6.0 Introduction

Here a collaborative inventory system consisting of single vendor and single buyer is developed to maximize the total profit of the supply chain. The model is developed for prevailing scenario of declining demand due to recession.

In section 6.1 model is developed for collaborative inventory system in which vendor offers different trade credit to buyers for settlement of accounts due against purchases. The units in inventory are subject to constant deterioration and replenishment rate is proportional to demand rate.

In section 6.2 article deals with formulation of optimal ordering and pricing policy with vendor-buyer integration when units in inventory deteriorate at a constant rate. Here the demand function is dependent on time and retail price. In collaborating
scenario, it is observed that the vendor is more beneficial compared to buyer, a quantity
discount pricing strategy is advantageous to attract the buyer to accept the joint
decision. A negotiation factor is incorporated to share profit between the vendor and the
buyer.

Sensitivity analysis is carried out in both the above models and the analysis
establishes that the integrated strategy with quantity is beneficial to increase the profit of
the supply chain.

6.1: A Collaborative Vendor-Buyer deteriorating inventory system in
declining market when trade credit is offered

Additional Assumptions and Notations which are used to derive model 6.1 is as follows:

6.1.1: Assumptions and Notations:

6.1.1.1 Assumptions

- A single – vendor single – buyer supply chain is under consideration.
- The vendor and buyer share each other’s information.
- The vendor’s replenishment rate is proportional to the demand rate.
• Demand rate is decreasing function of time and sales price.

• Shortages are not allowed.

• The product in inventory deteriorates at a constant rate $\theta(0 \leq \theta \leq 1)$. The deteriorated units can neither be repaired nor replaced during the cycle time.

• Holding costs apply to good units only.

• Vendor offers permissible delay period to attract the buyer to have joint decision.

6.1.1.2 Notations

$i = 1, 2, 3$

The buyer’s related parameters are:

$I_{bi}(t)$ inventory level for scenario i

$A_b$ Buyer’s ordering cost per order

$C_b$ Unit purchase cost

$I_b$ Inventory carrying charge fraction per $ per annum

$L_{mbi}$ Maximum lot-size per delivery for scenario i

$TP_{bi}$ Annual total profit for scenario i
Extra profit sharing for scenario 3 as compared to scenario 1 ($EP_b = TP_{b3} - TP_{b1}$)

The vendor’s related parameters are:

- $p(t,P)$: Annual production rate
- $I_{v1i}(t)$: Inventory level during the production period for scenario i
- $I_{v2i}(t)$: Inventory level during the non-production period for scenario i
- $A_v$: Set-up cost, $ per cycle
- $C_v$: Unit purchase cost
- $I_v$: Inventory carrying charge fraction per $ per annum
- $TP_{vi}$: Annual total profit for scenario i
- $EP_v$: Extra profit sharing for scenario 3 as compared to scenario 1 ($EP_v = TP_{v3} - TP_{v1}$)

The other related parameters for both the vendor and the buyer are

$R_i(t,P) = a(1 - bt) - dP$, $i = 1,2,3$; where $a$ is demand scale parameter, $b$ is rate of change of demand, $d$ is price-sensitive demand parameter and $P$ is end customer’s retail price $a, d > 0$ and $0 \leq b < 1$
\[ P_i(t, P) = \lambda R_i(t, P), \quad \lambda > 1 ; \] annual production rate

\[ \theta \] Deterioration rate, \( 0 \leq \theta < 1. \)

\[ TP_i \] Annual total profit of the system \( (= TP_{bi} + TP_{vi}) \) for scenario i

\[ \delta \] Negotiation factor of extra profit sharing between the vendor and the buyer

\[ r \] Continuous interest rate per annum

\[ e^{-rM_i} \] Present value of a unit cost after a time interval \( M_i \) for scenario i

The decision variables are

\[ T_{bi} \] Buyer's cycle time for scenario i

\[ n_i \] Number of shipments from the vendor to the buyer per cycle for scenario i

\[ P \] End customer retail price

\[ T_i \] Vendor's cycle time for scenario i

\[ T_{1i} \] Vendor's production period per cycle for scenario i

\[ T_{2i} \] Vendor's non-production period per cycle for scenario i

\[ M_i \] Permissible delay period offered by the vendor to the buyer for scenario i

We shall discuss three scenarios:
I. No vendor - buyer integration and permissible delay in payments

II. The vendor - buyer integration without permissible delay in payments

III. The vendor - buyer integration with permissible delay in payments

### 6.1.2 Mathematical formulation

The levels of inventory for the vendor and the buyer are as shown in above figure.

6.1.2.1. The buyer's inventory level is depleting due to a price-sensitive time dependent demand and a constant deterioration of units in the warehouse. Hence, the buyer's inventory level can be described by the following differential equation:
\[ \frac{d I_{bi}(t)}{dt} + \theta I_{bi}(t) = -R_i(t, P), \quad 0 \leq t \leq T_{bi}, \quad i = 1,2,3 \]  

(6.1.2.1)

The vendor’s total inventory system consists of production and non-production phases.

The vendor’s inventory level can be described by the following differential equations:

\[ \frac{d I_{v1i}(t)}{dt} + \theta I_{v1i}(t) = P_i(t, P) - R_i(t, P), \quad 0 \leq t \leq T_{1i} \]  

(6.1.2.2)

\[ \frac{d I_{v2i}(t)}{dt} + \theta I_{v2i}(t) = -R_i(t, P), \quad 0 \leq t \leq T_{2i} \]  

(6.1.2.3)

The initial and boundary conditions are

\[ I_{bi}(T_{bi}) = 0 \]  

(6.1.2.4)

\[ I_{v1i}(0) = \alpha \]  

(6.1.2.5)

\[ I_{v2i}(T_{2i}) = \beta \]  

(6.1.2.6)

\[ I_{v2i}(T_{1i}) = I_{v2i}(0) \]  

(6.1.2.7)

\[ T_i = T_{1i} + T_{2i} \]  

(6.1.2.8)

\[ T_{bi} = \frac{T_i}{n_i} \]  

(6.1.2.9)

The solutions of the differential equations (6.1.2.1) – (6.1.2.3) are

\[ I_{bi}(t) = \frac{ae^{\theta(T_{bi} - t)}}{\theta} \left\{ 1 - b \left( T_{bi} - \frac{1}{\theta} \right) \right\} - \frac{d}{\theta} \left\{ e^{\theta(T_{bi} - t)} - 1 \right\} - \frac{a}{\theta} \left\{ 1 - b \left( t - \frac{1}{\theta} \right) \right\}, \quad 0 \leq t \leq T_{bi} \]  

(6.1.2.10)
\[ I_{v1l}(t) = \frac{a(\lambda-1)}{\theta} \left(1 - b \left(t - \frac{1}{\theta}\right)\right) - (\lambda - 1) \frac{dP}{\theta} \left(1 - e^{-\theta t}\right) - \frac{a(\lambda-1)}{\theta} e^{-\theta t} \left\{1 + \frac{b}{\theta}\right\} + \alpha, \]

\[ 0 \leq t \leq T_{1l} \quad (6.1.2.11) \]

\[ I_{v2l}(t) = \frac{ae^{\theta(T_{2l}-t)}}{\theta} \left\{1 - b \left(T_{2l} - \frac{1}{\theta}\right)\right\} - \frac{dP}{\theta} \left\{e^{\theta(T_{2l}-t)} - 1\right\} - \frac{a}{\theta} \left\{1 - b \left(t - \frac{1}{\theta}\right)\right\} + \alpha, \]

\[ (6.1.2.12) \]

Using \( I_{bi}(0) = I_{mbi} \), the maximum inventory of the buyer is

\[ I_{mbi}(t) = \frac{ae^{\theta(T_{bi})}}{\theta} \left\{1 - b \left(T_{bi} - \frac{1}{\theta}\right)\right\} - \frac{dP}{\theta} \left\{e^{\theta(T_{bi})} - 1\right\} - \frac{a}{\theta} \left\{1 - \left(\frac{b}{\theta}\right)\right\}, \]

\[ (6.1.2.13) \]

During \([0, \beta]\), the vendor’s inventory level at any instant of time can be described by the differential equation

\[ \frac{dI_v(t)}{dt} + \theta I_v(t) = P(t, P), \quad 0 \leq t \leq \beta \quad (6.1.2.14) \]

Using, \( I_v(\beta) = 0 \), the solution of differential equation (6.1.2.14) is

\[ I_v(t) = \frac{\lambda ab}{\theta^2} \left(1 - e^{\theta(\beta-t)}\right) - \frac{\lambda e^{\theta(\beta-t)}}{\theta} [a(1 - b\beta) - dP] + \frac{\lambda e^{\theta t}}{\theta} [a(1 - bt) - dP] \]

\[ 0 \leq t \leq \beta \quad (6.1.2.15) \]

Clearly, the production quantity in \([0, \beta]\) is equal to the buyer’s lot-size. So, using log series expansion

\[ \beta = \frac{T_l(2an^3 - abn^2T_l + a\theta n^2T_l - ab\theta nT_l^2 - 2dPn^3 - dP\theta n^2T_l)}{2\lambda n^4(a(1-bT_l) - dP)} \quad (6.1.2.16) \]
and, hence

\[
\alpha = \frac{T_i(2an^2-abn^2T_i+a\theta n^2T_i-ab\theta nT_i^2-2dPn^3-dP\theta n^2T_i)}{2\lambda n^4} \quad (6.1.2.17)
\]

The buyer's inventory per time unit is

\[
\overline{I}_{bl} = \frac{1}{T_{bi}} \int_{0}^{T_{bi}} I_{bi}(t)dt 
\]

(6.1.2.18)

The vendor's inventory in the integrated two-echelon system is the difference between the vendor's total inventory and the buyer's average inventory. Therefore, the vendor's inventory per time unit is

\[
\overline{I}_{vi} = \frac{1}{T_i} \left[ \int_{0}^{T_1} I_{v1i}(t)dt + \int_{0}^{T_2} I_{v2i}(t)dt \right] - I_{bi} 
\]

(6.1.2.19)

So, the annual total inventory holding cost for the buyer and the vendor are as follows;

Buyer's inventory holding cost;

\[
BHC_i = C_b I_b \overline{I}_{bl} 
\]

(6.1.2.20)

and vendor's inventory holding cost;

\[
VHC_i = C_v I_v \overline{I}_{vi} 
\]

(6.1.2.21)

Respectively.

The annual deterioration costs for the buyer and the vendor are

Buyer's deterioration cost;
\[ BDC_i = C_B \overline{I_{bi}} \theta \]  \hspace{1cm} (6.1.2.22)

and vendor's deterioration cost;

\[ VDC_i = C_V \overline{I_{vi}} \theta \]  \hspace{1cm} (6.1.2.23)

The annual set-up costs for the buyer and the vendor are

Buyer's set-up cost;

\[ BSC_i = \frac{A_b}{T_{bi}} \]  \hspace{1cm} (6.1.2.24)

and vendor's set-up cost;

\[ VSC_i = \frac{A_v}{T_i} \]  \hspace{1cm} (6.1.2.25)

The present value of buyer's unit purchase price is \( e^{-rM_i} \) when the vendor offers credit period to settle the account against the purchases. Therefore, the annual purchase costs for the buyer and the vendor are

Buyer's Purchase cost;

\[ BPC_i = \frac{C_B I_{mb} e^{-rM_i}}{T_{bi}} \]  \hspace{1cm} (6.1.2.26)

and vendor's Purchase cost;

\[ VPC_i = \frac{C_V P(T_{yi}, P) T_{yi}}{T_i} \]  \hspace{1cm} (6.1.2.27)
For scenario I and II, $M_1$ and $M_2$ are zero and for scenario III, it is $M_3 > 0$ because here vendor offers credit period to the buyer.

The annual total profit for buyer and vendor are

$$TP_{bi} = \frac{P_{mbi}}{T_{bi}} - (BPC_i + BHC_i + BDC_i + BSC_i) \quad (6.1.1.28)$$

and

$$TP_{vi} = \frac{C_{bmbi}e^{-rM_i}}{T_{bi}} - (VPC_i + VHC_i + VDC_i + VSC_i) \quad (6.1.2.29)$$

Using Taylor’s series expansion and (6.1.2.7), we get

$$T_{1i} \approx T_{2i} \left( \frac{a\left(1 + \frac{(\theta - b)T_{2i}}{2} - \frac{b\theta T_{2i}^2}{2}\right)}{a - dP} \right) \quad (6.1.2.30)$$

From (6.1.2.8), we have

$$T_i \approx T_{2i} \left( \frac{\theta + \frac{(\theta - b)T_{2i}}{2} - \frac{b\theta T_{2i}^2}{2} - dP\left(\theta + \frac{\theta T_{2i}}{2}\right)}{a - dP} \right) \quad (6.1.2.31)$$

### 6.1.3 Theoretical results

**Scenario I:** *Inventory system without considering vendor- buyer integration and permissible delay in payments:*

Here, the buyer makes the decision independently.
So maximize \( TP_{b1} = TP_{b1}(T_{b1}, P) \) \( (6.1.3.1) \)

Here \( TP_{b1} \) is a function of two continuous variables \( T_{b1} \) and \( P \), solve

\[
\frac{\partial TP_{b1}}{\partial T_{b1}} = 0 \quad (6.1.3.2)
\]

\[
\frac{\partial TP_{b1}}{\partial P} = 0 \quad (6.1.3.3)
\]

for \( T_{b1} \) and \( P \), knowing the value of \( T_{b1} \), the vendor’s decision is

maximize \( TP_{v1} = TP_{v1}(n_1) \) \( (6.1.3.4) \)

where \( TP_{v1} \) is a function of discrete variable \( n_1 \). The optimal solution of \( n_1 \), say \( n_1^* \) must satisfy the following condition:

\[
TP_{v1}(n_1^* - 1) \leq TP_{v1}(n_1^*) \leq TP_{v1}(n_1^* + 1) \quad (6.1.3.5)
\]

Here, both the vendor and the buyer make their decisions independently.

Knowing the buyer’s cycle time and unit sale price and vendor’s number of shipments, total profit \( TP_1 = TP_{b1} + TP_{v1} \) can be computed.

**Scenario II:** Inventory system considering vendor-buyer integration without permissible delay in payments:
In this scenario, the joint agreeable decision is to taken by the vendor and the buyer. So the objective is to maximize

\[ TP_2(P, n_2, T_{22}) = TP_{b2}(P, T_{b2}(T_{22})) + TP_{v2}(P, n_2, T_{12}(T_{22}), T_2(T_{22}), T_{22}) \]  \hspace{1cm} (6.1.3.6)

where \(T_{b2}, T_{12}\) and \(T_{22}\) are functions of \(T_{22}\) (from (6.1.2.9), (6.1.2.30) and (6.1.2.31)).

Optimize total joint profit with respect to \(P, T_{22}\) and \(n_2\) where \(P\) and \(T_{22}\) are continuous variables and \(n_2\) is a discrete variable.

Now to maximize \(TP_2\) follows steps given below:

**Step1:** for discrete value of \(n_2\), to determine optimal values of \(P\) and \(T_{22}\) by equating partial derivatives of \(TP_2\) with respect to \(P\) and \(T_{22}\) to be zero. Designate the optimal values of \(P\) and \(T_{22}\) for each \(n_2\) by \(T_{22}(n_2)\) respectively.

**Step2:** Compute the optimal value of \(n_2\), (say) \(n_2^*\) which satisfies

\[ TP_2(T_{22}(n_2^* - 1), n_2^* - 1, P(n_2^* - 1)) \leq TP_2(T_{22}(n_2^*), n_2^*, P(n_2^*)) \]  \hspace{1cm} (6.1.3.7)

And \[ TP_2(T_{22}(n_2^*), n_2^*, P(n_2^*)) \leq TP_2(T_{22}(n_2^* + 1), n_2^* + 1, P(n_2^* + 1)) \]  \hspace{1cm} (6.1.3.8)

**Scenario III:** *Inventory system considering vendor-buyer integration permissible delay in payments:*
Under vendor-buyer integration, the vendor is more beneficial compared to the buyer. So obviously, the buyer will be reluctant to adopt joint decision. To overcome reluctant of the buyer, the vendor may offer some credit terms to attract the buyer for joint decision. A negotiation factor is incorporated to share the extra profit according to their contributions.

The buyer’s extra profit $EP_b$ is the difference between $TP_{b3}$ and $TP_{b1}$. i.e.

$$EP_b = TP_{b3} - TP_{b1}$$  \hspace{1cm} (6.1.3.9)

The Vendor’s extra profit, $EP_v$ is the difference between $TP_{v3}$ and $TP_{v1}$. i.e.

$$EP_v = TP_{v3} - TP_{v1}$$ \hspace{1cm} (6.1.3.10)

Clearly, the total joint profit in scenario 3 is more than the non-integrated total profit in scenario 1. So,

$$EP_v = \delta(EP_b), \hspace{0.5cm} \delta \geq 0$$  \hspace{1cm} (6.1.3.11)

where $\delta$ is the negotiation factor.
When $\delta = 0$, all extra profit is for buyer; when $\delta = 1$, the extra profit distributed equally among the vendor and the buyer. A large $\delta$ means that profit is for vendor. So the problem is

$$TP_3(P, n_3, T_{23}) = TP_{b3}(P, T_{b3}(T_{23})) + TP_{v3}(P, n_3, T_{13}(T_{23}), T_3(T_{23}), T_{23})$$

(6.1.3.12)

Subject to

$$EP_v = \delta(EP_b), \quad \delta \geq 0.$$ 

where, $T_{b3}, T_{13}, and T_3$ are functions of $T_{23}$ (from (6.1.2.9), (6.1.2.30) and (6.1.2.31)) and $n_3$ is discrete variable.

From following procedure we can determine the decision variables that maximizes $TP_3$

**Step1** From (6.1.3.11), obtain $M_3$ in terms of other model parameters including $n_3$, $P$ and $T_{23}$ substitute it into (6.1.3.12).

**Step2** For discrete values of $n_3$, set the partial derivative of $TP_3$ with respect to $P$ and $T_{23}$ equal to zero. Compute the values of $P$ and $T_{23}$ for each $n_3$ and denote it by $P(n_3)$ and $T_{23}(n_3)$ respectively.

**Step3** Compute the optimal value of $n_3$, (say) $n_3^*$ which satisfies
\[ TP_3(T_{23}(n^*_3 - 1), n^*_3 - 1, P(n^*_3 - 1)) \leq TP_3(T_{23}(n^*_3), n^*_3, P(n^*_3)) \] (6.1.3.13)

and \[ TP_3(T_{23}(n^*_3), n^*_3, P(n^*_3)) \leq TP_3(T_{23}(n^*_3 + 1), n^*_3 + 1, P(n^*_3 + 1)) \] (6.1.3.14)

### 6.1.4 Numerical example and observation:

Consider the following parametric values in proper units:

\[ [a, b, \delta, C_b, I_b, A_b, A_v, \lambda, C_v, I_v, \delta, \theta, r] \]

\[ = [3000, 2\%, 35, 35, 20\%, 100, 6000, 2, 20, 20\%, 1, 10\%, 12\%] \]

The optimal solution for three scenarios is exhibited in table 6.1.4.1

**Table 6.1.4.1: The optimal solution for various scenarios**

<table>
<thead>
<tr>
<th>Scenario i</th>
<th>i = 1</th>
<th>i = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P )</td>
<td>60.64</td>
<td>55.22</td>
</tr>
<tr>
<td>( d_i )</td>
<td>674</td>
<td>915</td>
</tr>
<tr>
<td>( \tau_i )</td>
<td>0</td>
<td>0.33</td>
</tr>
<tr>
<td>( n_i )</td>
<td>30</td>
<td>24</td>
</tr>
<tr>
<td>( T_{bi} )</td>
<td>0.107</td>
<td>0.105</td>
</tr>
<tr>
<td>( TP_{bi} )</td>
<td>20636</td>
<td>19104</td>
</tr>
<tr>
<td>( TP_{vi} )</td>
<td>9157</td>
<td>11625</td>
</tr>
<tr>
<td>( TP_i )</td>
<td>29793</td>
<td>30729</td>
</tr>
<tr>
<td>( P \cdot T_{pi} )</td>
<td>0</td>
<td>3.14</td>
</tr>
</tbody>
</table>
For scenario 1, the optimal sale price is $60.64 and buyer's cycle time 0.107 resulting annual demand of 674- units. The buyer's total profit is $20637. The optimal number of shipments from the vendor to the buyer per cycle is 30. Resulting the vendor's annual total profit to be $9157. hence the total profit of the vendor and the buyer is $29793.

In scenario 3, the vendor and the buyer make joint decision without allowable credit period. The optimal sale price is $55.22 and the annual demand is 915 units with buyer's cycle time of 0.105 years. The buyer's, the vendor's and total joint profits are $19104, $11625 and $30729 respectively. Here, the total joint profit increases by $936 with respect to scenario 1. Since the vendor gains by $2468 and the buyer's looses by $1532, the buyer's will be reluctant to adopt joint decision. To attract the buyer, the vendor may offer some credit period. Agreeing to equal sharing($\delta = 1$), the optimal credit period for the buyer is 0.33 years. The extra profit of $936 is to be distributed equally between the vendor and the buyer. When this negotiation factor is incorporated,
The percentage of extra profit is 3.14%. The concavity of the total profit is in fig.6.1.4.1 and fig.6.1.4.2 for scenario 1 and scenario 3 respectively.

**Fig6.1.4.1** concavity of total profit in scenario 1

**Fig6.1.4.2** concavity of total profit in scenario 3
### Table 6.1.4.2: Sensitive analysis for the demand scale parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>$P$</th>
<th>$d_3$</th>
<th>$n_3$</th>
<th>$\tau_3$</th>
<th>$TB_3$</th>
<th>$TP_1$</th>
<th>$TP_3$</th>
<th>$PETP_3$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>-10%</td>
<td>51.24</td>
<td>750</td>
<td>25</td>
<td>0.354</td>
<td>0.114</td>
<td>21339</td>
<td>22136</td>
<td>3.70</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>59.28</td>
<td>1077</td>
<td>23</td>
<td>0.319</td>
<td>0.099</td>
<td>39630</td>
<td>40659</td>
<td>2.59</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>63.40</td>
<td>1230</td>
<td>23</td>
<td>0.298</td>
<td>0.092</td>
<td>50814</td>
<td>51913</td>
<td>2.16</td>
</tr>
</tbody>
</table>

### Table 6.1.4.3: Sensitive analysis for the demand rate parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>$P$</th>
<th>$d_3$</th>
<th>$n_3$</th>
<th>$\tau_3$</th>
<th>$TB_3$</th>
<th>$TP_1$</th>
<th>$TP_3$</th>
<th>$PETP_3$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>-20%</td>
<td>55.05</td>
<td>964</td>
<td>21</td>
<td>0.369</td>
<td>0.109</td>
<td>29336</td>
<td>30411</td>
<td>3.66</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>55.12</td>
<td>944</td>
<td>22</td>
<td>0.355</td>
<td>0.108</td>
<td>29569</td>
<td>30565</td>
<td>3.37</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.33</td>
<td>822</td>
<td>26</td>
<td>0.323</td>
<td>0.103</td>
<td>30035</td>
<td>30904</td>
<td>2.89</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.47</td>
<td>848</td>
<td>28</td>
<td>0.300</td>
<td>0.102</td>
<td>30316</td>
<td>31094</td>
<td>2.57</td>
</tr>
</tbody>
</table>
Table 6.1.4.4: Sensitive analysis for the price-sensitive demand parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>P</th>
<th>$d_3$</th>
<th>$n_3$</th>
<th>$\tau_3$</th>
<th>$Tb_3$</th>
<th>$T_1$</th>
<th>$TP_3$</th>
<th>$PETP_3$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-20%</td>
<td>65.72</td>
<td>1023</td>
<td>23</td>
<td>0.268</td>
<td>0.100</td>
<td>44838</td>
<td>45666</td>
<td>1.85</td>
</tr>
<tr>
<td>d</td>
<td>-10%</td>
<td>59.86</td>
<td>973</td>
<td>23</td>
<td>0.304</td>
<td>0.104</td>
<td>36412</td>
<td>37304</td>
<td>2.45</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>51.43</td>
<td>858</td>
<td>24</td>
<td>1.790</td>
<td>0.109</td>
<td>24504</td>
<td>25457</td>
<td>3.89</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>48.33</td>
<td>799</td>
<td>25</td>
<td>2.495</td>
<td>0.112</td>
<td>20246</td>
<td>21167</td>
<td>4.55</td>
</tr>
</tbody>
</table>

Table 6.1.4.5: Sensitive analysis for the vendor set-up cost

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>P</th>
<th>$d_3$</th>
<th>$n_3$</th>
<th>$\tau_3$</th>
<th>$Tb_3$</th>
<th>$T_1$</th>
<th>$TP_3$</th>
<th>$PETP_3$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-20%</td>
<td>54.98</td>
<td>937</td>
<td>22</td>
<td>-0.10</td>
<td>0.102</td>
<td>30858</td>
<td>31235</td>
<td>1.22</td>
</tr>
<tr>
<td>$A_v$</td>
<td>-10%</td>
<td>55.07</td>
<td>933</td>
<td>22</td>
<td>0.155</td>
<td>0.107</td>
<td>29986</td>
<td>30974</td>
<td>3.29</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.33</td>
<td>900</td>
<td>25</td>
<td>0.324</td>
<td>0.106</td>
<td>29611</td>
<td>30497</td>
<td>2.99</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.45</td>
<td>888</td>
<td>26</td>
<td>0.311</td>
<td>0.107</td>
<td>29440</td>
<td>30278</td>
<td>2.85</td>
</tr>
</tbody>
</table>
Table 6.1.4.6: Sensitive analysis for the buyer's ordering cost

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>P</th>
<th>$d_3$</th>
<th>$n_3$</th>
<th>$\tau_3$</th>
<th>$Tb_3$</th>
<th>$TP_1$</th>
<th>$TP_3$</th>
<th>$PETP_3$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_b$</td>
<td>-20%</td>
<td>55.20</td>
<td>913</td>
<td>27</td>
<td>0.337</td>
<td>0.093</td>
<td>29996</td>
<td>30928</td>
<td>3.11</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>55.20</td>
<td>914</td>
<td>25</td>
<td>0.339</td>
<td>0.100</td>
<td>29892</td>
<td>30826</td>
<td>3.12</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.23</td>
<td>917</td>
<td>23</td>
<td>0.361</td>
<td>0.110</td>
<td>29700</td>
<td>30637</td>
<td>3.15</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.24</td>
<td>914</td>
<td>22</td>
<td>0.340</td>
<td>0.115</td>
<td>29611</td>
<td>30549</td>
<td>3.17</td>
</tr>
</tbody>
</table>

Table 6.1.4.7: Sensitive analysis for the buyer's holding cost

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>P</th>
<th>$d_3$</th>
<th>$n_3$</th>
<th>$\tau_3$</th>
<th>$Tb_3$</th>
<th>$TP_1$</th>
<th>$TP_3$</th>
<th>$PETP_3$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_b$</td>
<td>-20%</td>
<td>55.19</td>
<td>920</td>
<td>23</td>
<td>0.342</td>
<td>0.109</td>
<td>29820</td>
<td>30769</td>
<td>3.18</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>55.21</td>
<td>912</td>
<td>24</td>
<td>0.338</td>
<td>0.105</td>
<td>29813</td>
<td>30748</td>
<td>3.14</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.22</td>
<td>918</td>
<td>24</td>
<td>0.339</td>
<td>0.104</td>
<td>29775</td>
<td>30709</td>
<td>3.14</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.22</td>
<td>913</td>
<td>24</td>
<td>0.340</td>
<td>0.104</td>
<td>29756</td>
<td>30690</td>
<td>3.14</td>
</tr>
</tbody>
</table>
Table 6.1.4.8: Sensitive analysis for the vendor’s holding cost

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>P</th>
<th>$d_3$</th>
<th>$n_3$</th>
<th>$\tau_3$</th>
<th>$Tb_3$</th>
<th>$TP_1$</th>
<th>$TP_3$</th>
<th>$PETP_3$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_v$</td>
<td>-20%</td>
<td>55.21</td>
<td>904</td>
<td>25</td>
<td>0.339</td>
<td>0.106</td>
<td>30071</td>
<td>30956</td>
<td>3.08</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>55.20</td>
<td>916</td>
<td>24</td>
<td>0.340</td>
<td>0.107</td>
<td>29656</td>
<td>30861</td>
<td>4.06</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.21</td>
<td>923</td>
<td>23</td>
<td>0.340</td>
<td>0.106</td>
<td>29389</td>
<td>30601</td>
<td>4.12</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.23</td>
<td>925</td>
<td>23</td>
<td>0.336</td>
<td>0.104</td>
<td>29263</td>
<td>30475</td>
<td>4.14</td>
</tr>
</tbody>
</table>

Table 6.1.4.9: Sensitive analysis for the deterioration rate

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>P</th>
<th>$d_3$</th>
<th>$n_3$</th>
<th>$\tau_3$</th>
<th>$Tb_3$</th>
<th>$TP_1$</th>
<th>$TP_3$</th>
<th>$PETP_3$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta$</td>
<td>-20%</td>
<td>55.17</td>
<td>902</td>
<td>25</td>
<td>0.367</td>
<td>0.109</td>
<td>30226</td>
<td>31109</td>
<td>3.05</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>55.06</td>
<td>919</td>
<td>24</td>
<td>0.349</td>
<td>0.108</td>
<td>30004</td>
<td>30934</td>
<td>3.09</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.23</td>
<td>924</td>
<td>23</td>
<td>0.340</td>
<td>0.105</td>
<td>29591</td>
<td>30782</td>
<td>4.02</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.28</td>
<td>921</td>
<td>24</td>
<td>0.336</td>
<td>0.099</td>
<td>29396</td>
<td>30335</td>
<td>3.19</td>
</tr>
</tbody>
</table>
From Table 6.1.4.2, it is observed that when the demand scale parameter increases, the percentage of total extra profit decreases. It is seen in Table 6.1.4.4 that increase in price sensitive demand parameter increase the percentage of total extra profit seen in figure 6.1.4.3.
It is advised to the players of the supply chain to take the joint decision. From Table 6.1.4.5, it is seen that change in rate of demand parameter; \(b\) decreases delay period and the percentage of total extra profit significantly. In Tables 6.1.4.5 and 6.1.4.6, the vendor’s and the buyer’s ordering cost varied from -20% to 20%. Increase in vendor’s ordering cost reduces the percentage of total extra profit. Figure 6.1.4.4 shows that the value of the total extra profit is highly sensitive to the vendor’s set-up cost as compared to the buyer’s ordering cost. Hence it is advantageous to adopt integrated strategy when the vendor’s set-up cost and/or the buyer’s ordering cost increases.

![Graph showing % changes in \(A_v\) and \(A_b\)](image)

**Fig 6.1.4.4:** % changes in \(A_v\) and \(A_b\)
From Table 6.1.4.7, 6.1.4.8 and Table 6.1.4.9, it is seen that the increase in the vendor's inventory caring charge fraction; $A_b$, the buyer's inventory caring charge fraction; $A_v$, and the deterioration rate, increase percentage of total extra profit is almost linear. See figure 6.1.4.5.

The effect of negotiation factor is studied in table 6.1.4.10. It is seen that the total extra profit remains constant because, the negotiation factor relates two players extra profit sharing which does not affect the total profit of the supply chain.

In next section, we develop discounted selling price inventory policy for vendor to the retailer
6.2 Optimal Policies for integrated inventory system for deteriorating items

using quantity discount in price-sensitive declining market.

Additional assumption and Notations are as follows:

6.2.1 Assumptions

The proposed models are derived using following assumptions:

- The supply chain comprises of single vendor and the single buyer.
- The players have complete knowledge of each other’s information.
- Inventory system deals with stocking single item.
- The replenishment rate is instantaneous and lead-time is zero or negligible.
- The demand rate is linear decreasing function of time and retail price.
- All-unit quantity discount is considered.
- Shortages are not allowed.
- Carrying cost is applicable to good units only.
- Deterioration of the units is considered only after its arrival into the inventory.

There is no replacement or repair of deteriorated units.
Three scenarios are discussed. The first scenario does not consider the vendor buyer integration and quantity discount. The second scenario considers the vendor-buyer integration without quantity discount. The scenario 3 deals with the vendor-buyer integration and quantity discount simultaneously.

6.2.2. Notations

The variable parameters are as follows:

\[ i = 1, 2, 3 \]

\[ T_{bi} \]  
Buyer’s cycle time for scenario \( i \)

\[ n_i \]  
Number of shipments from the vendor to the buyer per cycle for scenario \( i \)

\[ T_{vi} \]  
Vendor’s cycle time for scenario \( i \)

\[ p \]  
Buyer’s retail price

\[ R(t, p) \]  
Annual price-sensitive declining demand (say) \( R(t, p) = a(1 - bt) - dP \)

Where \( a > 0 \) is scale parameter for demand, \( 0 < b < 1 \) denotes the rate of change of demand with respect to time and \( d > 0 \) denotes price-
sensitive demand parameter

\( c_{b3} \)  The buyer’s purchase unit cost for scenario 3

The buyer’s related parameters are as follows

\( I_{bi}(t) \)  Inventory level for scenario \( i \) at any instant of time \( t \)

\( A_b \)  Ordering cost for buyer, $ per order

\( c_{bi} \)  The buyer’s purchase unit cost for scenario \( i, i = 1,2 \)

\( I_b \)  Inventory carrying charge fraction per annum per dollar

\( Tc_{bi} \)  Annual total cost for scenario \( i \)

\( TP_{bi} \)  Annual total profit for scenario \( i \)

\( S_b \)  Extra profit sharing for scenario 3 as compared to scenario 1

\[ (S_b = TP_{b3} - TP_{b1}) \]

The vendor’s related parameters are as follows

\( I_{vi}(t) \)  Inventory level for scenario \( i \) at any instant of time \( t \)

\( A_v \)  Set-up cost, $ per cycle

\( c_{vb} \)  Fixed cost to process each buyer’s order
The other related parameter for both the vendor and the buyer are

$$\theta$$  Constant deterioration rate of on-hand-stock, $0 < \theta < 1$.

$$TC_i$$  Annual total cost ($TC_{vi} + TC_{bi}$) for scenario $i$

$$TP_i$$  Annual total profit ($TP_{vi} + TP_{bi}$) for scenario $i$

$$\alpha$$  Extra profit sharing negotiation factor between the vendor and the buyer

6.2.3. Mathematical formulation

The depletion of the inventory is due to the demand and the constant on-hand-stock deterioration. The buyer’s inventory level at any instant of time is governed by the differential equation

$$\frac{dl_{bi}(t)}{dt} = -R(t, P) - \theta l_{bi}(t), \quad i = 1, 2, 3 \quad (6.2.3.1)$$
With boundary condition $I_{bl}(T_{bl}) = 0$, the solution of differential equation is given by

$$ I_{bl}(t) = \frac{a-dP}{\theta} \left( e^{\theta(T_{bl}-t)} - 1 \right) - \frac{abT_{bl}e^{\theta(T_{bl}-t)}}{\theta} + \frac{abt}{\theta^2} + \frac{ab}{\theta^2} \left( e^{\theta(T_{bl}-t)} - 1 \right) $$

(6.2.3.2)

**Scenario 1: inventory system without considering integration and quantity discount**

The total cost per unit time is given by

$$ TC_{b1} = \frac{1}{T_{b1}} \left[ A_b + C_{b1} I_b \int_0^{T_{b1}} I_{b1}(t) \, dt + C_b I_{b1}(0) \right] $$

(6.2.3.3)

And the total buyer's profit is

$$ TP_{b1} = \frac{PI_{b1}(0)}{T_{b1}} - TC_{b1} $$

(6.2.3.4)

Taking the first derivatives of $TC_{b1}$ with respect to $T_{b1}$ and $P$, and setting it to zero, one has

$$ \frac{\partial TP_{b1}}{\partial T_{b1}} = 0 $$

(6.2.3.5)

$$ \frac{\partial TP_{b1}}{\partial P} = 0 $$

(6.2.3.6)

Since, (6.2.3.5) and (6.2.3.6) are highly non-linear, the two optimal variable $T_{b1}$ and $P$ denoted by $T_{b1}^*$ and $P^*$ are derived numerically.

The vendor's replenishment time is $T_{v1} = n_1 T_{b1}^*$ Where, $n_1$ is a positive integer.

(6.2.3.7)
Similarly, the vendor’s inventory level is

\[ I_{v1}(t) = \frac{a - dp}{\theta} \left( e^{\theta (n_1 T_{b1} - t)} - 1 \right) - \frac{ab n_1 T_{b1}^* e^{\theta (n_1 T_{b1}^* - t)}}{\theta} + \frac{abt}{\theta^2} + \frac{ab}{\theta^2} \left( e^{\theta (n_1 T_{b1}^* - t)} - 1 \right) \]  

(6.2.3.8)

Clearly, vendor’s inventory level in (6.2.3.8) is decreasing exponentially. From (6.3.2.8) and (6.3.2.2), the vendor annual cost is

\[ TC_{v1} = \frac{1}{n_1 T_{b1}} \left[ A_v + n_1 C_{vb} + C_v I_v \left( \int_0^{n_1 T_{b1}^*} I_{v1}(t) dt - n_1 \int_0^{T_{b1}} I_{b1}(t) dt \right) + C_v I_{v1}(0) \right] \]  

(6.2.3.9)

In the parenthesis of (6.2.3.9), the first two terms are related ordering costs, the third term is the inventory holding cost and the last term is the purchase cost.

The vendor’s annual total profit is

\[ TP_{v1} = \frac{C_{b1} I_{b1}^*(0)}{T_{b1}} - TC_{v1} \]  

(6.2.3.10)

The vendor’s total profit in (6.2.3.10) is a function of discrete variable, \( n_1 \). Thus, optimal policy is

\[ \max_{n_1} TP_{v1}(n_1) \]  

(6.2.3.11)

Since, \( n_1 \) is a discrete integer, the optimal value of \( n_1 \), denoted by \( n_1^* \), must satisfy the following condition:

\[ TP_{v1}(n_1^* - 1) \leq TP_{v1}(n_1^*) \geq TP_{v1}(n_1^* + 1) \]  

(6.2.3.12)
When the vendor-buyer integration and quantity discount are not considered, the total profit of the vendor and the buyer is

\[ TP_1 = TP_{b1}(T_{b1}^*, P^*) + TP_{v1}(n_1^*) \]  

(6.2.3.13)

**Scenario 2: Inventory system with integration but not quantity discount**

The aim of vendor-buyer integration is to maximize the integrated total profit.

The total cost of the buyer and the vendor are

\[ TC_{b2} = \frac{1}{T_{b2}} \left[ A_b + C_{b1} I_b \int_0^{T_{b2}} I_{b2}(t) \, dt + C_b I_{b2}(0) \right] \]  

(6.2.3.14)

And

\[ TC_{v2} = \frac{1}{n_2 T_{b2}} \left[ A_v + n_2 C_{vb} + C_v I_v \left( \int_0^{n_1 T_{b2}} I_{v2}(t) \, dt - n_2 \int_0^{T_{b2}} I_{b2}(t) \, dt \right) + C_v I_{v2}(0) \right] \]  

(6.2.3.15)

Respectively.

The integrated total cost is the sum of (6.2.3.14) and (6.2.3.15). The buyer and vendor profits are

\[ TP_{b2} = \frac{P_{b2}(0)}{T_{b2}} - TC_{b2} \]  

(6.2.3.16)

\[ TP_{v2} = \frac{C_{b2} I_{b2}(0)}{T_{b2}} - TC_{v2} \]  

(6.2.3.17)

respectively.
We want to maximize the integrated total profit as

$$\text{maxi}. \, TP_2(T_{b2}, P, n_2) = TP_{b2}(T_{b2}, P) + TP_{v2}(n_2)$$  \hspace{1cm} (6.2.3.18)$$

Thus, the three variables $T_{b2}, P$ and $n_2$ are to be optimized jointly rather than independently as in scenario 1.

**Scenario 3:** inventory system with integration and quantity discount.

It is assumed that the discount price, $C_{b3}$, is less than the original unit price, $C_{b1}$. The buyer purchases

$$l_{b3}(0) = \frac{a-dp}{\theta} \left( e^{\theta T_{b3}} - 1 \right) - \frac{abT_{b3}e^{\theta T_{b3}}}{\theta} + \frac{ab}{\theta^2} \left( e^{\theta T_{b3}} - 1 \right)$$  \hspace{1cm} (6.2.3.19)$$

per shipment. The buyer’s and vendor’s total annual costs are

$$TC_{b3} = \frac{1}{T_{b3}} \left[ A_b + C_{b3} l_b + \int_0^{T_{b3}} l_{b3}(t) \, dt + C_{b3} l_{b3}(0) \right]$$  \hspace{1cm} (6.2.3.20)$$

and

$$TC_{v3} = \frac{1}{n_3 T_{b3}} \left[ A_v + n_3 C_{vb} + C_{v} I_v \left( \int_0^{n_3 T_{b3}} l_{v3}(t) \, dt - n_3 \int_0^{T_{b3}} l_{b3}(t) \, dt \right) + C_{v} I_{v3}(0) \right] + \frac{(C_{b1}-C_{b3}) I_{b3}(0)}{T_{b3}}$$  \hspace{1cm} (6.2.3.21)$$

respectively.
The last term of (6.2.3.21) is the increased cost of vendor when quantity discount is offered.

The total profit of the vendor and the buyer are

\[ TP_{b3} = \frac{P_{b3}(0)}{T_{b3}} - TC_{b3} \quad (6.2.3.22) \]

and

\[ TP_{v3} = \frac{c_{b3}I_{b3}(0)}{n_{3}T_{b3}} - TC_{v3} \quad (6.2.3.23) \]

respectively.

The buyer’s extra profit, \( S_b \) is defined as

\[ S_b = TP_{b3} - TP_{b1} \quad (6.2.3.24) \]

and the vendor’s extra profit, \( S_v \) is defined as

\[ S_v = TP_{v3} - TP_{v1} \quad (6.2.3.25) \]

The integrated total profit in scenario 3 (\( TP_3 \)) is more than the scenario 1 (\( TP_1 \)) or scenario 2 (\( TP_2 \)). Their relationship for positive \( S_b \) and \( S_v \) values is defined as

\[ S_v = \alpha S_b, \quad \alpha \geq 0 \quad (6.2.3.26) \]

When \( \alpha = 0 \), all extra profit sharing is for the buyer; when \( \alpha = 1 \), the extra profit sharing is equally distributed, A large \( \alpha \), means that profit is in the favour of the vendor.
Thus, we have

\[ \max_i TP_3(T_{b3}, P, n_3) = TP_{b3}(T_{b3}, P) + TP_{v3}(n_3) \quad (6.2.3.27) \]

Subject to \( S_v = \alpha S_b, \ \alpha \geq 0. \)

From (6.2.3.26) and (6.2.3.27), it can be seen that \( C_{b3} \) and \( TP_3 \) are functions of three variables, \( n_3, T_{b3} \) and \( P \).

**6.2.4 Solution Procedure**

For scenario 1, to determine the value of \( n_1 \) to maximize \( TP_1 \) (6.2.3.13), follow steps stated in (6.2.3.11) and (6.2.3.12).

For scenario 2, to determine the discrete value \( n_2 \) to maximize \( TP_2 \) (6.2.3.18), follow the steps stated below:

a) For a given \( n_2 \), set the partial derivatives of \( TP_2 \) with respect to \( P \) and \( T_{b2} \) to zero and determine values of \( P \) and \( T_{b2} \). Denote it by \( P(n_2) \) and \( T_{b2}(n_2) \).

b) Derive the optimal values of \( n_2 \), denoted by \( n_2^* \) such that

\[ TP_2(T_{b2}(n_2^* - 1), P(n_2^* - 1), (n_2^* - 1)) \leq TP_2(T_{b2}(n_2^*), P(n_2^*), (n_2^*)) \quad (6.2.4.1) \]

and

\[ TP_2(T_{b2}(n_2^*), P(n_2^*), (n_2^*)) \geq TP_2(T_{b2}(n_2^* + 1), P(n_2^* + 1), (n_2^* + 1)) \quad (6.2.4.2) \]
For scenario 3, we outline following procedure:

a) From (6.2.3.26), \( C_{b3} \) can be expressed as function of three variables: \( T_{b3}, n_3 \) and \( P \). Then substitute \( C_{b3} \) into (6.2.3.27).

b) For a given \( n_3 \), set the partial derivatives of \( TP_3 \) with respect to \( P \) and \( T_{b3} \) to zero to determine values of \( P \) and \( T_{b3} \). Denote it by \( P(n_3) \) and \( T_{b3}(n_3) \).

c) Derive the optimal value of \( n_3 \), denoted by \( n_3^* \) such that

\[
TP_3(T_{b3}(n_3^* - 1), P(n_3^* - 1), (n_3^* - 1)) \leq TP_3(T_{b3}(n_3^*), P(n_3^*), (n_3^*)) \quad (6.2.4.3)
\]

And \( TP_3(T_{b3}(n_3^*), P(n_3^*), (n_3^*)) \geq TP_3(T_{b3}(n_3^* + 1), P(n_3^* + 1), (n_3^* + 1)) \quad (6.2.4.4)\]

6.2.5 Numerical example and observation

The derived model is illustrated by the following numerical example where the parametric values are as follows:

\[[A, b, d, A_b, I_b, C_b, A_v, C_vb, I_v, C_v, \alpha, \theta] = [3000, 10\%, 35, 100, 0.2, 35, 6000, 100, 0.2, 20, 1, 5\%] \]

By applying the solution procedure, a result is worked out and is given in Table 6.2.5.1-6.2.5.12.

Table 6.2.5.1 shows the optimal solution for various scenarios when \( \alpha = 1 \) and \( \theta = 5\% \).
Table 6.2.5.1: The optimal solution for various scenarios

<table>
<thead>
<tr>
<th>Scenario i</th>
<th>i =1</th>
<th>i=2</th>
<th>i=3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>60.304</td>
<td>55.82</td>
<td>55.77</td>
</tr>
<tr>
<td>$d_i$</td>
<td>851</td>
<td>972</td>
<td>681</td>
</tr>
<tr>
<td>$c_{bi}$</td>
<td>35</td>
<td>35</td>
<td>33.16</td>
</tr>
<tr>
<td>$n_i$</td>
<td>8</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>$T_{bi}$</td>
<td>0.126</td>
<td>0.243</td>
<td>0.245</td>
</tr>
<tr>
<td>$TP_{bi}$</td>
<td>20919</td>
<td>19886</td>
<td>21778</td>
</tr>
<tr>
<td>$TP_{vi}$</td>
<td>22524</td>
<td>24979</td>
<td>23383</td>
</tr>
<tr>
<td>$TP_i$</td>
<td>43443</td>
<td>45117</td>
<td>45162</td>
</tr>
<tr>
<td>$PETP_i$</td>
<td>0</td>
<td>3.85%</td>
<td>3.96%</td>
</tr>
</tbody>
</table>

For scenario 1, the optimal retail price is $60.30, cycle time is 0.126 years and the corresponding annual demand is 851 units. The unit purchase price of the buyer is $35. The buyer's total profit is $20919. There are eight shipments from the vendor to the buyer per cycle, resulting vendor’s profit to be $22524. The total profit of the supply chain is $43443.
For scenario 2, the vendor and the buyer take joint decision. The buyer’s optimal retail price is $55.82 and cycle time is 0.243 years. The corresponding annual demand is 972 units. The buyer’s, the vendor’s and the integrated total profit are $19886, $24979 and $45117 respectively. The increase in the integrated total extra profit in scenario 2 with respect to scenario 1 is $1,674. The vendor gains $2,455 and buyer loses $1,033. To attract the buyer, the vendor offers some discount in the retail price. In agreement of equal sharing of the extra profit ($\alpha = 1$), the optimal unit discount price is $33.16. The optimal cycle time is 0.245 years.

The increase in the integrated total profit from scenario 1 to scenario 3 is $1,719. Here each player shares the same cost saving of $859.5. The annual demand is of 681 units. Only integration results the percentage of extra total profit ($PETP_2$) to be 3.85%. If both the collaboration and the quantity discount are considered, the percentage of extra total profit ($PETP_3$) is 3.96%.
### Table 6.2.5.2 Sensitive analysis for the demand scale parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>P</th>
<th>d3</th>
<th>n3</th>
<th>Cb3</th>
<th>Tb3</th>
<th>TP1</th>
<th>TP2 (PETP2%)</th>
<th>TP3 (PETP3%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>-20%</td>
<td>47.2</td>
<td>959</td>
<td>4</td>
<td>32.5</td>
<td>0.316</td>
<td>16717</td>
<td>5827(-65.1)</td>
<td>22451(34.3)</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>51.5</td>
<td>822</td>
<td>4</td>
<td>32.9</td>
<td>0.275</td>
<td>28762</td>
<td>30404(5.70)</td>
<td>30332(5.46)</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>60.0</td>
<td>1126</td>
<td>4</td>
<td>33.2</td>
<td>0.221</td>
<td>60736</td>
<td>62442(2.80)</td>
<td>62480(2.87)</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>64.2</td>
<td>1290</td>
<td>5</td>
<td>33.6</td>
<td>0.167</td>
<td>80627</td>
<td>82364(2.15)</td>
<td>82394(2.19)</td>
</tr>
</tbody>
</table>

### Table 6.2.5.3 Sensitive analysis for the demand rate parameter

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>P</th>
<th>d3</th>
<th>n3</th>
<th>Cb3</th>
<th>Tb3</th>
<th>TP1</th>
<th>TP2 (PETP2%)</th>
<th>TP3 (PETP3%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>-20%</td>
<td>55.98</td>
<td>976</td>
<td>4</td>
<td>33.18</td>
<td>0.268</td>
<td>44761</td>
<td>46329(3.50)</td>
<td>46375(3.60)</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>55.87</td>
<td>975</td>
<td>4</td>
<td>33.13</td>
<td>0.256</td>
<td>44089</td>
<td>45709(3.67)</td>
<td>45755(3.78)</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.67</td>
<td>973</td>
<td>4</td>
<td>33.12</td>
<td>0.235</td>
<td>42834</td>
<td>44503(3.89)</td>
<td>44595(4.11)</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.58</td>
<td>972</td>
<td>4</td>
<td>33.08</td>
<td>0.227</td>
<td>42257</td>
<td>44023(4.18)</td>
<td>44049(4.24)</td>
</tr>
</tbody>
</table>
### Fig 6.2.5.1: % changes in rate of change of demand $b$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>$P$</th>
<th>$d_3$</th>
<th>$n_3$</th>
<th>$Cb_3$</th>
<th>$Tb_3$</th>
<th>$TP_1$</th>
<th>$TP_2$ (PETP$_2%$)</th>
<th>$TP_3$ (PETP$_3%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>d</td>
<td>-20%</td>
<td>66.33</td>
<td>1097</td>
<td>6</td>
<td>33.26</td>
<td>0.149</td>
<td>18944</td>
<td>72985(285.2)</td>
<td>72955(285.1)</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>60.45</td>
<td>1038</td>
<td>5</td>
<td>33.52</td>
<td>0.191</td>
<td>55742</td>
<td>57318(2.82)</td>
<td>57314(2.82)</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>51.97</td>
<td>935</td>
<td>5</td>
<td>33.17</td>
<td>0.213</td>
<td>33668</td>
<td>35440(5.26)</td>
<td>35470(5.35)</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>44.78</td>
<td>869</td>
<td>4</td>
<td>33.02</td>
<td>0.273</td>
<td>25793</td>
<td>27652(7.20)</td>
<td>27437(6.37)</td>
</tr>
</tbody>
</table>

**Table 6.2.5.4** Sensitive analysis for the price-sensitive demand parameter
**Fig6.2.5.2**: % changes in price sensitive parameter

**Table6.2.5.5** Sensitive analysis for the buyer's set-up cost

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>( P )</th>
<th>( d_3 )</th>
<th>( n_3 )</th>
<th>( C b_3 )</th>
<th>( Tb_3 )</th>
<th>( T P_1 )</th>
<th>( T P_2 ) (PET( T_2 )% )</th>
<th>( T P_3 ) (PET( T_3 )% )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_b )</td>
<td>-20%</td>
<td>55.79</td>
<td>987</td>
<td>5</td>
<td>33.25</td>
<td>0.200</td>
<td>43495</td>
<td>45217(3.95)</td>
<td>45252(4.04)</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>55.79</td>
<td>986</td>
<td>5</td>
<td>33.31</td>
<td>0.201</td>
<td>43490</td>
<td>45167(3.85)</td>
<td>45201(3.93)</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.76</td>
<td>974</td>
<td>4</td>
<td>33.17</td>
<td>0.245</td>
<td>43366</td>
<td>45076(3.44)</td>
<td>45121(4.04)</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.76</td>
<td>974</td>
<td>4</td>
<td>32.27</td>
<td>0.247</td>
<td>43406</td>
<td>45035(3.75)</td>
<td>45086(3.87)</td>
</tr>
</tbody>
</table>
Table 6.2.3.6: Sensitive analysis for the buyer’s holding cost

| Parameter | % changes | P  | $d_3$ | $n_3$ | $C_b$ | $T_b$ | $T_P$ | $T_{P_2}(P_{ETP_2}\%)$ | $T_{P_3}(P_{ETP_3}\%)$
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_b$</td>
<td>-20%</td>
<td>55.73</td>
<td>975</td>
<td>4</td>
<td>33.23</td>
<td>0.247</td>
<td>43622</td>
<td>45289(3.82)</td>
<td>45418(3.9)</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>55.75</td>
<td>974</td>
<td>4</td>
<td>33.19</td>
<td>0.246</td>
<td>43529</td>
<td>45203(3.84)</td>
<td>45243(3.93)</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.80</td>
<td>986</td>
<td>5</td>
<td>33.32</td>
<td>0.201</td>
<td>43389</td>
<td>45046(3.82)</td>
<td>45086(3.90)</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.82</td>
<td>986</td>
<td>5</td>
<td>33.30</td>
<td>0.199</td>
<td>43330</td>
<td>44976(3.79)</td>
<td>45016(3.89)</td>
</tr>
</tbody>
</table>

Figure 6.2.5.3: % Changes of buyer’s holding cost
### Table 6.2.5.7 Sensitive analysis for the buyer’s purchase cost

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>P</th>
<th>d₃</th>
<th>n₃</th>
<th>Cₜ₃</th>
<th>Tₜ₃</th>
<th>P₁</th>
<th>T₂₂(PEₜ₂%)</th>
<th>T₃₃(PEₜ₃%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C_b</td>
<td>-20%</td>
<td>55.76</td>
<td>986</td>
<td>4</td>
<td>30.80</td>
<td>0.245</td>
<td>51844</td>
<td>52626(1.50)</td>
<td>45198(12.8)</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>55.76</td>
<td>974</td>
<td>4</td>
<td>32.08</td>
<td>0.245</td>
<td>47646</td>
<td>48816(2.45)</td>
<td>45188(5.16)</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.79</td>
<td>986</td>
<td>5</td>
<td>34.20</td>
<td>0.201</td>
<td>39263</td>
<td>41534(5.78)</td>
<td>45133(14.9)</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.79</td>
<td>987</td>
<td>5</td>
<td>34.87</td>
<td>0.201</td>
<td>35088</td>
<td>38062(8.47)</td>
<td>45119(28.5)</td>
</tr>
</tbody>
</table>

### Table 6.2.5.8 Sensitive analysis for the vendor’s ordering cost

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>P</th>
<th>d₃</th>
<th>n₃</th>
<th>Cₜ₃</th>
<th>Tₜ₃</th>
<th>P₁</th>
<th>T₂₂(PEₜ₂%)</th>
<th>T₃₃(PEₜ₃%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aᵥ</td>
<td>-20%</td>
<td>55.85</td>
<td>978</td>
<td>4</td>
<td>33.30</td>
<td>0.221</td>
<td>44783</td>
<td>46409(3.63)</td>
<td>43501(2.86)</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>55.81</td>
<td>976</td>
<td>4</td>
<td>33.23</td>
<td>0.233</td>
<td>44108</td>
<td>45747(3.71)</td>
<td>45788(3.80)</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.75</td>
<td>985</td>
<td>5</td>
<td>33.29</td>
<td>0.211</td>
<td>42852</td>
<td>44533(3.92)</td>
<td>44569(4.01)</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.71</td>
<td>984</td>
<td>5</td>
<td>33.24</td>
<td>0.219</td>
<td>42261</td>
<td>43972(4.04)</td>
<td>44011(4.14)</td>
</tr>
</tbody>
</table>
Table 6.2.5.9 Sensitive analysis for the fix rate \( C_{vb} \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>( P )</th>
<th>( d_3 )</th>
<th>( n_3 )</th>
<th>( C_b _3 )</th>
<th>( T_b _3 )</th>
<th>( T_P _1 )</th>
<th>( T_P_2(PETP_2%) )</th>
<th>( T_P_3(PETP_3%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_{vb} )</td>
<td>-20%</td>
<td>55.79</td>
<td>987</td>
<td>5</td>
<td>33.37</td>
<td>0.200</td>
<td>43601</td>
<td>45217(3.71)</td>
<td>45225(3.72)</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>55.79</td>
<td>986</td>
<td>5</td>
<td>33.35</td>
<td>0.201</td>
<td>43522</td>
<td>45167(3.78)</td>
<td>45200(3.85)</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.76</td>
<td>974</td>
<td>4</td>
<td>33.14</td>
<td>0.245</td>
<td>43365</td>
<td>45076(3.94)</td>
<td>45122(4.05)</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.76</td>
<td>974</td>
<td>4</td>
<td>33.11</td>
<td>0.246</td>
<td>43286</td>
<td>43286(4.04)</td>
<td>45082(4.15)</td>
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</table>

Table 6.2.5.10 Sensitive analysis for the vendor’s holding cost

<table>
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<th>Parameter</th>
<th>% changes</th>
<th>( P )</th>
<th>( d_3 )</th>
<th>( n_3 )</th>
<th>( C_b _3 )</th>
<th>( T_b _3 )</th>
<th>( T_P _1 )</th>
<th>( T_P_2(PETP_2%) )</th>
<th>( T_P_3(PETP_3%) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_v )</td>
<td>-20%</td>
<td>55.69</td>
<td>989</td>
<td>4</td>
<td>33.29</td>
<td>0.205</td>
<td>43683</td>
<td>45383(3.90)</td>
<td>45374(3.97)</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>55.74</td>
<td>988</td>
<td>4</td>
<td>33.31</td>
<td>0.203</td>
<td>43563</td>
<td>45249(3.87)</td>
<td>45284(3.95)</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>55.81</td>
<td>973</td>
<td>5</td>
<td>33.18</td>
<td>0.243</td>
<td>43326</td>
<td>44998(3.85)</td>
<td>45042(3.96)</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>55.85</td>
<td>972</td>
<td>5</td>
<td>33.20</td>
<td>0.242</td>
<td>43220</td>
<td>44879(3.84)</td>
<td>44922(3.94)</td>
</tr>
</tbody>
</table>
**Fig6.2.5.4:** % changes in vendor's holding cost

**Table6.2.5.11** Sensitive analysis for the vendor's purchase cost

<table>
<thead>
<tr>
<th>Parameter</th>
<th>% changes</th>
<th>P</th>
<th>$d_3$</th>
<th>$n_3$</th>
<th>$C_b_3$</th>
<th>$T_b_3$</th>
<th>$T_P_1$</th>
<th>$T_P_2(PE_T_{P_2}%)$</th>
<th>$T_P_3(PE_T_{P_3}%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_v$</td>
<td>-20%</td>
<td>54.69</td>
<td>1013</td>
<td>4</td>
<td>32.54</td>
<td>0.241</td>
<td>46733</td>
<td>49133(5.14)</td>
<td>49186(5.25)</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>55.23</td>
<td>993</td>
<td>4</td>
<td>32.86</td>
<td>0.243</td>
<td>45083</td>
<td>47103(4.48)</td>
<td>47152(4.59)</td>
</tr>
<tr>
<td></td>
<td>10%</td>
<td>56.31</td>
<td>954</td>
<td>4</td>
<td>33.43</td>
<td>0.247</td>
<td>41814</td>
<td>43177(3.26)</td>
<td>43215(3.35)</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>56.85</td>
<td>935</td>
<td>4</td>
<td>33.66</td>
<td>0.249</td>
<td>40185</td>
<td>41280(2.72)</td>
<td>41312(2.80)</td>
</tr>
</tbody>
</table>
**Table 6.2.5.12** Sensitive analysis for the deterioration rate

<table>
<thead>
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<th>% changes</th>
<th>P</th>
<th>$d_3$</th>
<th>$n_3$</th>
<th>$Cb_3$</th>
<th>$Tb_3$</th>
<th>$TP_1$</th>
<th>$TP_2$($PET_{P_2}%$)</th>
<th>$TP_3$($PET_{P_3}%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-20%</td>
<td>56.87</td>
<td>935</td>
<td>4</td>
<td>32.54</td>
<td>0.247</td>
<td>43287</td>
<td>44945(3.83)</td>
<td>41117(-5.01)</td>
</tr>
<tr>
<td></td>
<td>-10%</td>
<td>56.86</td>
<td>935</td>
<td>4</td>
<td>32.86</td>
<td>0.248</td>
<td>43365</td>
<td>45031(3.84)</td>
<td>41194(-5.00)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>10%</td>
<td>56.85</td>
<td>935</td>
<td>4</td>
<td>33.43</td>
<td>0.251</td>
<td>43522</td>
<td>45204(3.86)</td>
<td>41351(-4.98)</td>
</tr>
<tr>
<td></td>
<td>20%</td>
<td>56.85</td>
<td>935</td>
<td>4</td>
<td>33.66</td>
<td>0.249</td>
<td>43601</td>
<td>45292(3.87)</td>
<td>41271(-5.34)</td>
</tr>
</tbody>
</table>

**Figure 6.2.5.5:** % Changes of deterioration rate
In Table 6.2.5.2, the demand scale parameter is changed. It is observed that PETP₃ decreases significantly. Increase in demand rate parameter in Table 6.2.5.3
shows that the integrated profit decreases. This is because of decrease in the discounted unit price offered to the buyer by the vendor. From Table 6.2.5.4, when the demand price-sensitive parameter increases, $PETP_3$ increases as well. This suggests logistic manager to approach for the integration and quantity discount when price-sensitive demand parameter increases.

From Table 6.2.5.5 to 6.2.5.11 it is observed that the integrated total profit decreases significantly. This suggests that the responsible player should try to control these factors. Increase in buyer's purchase cost (Table 6.2.5.7), the vendor's ordering cost (Table 6.2.5.8), we observe that the integrated total profit increases. The change in the negotiation factor does not have any effect on the percentage change in extra profit.

6.2.6 Conclusion

A collaborative optimal policy is developed for the two players of the supply chain. The demand is assumed to be decreasing function of time and selling price. The production rate is promotional to the demand rate. To encourage the buyer for a joint decision, the vendor offers either a credit period or a quantity discount to the buyer.
From the sensitivity analysis it is seen that the rate of change of demand parameter and the deterioration rate are critical parameters. The increase in price sensitive parameter and decrease in demand scale parameter results significant increase in profit when the players of the supply chain take decision jointly. This study helps in constructing an efficient supply chain in present market as demand is decreasing.