CHAPTER 3

LITERATURE SURVEY

3.1 TE mode interaction

The mechanism of small-signal amplification in gyro-TWTs and cyclotron resonance masers (CRMs) [49-55, 58, 59] was actively studied by many researchers in the 1980s [17-20, 71, 72] culminating in the derivation of the dispersion relation in the form of an infinite series. The celebrated (Doppler-shifted) cyclotron resonance condition

\[ \omega - v_z \beta_{mn}(\omega) - s \Omega_e / \gamma = 0 \]  

(3.1)

was identified as a necessary requirement for small-signal amplification. In (3.1), \( \omega \) is the operating (radian) frequency, \( \beta_{mn}(\omega) \) is the unperturbed propagation phase constant of the \( mn^{th} \) waveguide mode, \( v_z \) is the axial speed of the electrons, \( \Omega_e = eB_o / m_e \) is the electron cyclotron frequency corresponding to an applied uniform magnetic field \( \hat{z}B_o \) in the axial direction and \( \gamma \) is the relativistic factor. In the expression for \( \Omega_e \) and in the sequel, \( -e \) and \( m_e \) are respectively the charge and the rest mass of an electron. When the cyclotron resonance condition is satisfied by a particular waveguide mode for a given \( s = s_o \), the dispersion relation may be reduced to an algebraic equation by retaining only the significant contribution from the \( s_o^{th} \) term of the infinite series. The resulting biquadratic algebraic equation may be solved for the (complex) propagation phase constant as a function of the operating frequency.

The works of Edgcombe [17], Chu et al. [18], Fliflet [19] and Chu and Lin [20] may be cited as being specifically directed towards a derivation of the linear dispersion relation for circular cylindrical waveguide modes. Edgcombe [17] assumes without adequate justification that the r.f. charge and current densities depend linearly on the electric field of the TE mode in the derivation of the dispersion relation. Also, the need for maximizing

\[ I_{ms} \Delta \frac{1}{4} (1 + \delta_{mo}) \left[ J_{s+m}^2(k_c r_g) + J_{s-m}^2(k_c r_g) \right] \]
where \( r_g \) is the gyro-radius and \( k_c \) is the mode cut-off wave number, with respect to \( k_c r_g \) to arrive at the biquadratic algebraic equation satisfied by the normalized phase constant \( k/k_c \) has not been brought out clearly by him. Although the derivation of the dispersion relation by Chu et al. [18] for TE modes is free from any of these drawbacks, their treatment is too sketchy leaving out many details of analysis. However, a complete derivation with all details filled in has been provided by Chu et al. in a subsequent paper [20]. Fliflet [19], on the other hand, gives a detailed derivation of the linear dispersion relation together with a description of the single-particle quasilinear theory for both TE and TM modes. Finally, Kou et al. [21] have, in the recent past, presented a linear theory, using Laplace transforms that is applicable to both gyro-TWTs and gyro-BWOs, and used this theory to study the stability of harmonic gyro-TWTs.

All of the above derivations of the dispersion relation make use of kinetic theory based on linearized Vlasov equation and a transformation from the polar coordinates \( (r, \theta, p_r, \phi) \) of the transverse position and momentum to the gyro-co-ordinates \( (r_o, \theta, r_L, \tilde{\phi}) \) (Edgcombe starts initially with Cartesian coordinates of position and momentum) where

\[
p_t = \left( p_r^2 + p_\theta^2 \right)^{\frac{1}{2}} = \left( p_r^2 + p_\phi^2 \right)^{\frac{1}{2}}\]

is the magnitude of the transverse momentum, 
\[
\phi = \arctan(p_\theta/p_\phi), \]
\[
r_o \text{ is the distance of the electron guiding centre from the waveguide axis,}
\]
\[
r_L = p_r/m_e \Omega_e \]
\[
is the gyro-radius and \( \tilde{\phi} \) is the gyro-phase.

It is well known that the functional dependence of the axially symmetric equilibrium distribution function \( f_o \) on the position and the momentum variables can only be through the single-particle constants of motion. For a \( z \)-directed uniform magnetic field, \( z B_z \) such constants of motion in absence of space-charge fields are the total energy

\[
H = \left[ m_e c^4 + c^2 \left( p_t^2 + p_\phi^2 \right) \right]^{\frac{1}{2}} - m_e c^2
\]

the canonical angular momentum \( P_\theta = r p_\theta - \frac{1}{2} e B_s r^2 \) and the \( z \)-component of the linear
momentum $p_z$ [22] where $c$ is the vacuum speed of light. The relation

$$P_\theta = \frac{1}{2} m_r \Omega_c \left( r_L^2 - r_o^2 \right)$$

implies that the axisymmetric equilibrium distribution function

$$\tilde{f}_o(r, p_t, \phi, p_z) \Delta f_o(H(p_t, p_z), P_\theta(p_t, r_o), p_z) \Delta \tilde{f}_o(r, p_t, p_z)$$

is a function only of $r_o, p_t$ (or equivalently $r_L$) and $p_z$.

Fliflet's derivation of the dispersion relation is based on the observation that, “for configuration which optimize the cyclotron maser interaction, $\hat{f}_o$ may be assumed to be independent of $r_o$ with negligible error”, and he sets $\partial \hat{f}_o / \partial r_o$ equal to zero on this ground.

However, he contradicts this hypothesis in a subsequent step by assuming a 'delta-function' dependence for $\hat{f}_o$ on $r_o$. Chu and Lin [20], on the contrary, do no drop the $\partial \hat{f}_o / \partial r_o$ terms at any stage of their derivation of the dispersion relation except that they too assume a 'delta-function' dependence for a differentiable function of $r_o$!

### 3.2 TM mode interaction

Along with the problem mentioned in previous section, there is a more serious fundamental error in the kinetic-theory based approach adopted in every one of the analyses of the linear interaction with a gyrating electron beam reported in the literature when the interaction is with a TM mode.

The small-signal theory of the linear interaction between a TM mode of a circular cylindrical waveguide and a relativistic beam of gyrating electrons in a gyro-TWT configuration appears in a number a papers [19,23-26] published between 1986 and 2009; however, every one of the reported derivations of the dispersion relation governing small-signal amplification without exception has failed to make use of the correct form of the electron phase-space density function in the kinetic-theory based approach adopted by them ending up thereby with erroneous results not only for TM-mode but also for TE-mode interaction. Moreover, there is a common fundamental error specific to TM-mode interaction made in every one of the
kinetic-theory based approaches reported so far in the literature. The starting point of the kinetic theory based linear analyses of the TM-mode interaction presented in the papers of Fliflet[19], Chen and Wurtele[23] and Jiao and Luo[24] is the incorrect axial profile relations which would be discussed in detail in a subsequent chapter. The modal expansions of the transverse electric and magnetic fields used by Chen and Wurtele for a general linear analysis of cyclotron autoresonance masers (CRMs) with multiple wave-guide modes in [23] are also based on the incorrect relations among the axial profile functions. Although the correct nonhomogeneous scalar Helmholtz equation for the electric field component $E_z$

$$\nabla_z^2 + \partial^2 E_z / \partial z^2 + (\omega^2 / c^2) E_z = j \omega \mu_0 J_z + \epsilon_0^{-1} \partial p / \partial z$$

which may be deduced from the Maxwell’s equations and has subsequently been made use of in [19], [23] and [24] to derive a dispersion equation (using the machinery of Laplace transforms in [23] and [24]) governing the linear interaction of a TM mode with a beam of gyrating electrons. The derived dispersion equation cannot be correct due to the initial use of the incorrect relations in the derivation. The study of cyclotron autoresonance with TE and TM guided modes conducted by Sabchevski and Idehara in [25] is mainly confined to quoting results only without any accompanying analysis. The paper on TM –mode interaction by Yang and Zhang [26] is nothing more than a slightly expanded version of the paper by Jiao and Luo[24] on the linear analysis of a CARM operating in a TM mode differing from the latter only in the numerical results for the TM$_{11}$ mode. It is thus seen that a small-signal kinetic theory of TM-mode interaction making use of the correct form for the equilibrium phase-space density function is yet to emerge.