Chapter 2  THEORY OF DIELECTRICS

2.1 Theory of Homogeneous Dielectrics
2.2 Theory of Heterogeneous Dielectrics
2.3 Dielectric Loss and Dissipation Factor
2.4 Attenuation Factor and Penetration Depth
2.5 Frequency Dependence of Tissue Conductivity and Relative Permittivity
2.6 Importance of Dielectric Properties of Tissue

2.1. Theory of Homogeneous Dielectrics

The dielectric permittivity ($\varepsilon$) and conductivity ($\sigma$) of a material are, respectively the charge and the current densities induced in the response to an applied electric field of unit amplitude. It can be illustrated by considering an idealized parallel plate capacitor, of plate area ($A$) and separation between the plates ($d$), which contains the material investigation (Fig. 2.1). Initially, let the region between the plates be a vacuum. A constant potential difference of ($V$) between the plates will induce a charge density ($D$) given by

$$D = \varepsilon_0 \frac{V}{d}$$

(2.1)

where $\varepsilon_0$ is permittivity of free space ($8.85 \times 10^{-12} \text{F/m}$)

The capacitance ($C$) is the ratio of the total induced charge on the plates to the applied voltage. Ignoring the effects from the fringing fields outside of the region between the plates, the capacitance is given by

$$C = \frac{\varepsilon_0 A}{d}$$

(2.2)
Fig. 2.1 Equivalent circuit of an idealized parallel-plate capacitor filled with material of dielectric constant $\varepsilon'$ and conductivity $\sigma$. The plate of area $A$ and the distance between the plates is $d$. In a real circuit a small additional capacitance would occur from the stray fields in air outside of the material.

If a material is now introduced between the plates, the additional charge density $P_0$ will be induced on the plates from the polarization of charges within the material. For sufficiently low voltages and in the limit of long times, this charge density is proportional to $E$ and can be written as

$$D = \varepsilon_0 E + P_0 = \varepsilon_1 \varepsilon_0 E$$  \hspace{1cm} (2.3)

and the capacitance is now represented by

$$C = \frac{\varepsilon_1 \varepsilon_0 A}{d}$$  \hspace{1cm} (2.4)

where $\varepsilon_0$ is static permittivity of the material.

The circuit might also include a conductance $G$ in parallel with the capacitance and is given by

$$G = \frac{\sigma_0 A}{d}$$  \hspace{1cm} (2.5)

where $\sigma_0$ is static conductivity of the material.

For sinusoidally oscillating fields, the electrical properties of the circuit (Fig.2.1) can be specified in several ways.

At angular frequency ($\omega$) the complex admittance can be written as

$$Y' = G + j\omega C = \frac{A}{d} (\sigma + j\omega \varepsilon_0 \varepsilon)$$  \hspace{1cm} (2.6)

Then the complex conductivity ($\sigma'$) the material can be written as
The complex capacitance ($C^*$) can be defined as

$$C^* = \frac{Y^*}{j\omega}$$

(2.8)

Which leads to the definition of the complex relative permittivity ($\varepsilon^*$).

$$\varepsilon^* = \varepsilon' - j\varepsilon'' = \varepsilon' - j\frac{\sigma}{\omega\varepsilon_0}$$

(2.9)

The complex conductivity and permittivity are related by

$$\sigma^* = j\omega\varepsilon^*$$

(2.10)

Finally, the series equivalent impedance ($Z^*$) can be written as

$$Z^* = \frac{1}{Y^*} = R + jX = \frac{G - j\omega C}{G^2 + (\omega C)^2}$$

(2.11)

from which the complex specific impedance of the material ($Z^*_i$) is defined as

$$Z^*_i = \frac{1}{\sigma^*} = \frac{\sigma - j\omega\varepsilon_0\omega}{\sigma^2 + (\omega\varepsilon_0\omega)^2}$$

(2.12)

**Dielectric Relaxations**

The rates with which polarization can occur are limited, so that as the frequency of the applied electric field is increased some polarization will no longer be able to attain their low frequency or d.c values. The dipole moments are just not able to orient fast enough to keep in alignment with the applied field and the total polarisability falls from $\alpha_T$ to ($\alpha_T - \alpha_0$). This fall, with its related reduction of permittivity, and the occurrence of energy absorption, is referred to as dielectric relaxation or dispersions.

The dielectric response of a first order system in the frequency domain can be obtained by Laplace transformation as

$$\varepsilon^* = \varepsilon_\infty + \frac{\varepsilon_1 - \varepsilon_\infty}{1 + j\omega\tau}$$

(2.13)

where $\varepsilon_\infty$ = Dielectric constant measured at high frequency.
\( \varepsilon_s \) = Limiting low frequency dielectric constant.
\( \omega = 2\pi f \) Angular frequency of the applied electric field.
\( \tau = \text{Relaxation time and} \)
\( j = \sqrt{-1}. \)

But the real and imaginary components of complex permittivity is given by

\[
\varepsilon^* = \varepsilon' - j\varepsilon''
\]

(2.14)

The real part \( \varepsilon' \) represents the dielectric constant and is given by

\[
\varepsilon' = \varepsilon_s + \left( \frac{\varepsilon_s - \varepsilon_\infty}{1 + \omega^2 \tau^2} \right)
\]

(2.15)

The imaginary component \( \varepsilon'' \) represents dielectric loss and can be written as

\[
\varepsilon'' = \left( \frac{\varepsilon_s - \varepsilon_\infty}{1 + \omega^2 \tau^2} \right) \omega \tau
\]

(2.16)

both \( \varepsilon' \) and \( \varepsilon'' \) are frequency dependent. Their variations with the frequency of applied electric field are shown in (Fig. 2.2). The equations (2.15) and (2.16) are commonly known as Debye Dispersion formulae which are referred specially to the situation where equilibrium is attained exponentially with time when a constant external electric field is imposed on a dielectric.

The relationship between the relaxation \( \tau \), the frequency of maximum loss and the magnitude of maximum loss can be obtained by differentiating equation (2.16) with respect to \( \omega \) and equating to zero.

\[
\frac{d\varepsilon''}{d\omega} = 0 \text{ at } \omega_{\max} = \frac{1}{\tau}
\]

and

(2.17)

\[
\varepsilon_{\max}'' = \frac{\varepsilon_s - \varepsilon_\infty}{2}
\]

(2.18)

From the equations (2.17) and (2.18) it is obvious that loss peak \( \varepsilon_{\max}'' \) gives the relaxation time \( \tau \) and occurs at half the length of dielectric dispersion.
Fig. 2.2. Variation of the dielectric parameter $\varepsilon'$ and $\varepsilon''$ with frequency or a Debye-type relaxation process

**Cole – Cole Plots**

An ideal Debye type single relaxation time can be obtained by rearranging equations (2.15) and (2.16) to eliminate $\omega \tau$ giving

$$
\left( \varepsilon' - \frac{(\varepsilon_s - \varepsilon_{\infty})}{2} \right)^2 + (\varepsilon'')^2 = \left( \frac{(\varepsilon_s - \varepsilon_{\infty})}{2} \right)^2
$$

(2.19)

This equation is of the form $x^2 + y^2 = a^2$ i.e, the equation for a circle of radius (a). Then a plot $\varepsilon'$ against $\varepsilon''$ produces a semi-circle of radius $\frac{(\varepsilon_s - \varepsilon_{\infty})}{2}$ with the centre at $\left[ \frac{(\varepsilon_s - \varepsilon_{\infty})}{2}, 0 \right]$ such a plot of $\varepsilon'$ & $\varepsilon''$ refers to as a Cole–Cole plot. (Fig. 2.3).

Fig. 2.3 The Cole-Cole plot showing semi circle
If a symmetrical distribution of relaxation times occur about a mean relaxation time, then a **depressed semicircle** is found in the Cole- Cole plot of $\varepsilon'$ against $\varepsilon'$ (Fig 2.4) to account for such a symmetrical distribution of relaxation times, Cole and Cole modified equation (2.13) to the form.

$$
\varepsilon^* = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{(1 + j\omega \tau)^\beta}
$$

(2.20)

where $\beta$ is a parameter $0 < \beta \leq 1$, for a single relaxation $\beta=1$ and $\beta$ tends to zero as the distribution tends to an infinite one.

Davids and Cole later proposed another modification to equation (2.13).

$$
\varepsilon' = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{(1 + j\omega \tau)^\gamma}
$$

(2.21)

Which gives a skewed arc shaped $\varepsilon'$ and $\varepsilon''$ curve and is characteristic of a non uniform distribution of relaxation mechanisms where the distributions on the higher frequency side of the principal relaxation time decreases more rapidly than those on the low frequency side (Fig.2.5)
In general, dielectric behaviour of most substances is more complicated than the single relaxation system. This may be due to the fact that if there are multiple relaxation processes each with different relaxation time, or if relaxation process has kinetics that are of not first order. Hence, the dielectric response to a step change in field strength can be represented as the superposition of first order processes.

\[
\varepsilon'(\omega) = \varepsilon_x + \frac{\Delta\varepsilon_1}{1 + j\omega\tau_1} + \frac{\Delta\varepsilon_2}{1 + j\omega\tau_2} + \frac{\Delta\varepsilon_3}{1 + j\omega\tau_3} \quad (2.22)
\]

If the relaxation times are well separated (i.e.), if \(\tau_1 \ll \tau_2 \ll \tau_3\), a plot (Fig. 2.6) of permittivity and conductivity versus frequency will exhibit clearly resolved dispersions, separated by plateaus at intermiediate frequencies. A plot between \(\varepsilon'\) and \(\varepsilon''\) in the complex permittivity plane will then appears as a series of semi-circles each with a centre on the real permittivity axis and they intersect the real axis at the points.

\[
\varepsilon_x, \varepsilon_x + \Delta\varepsilon_1, \varepsilon_x + \Delta\varepsilon_1 + \Delta\varepsilon_2, \varepsilon_x + \Delta\varepsilon_1 + \Delta\varepsilon_2 + \Delta\varepsilon_3 \quad (2.23)
\]

Fig. 2.6. Cole-Cole plot showing complex plane loci. of multiple relaxations.

1.2 Theory of Heterogeneous Dielectrics
By their very nature biological materials are not homogenous. A simple example of a heterogeneous biological system is that of blood corpuscles suspended in blood serum.
In the past, an empirical relation was proposed for finding the percentage volume ‘P’ of serum in dog’s blood as
\[
P = \frac{\sigma_b}{\sigma_s} \left( 180 - \sigma_b - \sqrt{\sigma_b} \right)
\] (2.4a)

where \( \sigma_b \) and \( \sigma_s \) are the conductivities of the red blood cells and of the blood serum respectively. A little extensive measurement, for dog’s blood, and empirical equation was made.

By this time (Maxwell, 1881) and (Lord Rayleigh, 1892) also had considered the theoretical aspects of heterogeneous dielectrics their work being extended later by (Wagner, 1915; 1924).

**Dielectric Relaxation Mechanisms**

All of the theories which have been developed in the literature on the nature of polarization phenomena in living matter can be reduced to the following six basic physical mechanisms.

1. Dipolar orientation
2. Macrostructural polarsatons
3. Ordering of water structure
4. Counter ion polarization
5. Interfacial polarization
6. Delocalisation of electrons.

Among these, three relaxation processes, that are chiefly responsible for the dielectric properties of tissues are

1). Interfacial polarization
2). Dipolar orientation and
3). Counter ion polarization.

1). **Interfacial Polarization**

In the electrically heterogeneous material or dispersion occurs in the bulk properties from the charging of the interfaces within the material. This phenomenon does not arise from dielectric relaxation in the bulk
phases of the material, but it is a consequence of the boundary conditions on the field at the interfaces between phases. Here it is considered the dielectric properties of a simple model system which can be extended to complex biological system.

Consider two slabs of thickness $d_1$ and $d_2$ in contact with their interface oriented perpendicular to the external field (Fig 2.7). If the conductivity and permittivity $(\sigma_1, \sigma_2 \varepsilon_1, \varepsilon_2)$ independent of frequency, the boundary condition on the electric field component normal to the interface is

$$\varepsilon_1 E_1 = \varepsilon_2 E_2 \quad (2.24\text{b})$$

If the interface is free of charge.

The current densities $(j_1, j_2)$ are therefore, related by

$$\frac{j_1}{j_2} = \frac{\sigma_1 \varepsilon_1}{\sigma_2 \varepsilon_2}$$

Thus if $\sigma_1 \varepsilon_2 = \sigma_2 \varepsilon_1$ the interface will be charged at a rate proportional to the difference between $j_1$ and $j_2$.

**Fig. 2.7 The two-layer heterogeneous system**

The permittivity $\varepsilon$ and conductivity $\sigma$ of the composite can be found by regarding it as two capacitors in series and using the well known rule for series addition of two capacitances. The two layer model effectively represents two capacitors connected in series, with the two complex capacitances given by

$$C_1 = A \varepsilon_0 \left[ \frac{\varepsilon_1 - j \sigma_1/\omega \varepsilon_0}{d_1} \right]$$

$$C_2 = \frac{A \varepsilon_0 \varepsilon_2}{d_2} \quad (2.26)$$

$$C_2 = \frac{A \varepsilon_0 \varepsilon_2}{d_2} \quad (2.27)$$
where \( A \) is the cross-sectional area of the two slabs. The total complex capacitance is given by

\[
\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}
\]

Which from equations (2.26) and (2.27) gives

\[
C = \frac{A\varepsilon_0 \varepsilon_2 \left(\frac{\varepsilon_1}{j\sigma_1} - \frac{1}{\omega\varepsilon_0}\right)}{d_2 \left(\frac{\varepsilon_1}{j\sigma_1} - \frac{1}{\omega\varepsilon_0}\right) + d_1 \varepsilon_2}
\]

(2.28)

This shows that the effective permittivity of the two layer system is a function of frequency.

At low frequencies, as \( \varepsilon \to 0 \), the limiting low frequency permittivity \((\varepsilon_s)\) is given by

\[
\varepsilon_s = \frac{\varepsilon_2 d}{d_2}
\]

(2.29)

where \( d = d_1 + d_2 \)

Also, at high frequencies as \( \omega \to \infty \), we have

\[
\varepsilon_\infty = \frac{\varepsilon_1 \varepsilon_2 d}{d_2 \varepsilon_1 + d_1 \varepsilon_2}
\]

(2.30)

from this we see that \( \varepsilon_s > \varepsilon_\infty \), showing that the two layer system inhibits a dielectric dispersion.

The dielectric behavior, which is associated with the non-uniform distribution of free electronic charges across the interface between the dissimilar dielectric materials, can best be understood by considering the situation under dc equilibrium conditions where there must be electrical current continuity across the boundary between material 1 and 2. Each material has its own free charge carriers concentration and associated charge carries mobility. To achieve current continuity through materials 1 and 2 there will have to be a charge carrier concentration discontinuity across the interface. This interfacial charge build up or polarization, as function of frequency
that give rise to the dielectric dispersion exhibited by inhomogeneous system.

For the full analysis, the dielectric dispersions of two layer system have been described in terms of the Debye equations as (Van Beek, 1960)

\[
\varepsilon' = \varepsilon_\infty + \frac{\varepsilon_s - \varepsilon_\infty}{1 + \omega^2 \tau^2} \\
\varepsilon'' = \left(\frac{\varepsilon_s - \varepsilon_\infty}{1 + \omega^2 \tau^2}\right) \omega \tau
\]

Where

\[
\varepsilon_s = \frac{d \left( \varepsilon_{1d1}^2 \sigma_1^2 + \varepsilon_{2d2}^2 \sigma_1^2 \right)}{\left( \sigma_{1d2}^2 + \sigma_{2d1}^2 \right)^2}
\]

\[
\varepsilon_\infty = \frac{d \varepsilon_{1d2} \varepsilon_{2d2}}{\left( \varepsilon_{1d2} + \varepsilon_{2d2} \right)}
\]

\[
\tau = \frac{\varepsilon_0 \left( \varepsilon_{1d1} + \varepsilon_{2d2} \right)}{\sigma_{1d2}^2 + \sigma_{2d1}^2}
\]

The conductivity of a two layer system as a whole is given by

\[
\sigma = \frac{d \sigma_{1d2} \sigma_{2d2}}{\left( \sigma_{1d2}^2 + \sigma_{2d1}^2 \right)}
\]

So that the dielectric loss factor contains an extra term because of this conductivity (i.e.)

\[
\sigma (\omega) = \sigma + \omega \varepsilon_0 \varepsilon_1^{11}, \text{ and we have } \varepsilon' = \frac{\sigma}{\varepsilon_0 \omega} + \frac{\left( \varepsilon_0 - \varepsilon_\infty \right) \omega \tau}{1 + \omega^2 \tau^2}
\]

A system composed of a number of layers of the same material ($\varepsilon_1 \sigma_1$) alternating with layers of another material ($\varepsilon_2 \sigma_2$) will have the same dielectric characteristics as the two-layer system, where as a many layer system of a multitude of materials will have the limiting permittivities in the Debye equations according is given by
In general, large interfacial dispersion, sometimes termed as **Maxwell-Wagner** polarisation occurs for layered system when one of the layer is very thin and of high resistivity compared to the other component.

Another heterogeneous model, which can be considered relevant to many real biological situations, is that of a porous dielectric, which in its simplest form would be as shown in (Fig. 2.8). For such a heterogeneous system, with \( \sigma_2 > \sigma_1 \), then (Van Beek, 1960) has shown that the dielectric dispersion is characterized by the dielectric parameters

\[
\varepsilon' = (1 - \nu)\varepsilon_1 + \nu\varepsilon_2 \quad \text{and} \quad \varepsilon'' = \frac{\sigma^2}{\varepsilon_0\omega}
\]

(2.39) \hspace{10cm} (2.40)

Fig. 2.8. A porous heterogeneous system

1) **Dipolar relaxation mechanism**

It is the polarization mechanism, in which the partial orientation of dipoles is considered. A dipole of dipole movement \( \mu \) exerts a torque \( \mu F \sin \theta \), when \( \theta \) is the angle between dipole moment and field, when an external field of intensity \( F \) is imposed on it. The torque will act to orient the dipole, but will be opposed by random thermal agitation. At
equilibrium the average orientation is evaluated on the basis of thermodynamic partition function as

$$<\cos \theta> = \coth \left(\frac{\mu F}{kT} - \frac{kT}{\mu F}\right)$$

(2.41)

where \(k\) is the Boltzmann’s constant and \(T\) is the absolute temperature.

For reasonable field strengths, the maximum orientational energy of a dipole of a dipole (\(\mu F\)) is much less than its mean kinetic energy (which is of the order of \(kT\)), and the above equation reduces to

$$<\cos \theta> = \frac{\mu F}{3kT}$$

(2.42)

The net polarization of a sample, \(P\) is given by

$$P = N\mu <\cos \theta> = N\frac{\mu^2 F}{3kT}$$

(2.43)

where \(N\) is the number of molecules per unit volume, or approximately

The local field \(F\) at each molecule can be related to the external field strength \(E\) by the Lorentz expression

$$F = \frac{2 + \varepsilon_0}{3} E$$

(2.44)

which leads to relative static permittivity

$$\frac{(\varepsilon_0 - 1)}{(\varepsilon_0 + 2)} = N_0 \frac{\mu^2}{kTV\varepsilon_0}$$

(2.45)

where \(N_0\) is Avagadro’s number and \(V\) is the molar volume equaling \(N/N_0\). Equation (2.45) holds for a lossless medium in which the concentration of dipoles is very small.

After the application of the electric field, thermal equilibrium is achieved with first order kinetics, leading to dielectric relaxation behavior of the Debye type. The relaxation time for this process is the rotational diffusion time \(\tau\), which can be estimated from stokes’s law.

$$\tau = \frac{4\pi\eta a^3}{kT}$$

(2.46)

where \(a\) is the radius of dipole and \(\eta\) is the viscosity of the medium.
2) Counter-Ion Polarization Effects

One of the major types of dielectric phenomena arises from ionic diffusion in the electrical double layer adjacent to charged surfaces. The time constants for such effects are of the form $L^2/D$ where $L$ is the length over which diffusion occurs, and $D$ is diffusion coefficient, in contrast, the time constant for the Maxwell – Wagner effect is of the form of $RC$, Where $R$ is resistance and $C$ represents capacitance.

In biological system, counter ion polarization effects are typically prominent at audio frequencies, while Maxwell – Wagner effects are pronounced at radio frequencies. Counter ion polarization effects can be large at low frequencies in the Kilo – Hertz range.

Recently a model is proposed (Grosse and Greffe, 1979) which assumes that the counterion layer is thin and contains only ions that are opposite in sign to that of fixed charges on the particles, and the counterions can exchange freely only with ions of the same sign in the bulk electrolyte. The surface conductivity $G$ was assumed to be high.

In this model, when an electric field is applied, the ions in the system will redistribute under the influence of both the field and diffusion. Grosse obtained a set of coupled differential equations gives broad, asymmetrical, low frequency dispersion in permittivity of the particle. The time constant $\tau$ of this dispersion is similar to that derived by Schwaez.

$$T = \frac{a^2}{D}$$  \hspace{1cm} (2.47)

where ‘$a$’ is the size of the particle and $D$ is diffusion coefficient of ion in bulk electrolyte.

The parameter that determines the low frequency limit of permittivity is $\psi a$, where $\psi$ inverse of the Debye screening length of the electrolyte is.

$$\psi = \frac{\sigma}{\varepsilon \varepsilon_0 D}$$  \hspace{1cm} (2.48)

where $\sigma$ and $\varepsilon$ are the conductivity and permittivity of the bulk electrolyte.
For particles with high surface conductivity, the magnitude of dispersion is approximately

\[ \frac{\varepsilon_r - \varepsilon_\infty}{\varepsilon_\infty} = \frac{9(\psi a)^2}{16} \]  

(2.49)

This dispersion in \( \varepsilon \) is quite broad and closely resembles the Cole-Cole relaxation.

### 2.3 Dielectric Loss and Dissipation Factor

When a dielectric is subjected to the a.c voltage, the electrical energy is absorbed by the material and is dissipated in the form of heat. This dissipation of energy is known as dielectric loss.

When A.C voltage is applied to a perfect insulator, there is no consumption of energy and the charging current leads the applied voltage by 90°. This phase angle \( \Phi \) is less than 90° in commercial dielectrics. Complementary angle \( \delta = 90° - \Phi \) is called the dielectric loss angle. The loss angle is a measure of power dissipated in each cycle (Fig. 2.9).

An oscillating field \( E = E_0 e^{i\omega t} \) is applied across the plates. All the displacement field \( D = D_0 e^{j(\omega t - \delta)} \) with phase difference with respect to \( E \) – field. These equation leads to a complex quantity as

\[ \frac{D_0}{E_0} e^{-j\delta} = \varepsilon_r - j\varepsilon_i = \varepsilon^* \]  

(2.50)

where the relative permittivity or dielectric constant \( \varepsilon_r = \frac{D_0}{E_0} \cos \delta \),

dielectric loss \( \varepsilon_i = \frac{D_0}{E_0} \sin \delta \) and \( \tan \delta = \frac{\varepsilon_i}{\varepsilon_r} \) is called **loss tangent or dissipation** of the dielectric. It indicates the amount of energy that is dissipated in the dielectric when an electric field is applied across it.

When an a.c voltage of \( V \) is applied then the dielectric power loss is expressed as \( \omega \varepsilon_0 \varepsilon_r V^2 \tan \delta \)......watts/cycle.
Fig. 2.9 (a) Phase relation between charging current and applied voltage in perfect insulator, (ideal dielectric material) (b) Phase relation between charging current and applied voltage in commercial dielectric material (lossy dielectric material) (c) Current density components in a lossy dielectric material with respect to the field direction.

### 2.4 Attenuation factor and Penetration depth

In dielectric materials, the electric field strength decreases with distance $z$ from the surface and it is written as

$$E = E_0 e^{-\alpha z}$$

(2.51)

Where $\alpha$ the attenuation factor is depends on the dielectric properties of the material (von- Hippel, 1954) and is given by

$$\alpha = \frac{2\pi}{\lambda_0} \left[ \frac{1}{2} \varepsilon' \left( \sqrt{1 + \left( \frac{\varepsilon''}{\varepsilon'} \right)^2} - 1 \right) \right]^{\frac{1}{2}}$$

(2.52)

where $\lambda_0 = \frac{C}{f}$ is the free space wavelength.

Penetration depth of microwave and RF Power is defined as the depth where the power is reduced to $1/e$ ($e=2.718$) of the power entering the surface (Fig.2.10). The penetration depth ($d_p$) in meters of RF and microwave energy in a lossy material can be calculated, von-Hippel (1954) by the relation

$$d_p = \frac{C}{2\pi f \sqrt{2\varepsilon' \left[ \sqrt{1 + \left( \frac{\varepsilon''}{\varepsilon'} \right)^2} - 1 \right]}}$$

(2.53)
where $C$ is the speed of light in free space ($3 \times 10^8 \text{m/s}$)

![Diagram of EM radiation penetration depth into material](image)

Fig. 2.10 Typical penetration depth inside a large-sized material (larger than the wavelength).

After obtaining the dielectric properties, the penetration depths of electromagnetic energy into selected materials can be calculated at the required frequency. Given fixed dielectric properties, the penetration depth of a material is inversely proportional to frequency. It is, therefore, expected that in general deeper penetration corresponds to lower frequencies, and that higher frequencies result in greater surface heating. It should be noted that the dielectric properties of lossy materials vary with frequency but penetration depth does not vary exactly as $1/f$. Nevertheless, EM waves with short wavelength do not penetrate deeply into most moist media where the dielectric constants and loss factors are relatively high.

### 2.5 Frequency Dependence of Tissue Conductivity and Relative Permittivity

Biological tissues injected with alternating currents exhibit frequency dependent impedance characteristics. These impedance characteristics are complex (i.e., they possess both a real and an imaginary part) and vary as a function of frequency. (Fig. 2.11) presents impedance magnitude and phase values for liver over the frequency range $10$ to $10^9 \text{Hz}$, (Gabriel, et al. 1996).

In general, biological tissues have decreasing impedance magnitude with increasing frequency, three phase local minima, and a negative impedance phase (implying capacitive properties).
Dispersion Regions
The three phase minima correspond to the α, β, and γ dispersion regions. At very low frequencies (f < 1 kHz) α-dispersion dominates, attributed to electrochemical polarizations and an ionic atmosphere surrounding the cells, conditions that produce very large relative permittivity values (Rigaud et.al., 1996; Pething, 1987). At frequencies between 1 kHz and 100 MHz β-dispersion dominates, attributed to Maxwell-Wagner structural relaxation and the cell membrane capacitive response. Cell membrane degradation resulting from cellular death greatly reduces the presence of this dispersion region, an important effect exploited in biomedical diagnostic instrumentation based on measurements of tissue impedance. At frequencies greater than 100 MHz. γ-dispersion dominates, caused by the relaxation of water molecules (Debye effect) (Rigaud, et.al., 1996; Pething, 1987; Raicu, 1999). All three of these dispersions generally depend on cellular quantity, shape, and orientation, as well as the tissue chemical composition (i.e., the composition and ionic concentrations of interstitial space and cytoplasm) Rigaud, et.al., 1996; Pething, 1987)

Fig. 2.11 Dispersion regions for liver. Liver impedance and phase for frequencies from 10HZ.to10GHz computed for geometric factor of 1.0[1/m]. (Gabrial et.al., 1996)
(Fig. 2.12) presents an example of tissue cross-section at the cellular level, illustrating the behavior of electric current flow across tissue transitions in the β-dispersion frequency range. The figure illustrates a cell enclosed by its lipid bi-layer membrane and filled with intracellular fluid, surrounded by extra-cellular fluid. An applied electric field oriented from top to bottom of the figure induces an electrical current, which flows through the local cellular environment.

![Fig. 2.12 Tissue current paths](image)

*Fig. 2.12 Tissue current paths*  
Illustration of a basic cell with intra and extra cellular membrane. Current Paths at low and high frequencies are illustrated (Webster, 1990)

![Fig. 2.13 Tissues equivalent circuit](image)

*Fig. 2.13 Tissues equivalent circuit*  
An equivalent circuit, models the intra and extra cellular fluid with resistors and cell membrane with parallel resistor/capacitor circuit. (Webster, 1990)

At the low end of the β-dispersion frequency range \( f \approx 1 \text{ kHz} \), the applied electric field (E-field) induces a charge on the cell membrane, which charges and discharges fully during each cycle of the alternating externally-applied E-field. When fully charged, no current flows into the interior of the cell, resulting in a current flow.
mainly in the extra-cellular fluid (illustrated as the dashed current flow in the figure). Impedance measurements are almost entirely resistive and reflect only the extra-cellular impedance (Rigaud, et.al. 1996).

Increasing the applied frequency results in incomplete charging of the cell membrane, leading to inter-cellular current flow. The measured impedance is influenced by the cell membrane (which is highly resistive as a result of its nonpolar lipid core), the intra-cellular fluid (highly conductive), and the capacitive nature of the cell membrane (resulting from the phenomenon of charge accumulation at the membrane interface). Accordingly, measured impedance is both resistive and reactive (Rigaud, et.al., 1996).

At the upper end of the β-dispersion frequency range (f ≅ 10 MHz), charge has insufficient time to accumulate on the cell membrane. Current flows freely and penetrates the cells easily, resulting in a nearly purely resistive impedance component. This is illustrated as the solid line current flowing directly through the cell in (Fig. 2.12).

Fig.(2.13) presents an equivalent circuit that models the impedance behavior of the cell. The circuit is composed of 4 different elements: a resistor, $R_{\text{extra-cellular}}$, associated with the extra-cellular fluid, a resistor, $R_{\text{intra-cellular}}$, associated with the intra-cellular fluid, and a resistor and capacitor, $R_{\text{cell membrane}}$ and $C_{\text{cell membrane}}$ in parallel associated with the cell membrane. The resistive and capacitive characteristics of cells on a microscopic level are also observed on a macroscopic tissue level, and are described by tissue conductivity and relative permittivity, respectively.

(Fig. 2.14) and (Fig. 2.15) present conductivity and relative permittivity values, respectively, for various tissues (liver, kidney, muscle, blood, breast fat) over the frequency range from 1 kHz to 1 MHz, the β-dispersion region (Gabriel, et.al. 1996). Tissue
conductivities range in value from 0.01 to 1 S/m while relative permittivities range from approximately $10^2$ to $10^6$. At very low frequencies the tissue relative permittivity is very large. The conductivity for each tissue.

**Fig. 2.14** Frequency dependency of tissues conductivities conductivity values for various biological tissues over the frequency range 1 KHz to 10 MHz (Gabriel, et.al. 1996)

**Fig. 2.15** Frequency dependency of tissues conductivities relative permittivity values for various biological tissues over the frequency range 1 KHz to 10 MHz (Gabriel, et.al. 1996)
Table 2.1, it is evident that the relative permittivity values for biological media in the α- and β-dispersion regions are much greater than for non-biological medias.

Table 2.1 Permittivity Values of biological and non biological substances Relative permittivity values of various biological (Gabriel, et.al. 1996) and non-biological substances (Weast et.al., 1984).

<table>
<thead>
<tr>
<th>Substance</th>
<th>Relative Permittivity</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water (H₂O)</td>
<td>78.54</td>
<td>Temperature = 25°C, ( f \approx 10^8 ) Hz, Liquid</td>
</tr>
<tr>
<td>Benzene (C₆H₆)</td>
<td>2.274</td>
<td>Temperature = 25°C, ( f \approx 10^6 ) Hz, Liquid</td>
</tr>
<tr>
<td>Methanol (CH₃OH)</td>
<td>32.63</td>
<td>Temperature = 25°C, ( f \approx 10^6 ) Hz, Liquid</td>
</tr>
<tr>
<td>Glycol (C₃H₆O₃)</td>
<td>37.0</td>
<td>Temperature = 25°C, ( f \approx 10^6 ) Hz, Liquid</td>
</tr>
<tr>
<td>Glycerol (C₃H₈O₃)</td>
<td>42.5</td>
<td>Temperature = 25°C, ( f \approx 10^6 ) Hz, Liquid</td>
</tr>
<tr>
<td>Calcium Carbonate</td>
<td>6.14</td>
<td>Temperature between 17 and 22 °C, ( f = 10^5 ) Hz, Solid</td>
</tr>
<tr>
<td>Ferrous Oxide</td>
<td>14.2</td>
<td>Temperature 15 °C, ( f = 10^8 ) Hz, Solid</td>
</tr>
<tr>
<td>Potassium Sulphate</td>
<td>5.9</td>
<td>Temperature between 17 and 22 °C, ( f = 6 \times 10^3 ) Hz, Solid</td>
</tr>
<tr>
<td>Sodium Chloride</td>
<td>6.12</td>
<td>Temperature between 17 and 22 °C, ( f = 6 \times 10^3 ) Hz, Solid</td>
</tr>
<tr>
<td>Lucite</td>
<td>2.58</td>
<td>Temperature 23 °C, ( f = 10^5 ) Hz, Plastic</td>
</tr>
<tr>
<td>Breast Fat</td>
<td>543</td>
<td>( f = 10 ) kHz</td>
</tr>
<tr>
<td>Liver</td>
<td>28930</td>
<td>( f = 10 ) kHz</td>
</tr>
<tr>
<td>Kidney</td>
<td>38750</td>
<td>( f = 10 ) kHz</td>
</tr>
<tr>
<td>Pancreas</td>
<td>9190</td>
<td>( f = 10 ) kHz</td>
</tr>
</tbody>
</table>

2.6 Importance of Dielectric Properties of Tissue

High-frequency EM energy is widely used in medical treatments, including physiotherapeutic, diagnostics, rapid rewarming of cryopreserved tissues, pharmacology, reflex therapeutic, blood sterilization, and hyperthermic treatment of cancer and stimulation of bone healing. EEG recording electroshock therapy and the performance of cardiac pacemakers and implanted transponders.

In the agricultural sector, EM energy is widely used the electrical measurement of moisture content of the material, Dielectric heating in agricultural industry, drying, in telecommunication and in insulators industries.