{202} = 1.0599$ as $\tilde{\omega}_M$ is varied. This constant value of $X_{202}$ will be made use of in the rest of the design calculations.
Replacing $X_M$ by $X_{202}$ in the expressions (6.48) for the coefficients of the biquadratic (6.47), the resulting biquadratic is seen to possess a pair of real roots and a second pair of complex conjugate roots for all $\tilde{\omega} \in (\tilde{\omega}_{lo}, \tilde{\omega}_{uo})$ where

$$\tilde{\omega}_{lo,uo} = \tilde{\omega}_M (1 \mp \hat{v}_{oz} X_{202} / s) \sigma_s (X_{202})$$

A plot of the normalized initial growth rate $|\text{Im}(\Delta \beta)|/\beta_{02}$ of the exponentially growing wave as a function of $\tilde{\omega} \in (\tilde{\omega}_{lo}, \tilde{\omega}_{uo})$ for the choice of $\tilde{\omega}_M = 1.53184$ is shown in Figure 7.7. It may be observed from the plot in Figure 7.7 that the initial growth rate exhibits a shallow minimum at $\tilde{\omega}_{min}$, which is shifted slightly to the right with respect to $\tilde{\omega}_M$ and two maxima on either side of the minimum, a sharp but large maximum very close to the lower cut-off frequency $\tilde{\omega}_{lo}$ and a flat but smaller maximum at $g(\tilde{\omega}_M)$ between $\tilde{\omega}_{min}$ and the upper ‘cut-off frequency’ $\tilde{\omega}_{uo}$ where $g(\tilde{\omega}_M)$ is the value of the strictly monotone increasing function $g$ (of $\tilde{\omega}_M$) specifying the location of the flat maximum on the $\tilde{\omega}$-axis. When $\tilde{\omega}_M = 1.53184$, $\tilde{\omega}_{min} = 1.54740$ and $g(\tilde{\omega}_M) = \tilde{\omega}_{do} = 1.92722$ which corresponds to the design frequency of $f_{do} = 94$GHz.

**Figure 7.7:** Variation of the initial growth rate when cyclotron resonance is maintained throughout

Similar ‘growth rate vs frequency’ curves may be plotted for any $\tilde{\omega}_M > 1$; the peak value of the initial growth rate at $g(\tilde{\omega}_M)$ decreases progressively with increasing values of $\tilde{\omega}_M$. It should be emphasized that such growth rate vs. frequency curves are obtained by varying
\( \gamma_o(\tilde{\omega}) \) and \( X(\tilde{\omega}) \) with respect to \( \tilde{\omega} \) in such a way as to maintain cyclotron resonance throughout the frequency range \( [\omega_{1o}, \omega_{uo}] \). However, it is impractical to allow any such variation of the amplifier parameters with respect to the frequency of the input signal in a gyro-TWT amplifier designed for operation around a fixed frequency \( f_{do} \). Thus, it is mandatory to keep the values of the relativistic factor and the normalized gyro-radius constant at

\[
\gamma_o(\tilde{\omega}_d) = 1/\left( \tilde{\omega}_d - \tilde{\beta}_{mn}(\tilde{\omega}_d)\tilde{v}_{oz} \right) \sigma_s(X_{202})
\]

and

\[
X(\tilde{\omega}_d) = X_{202}\left[ 1 - (s/X_{202})^2\left( (\tilde{\beta}_{mn}(\tilde{\omega}_d) - \tilde{\omega}_d \tilde{v}_{oz})/(\tilde{\omega}_d - \tilde{\beta}_{mn}(\tilde{\omega}_d) \tilde{v}_{oz}) \right)^2 \right]^{1/2}
\]

respectively. With \( \gamma_o(\tilde{\omega}) \) and \( X(\tilde{\omega}) \) frozen at \( \gamma_o(\tilde{\omega}_d) \) and \( X(\tilde{\omega}_d) \), the expressions (6.48) for the coefficients of the biquadratic (6.47) become

\[
a_{d3}(\tilde{\omega}) = 2 - \Lambda_o \sigma_s(X_{202}) \left[ \Delta^2(\tilde{\omega}_d)H_{smn}(X(\tilde{\omega}_d)) + s^2K_{smn}(X(\tilde{\omega}_d)) \right] / \tilde{\beta}_{mn}(\tilde{\omega}) \quad (7.4a)
\]

\[
a_{d2}(\tilde{\omega}) = \Lambda_o \sigma_s(X_{202}) / \tilde{\beta}_{mn}(\tilde{\omega}) \left[ 2s^2K_{smn}X(\tilde{\omega}_d)/\tilde{\omega}\tilde{\beta}_{mn}(\tilde{\omega})\tilde{v}_{oz} - \Delta^2(\tilde{\omega}_d)H_{smn}(X(\tilde{\omega}_d)) + \chi(\tilde{\omega})(2s^2K_{smn}X(\tilde{\omega}_d)) + (\Delta(\tilde{\omega}_d)\Delta(\tilde{\omega}_d)X(\tilde{\omega}_d) - 1)/\tilde{\beta}_{mn}(\tilde{\omega})\tilde{v}_{oz} H_{smn}(X(\tilde{\omega}_d))/\tilde{\omega}\tilde{v}_{oz} \right] \quad (7.4b)
\]

\[
a_{d1}(\tilde{\omega}) = (\Lambda_o \sigma_s(X_{202}) / \tilde{\beta}_{mn}^2(\tilde{\omega})) \left[ \Delta(\tilde{\omega}_d) \left( 2H_{smn}(X(\tilde{\omega}_d))/\tilde{\omega}\tilde{\beta}_{mn}(\tilde{\omega})\tilde{v}_{oz} + \chi(\tilde{\omega})(\Delta(\tilde{\omega}_d)X(\tilde{\omega}_d))/\tilde{\omega}\tilde{v}_{oz} + \tilde{\beta}_{mn}(\tilde{\omega}) \right) H_{smn}(X(\tilde{\omega}_d)) \right]
\]

\[
- s^2 \chi^2(\tilde{\omega})K_{smn}(X(\tilde{\omega}_d))/\tilde{\beta}_{mn}(\tilde{\omega})\tilde{v}_{oz}^2 \quad (7.4c)
\]

\[
a_{do}(\tilde{\omega}) = \Lambda_o \sigma_s(X_{202}) \Delta(\tilde{\omega}_d)\chi^2(\tilde{\omega})H_{smn}(X(\tilde{\omega}_d))/\tilde{\beta}_{mn}^4(\tilde{\omega})\tilde{v}_{oz}^2 \quad (7.4d)
\]

When the normalized growth rate corresponding to the expressions (7.4) for the biquadratic is computed and plotted against \( \tilde{\omega} \in [\tilde{\omega}_{min}, \tilde{\omega}_{M}] \) for a fixed choice of value of \( \tilde{\omega}_M > 1 \) and a corresponding choice of value \( \tilde{\omega}_d^{(o)} \) of \( \tilde{\omega}_d \) lying strictly between \( \tilde{\omega}_{min} \) and \( \tilde{\omega}_{uo} \), the resulting plot is found to have a peak at a value \( \tilde{\omega}_d^{(1)} \) of \( \tilde{\omega}_d \) differing in general from \( \tilde{\omega}_d^{(o)} \). The procedure is repeated with \( \tilde{\omega}_d^{(n)} \) replacing \( \tilde{\omega}_d^{(n-1)} \) at the nth stage of the iteration for
n=1,2,3,... The resulting sequence of values of $\hat{\omega}_d^{(n)}$, n=1,2,... thus generated converges rapidly to a number $h(\hat{\omega}_M)$ which may be taken to be value of a strictly monotone increasing function $h$ at $\hat{\omega}_M$. In other words, we are seeking by the method of successive approximations, for each $\hat{\omega}_M > 1$, the fixed point $h(\hat{\omega}_M)$ of the function $F$ mapping $\hat{\omega}_d \mapsto F(\hat{\omega}_d, \hat{\omega}_M)$ (i.e., $F(h(\hat{\omega}_M), \hat{\omega}_M) = h(\hat{\omega}_M)$), where $F(\hat{\omega}_d, \hat{\omega}_M)$ is the point on the $\hat{\omega}$-axis corresponding to the peak of the growth-rate curve. Our objective is to determine the value of $\hat{\omega}_M$ to be the unique solution of the equation $h(\hat{\omega}_M) = \hat{\omega}_d$, for $\hat{\omega}_M > 1$, where $\hat{\omega}_{do} = 1.92722$ is the normalized design frequency. Thus, we need only choose $\hat{\omega}_M = h^{-1}(\hat{\omega}_{do})$ in order to meet the design objective where $h^{-1}$ denotes the function inverse to $h$.

![Figure 7.8: Plot of the function h](image)

A plot of $h(\hat{\omega}_M)$, computed using the method outlined above, against $\hat{\omega}_M \in [1.3,1.4]$ is shown in Figure 7.8. The required value of $h^{-1}(1.92722)$ may be read off from the almost straight-line plot of Figure 7.8 to be $\hat{\omega}_M = 1.34610$. The design of the gyro-TWT amplifier may now be completed by computing the values of remaining amplifier parameters:

$$\hat{v}_{oz} = \frac{v_{oz}}{c} = \beta_{02} (\hat{\omega}_M)/\hat{\omega}_M = 0.669417$$
If the dominant TM$_{01}$ had been chosen as the operating mode instead of TM$_{02}$, the value of $\hat{v}_{oz}$ would have been considerably larger. The required value of the applied (uniform) magnetic field works out to be

$$B_0 = (m_e k_{02} c/e) \hat{\Omega}_e = (m_e k_{02} c/e) \left( s^2 - \hat{X}_{202}^2 \right)^{-1/2} = 1.0273 \text{T}$$

The choice of $s=1$ for the TM$_{02}$ would have resulted in an inconveniently large value for the magnetic field strength. The variation of the normalized initial growth rate $|\text{Im}(\Delta \beta(\omega))/\beta_{02}(\omega)|$ of the exponentially growing wave and the normalized deviation $\text{Re}(\Delta \beta(\omega)/\beta_{02}(\omega))$ of this wave from the unperturbed propagation phase constant $\beta_{02}(\omega)$ of the TM$_{02}$ mode are computed for gyro-TWT amplifier optimally designed according to the specifications of Table 7.1 for operation in a ‘small’ frequency band $[f_1, f_2]$ around the design frequency of $f_{do}=94\text{GHz}$, subject to the condition $(f_2 - f_1)/f_{do} \ll 1$, is plotted against $f/f_{do}$ for the choice of $f_1/f_{do}=0.98195$ and $f_2/f_{do}=1.01945$ in Figures 7.9.a and 7.9.b respectively. The ‘small-frequency deviation’ condition $(f_2 - f_1)/f_{do} \ll 1$ is required to ensure that cyclotron resonance is maintained at least approximately over the entire frequency band $[f_1, f_2]$. 
Figure 7.9.a: Variation of the initial growth rate around the design frequency

Figure 7.9.b: Variation of the normalized deviation of the perturbed propagation constant from $\beta_{02}$ around the design frequency
It may be seen from the frequency-response curves of Figure 7.9 that a gyro-TWT amplifier optimally designed for interaction with a TM mode of a circular cylindrical waveguide is capable of decent small-signal gain over a broad band of frequencies around the design frequency. As a result, the frequency distortion suffered by a passband input signal is negligibly small.