CHAPTER 3

WAVELET AND CURVELET BASED IMAGE COMPRESSION WITH DEAD ZONE QUANTIZATION

3.1 INTRODUCTION

An image file contains huge amounts of information that requires high storage space, large transmission bandwidths and long transmission times. So it is useful to compress the image by storing only the required information to reconstruct the original image. An image can be considered as a matrix of pixel or intensity values. In order to compress an image redundancies must be evaluated, for example, several areas where there is small change or there is no change among pixel values Hariom Yadav & Hitesh Gupta (2013). So the images having the consistent color with larger areas will have large redundancies, and on the other hand an image having more changes in color by means of less redundant becomes more difficult to compress.

Wavelet analysis can be used to separate the information of an image into determination and aspect subsignals Karen Lees (2002). The estimation of subsignal shows the universal tendency of pixel values and these subsignals shows that results of the information or changes in the images according to vertical, horizontal and diagonal regions. If the result becomes a small value it can automatically set to zero without considerably altering the image. The value which is set to zero is called as threshold.
value. More number of zeroes in the result show that the compression achieves better value. Energy retained is defined as the amount of information retained by an image after compression and decompression and it is proportional to the sum of the squares of the pixel values in the image. If there is no change in the image, the energy retained achieves 100% result. This type of compression is known as lossless; Lossless compression result the reconstruction of image that will produce exact image before and after compression. Reconstruction of the images is exact when the threshold value is zero. If any changes made in image, the energy retrained will be lost and it is known as lossy compression. During the compression process, initially of zeros and the energy maintenance motivation is as high as probable. However, as more zeros are obtained, more energy is lost, so stability among the two requirements are to be established.

3.2 WAVELET TRANSFORMATION IN IMAGE PROCESSING

Wavelet transforms and further multi-scale examination functions have been secondary for compacted signal and image representation in de-noising, compression and feature detection processing problems for concerning 20 years (Ebrahimi & Pazirandeh 2011). Various research works have demonstrated that space-frequency and space-scale expansions with this family of study functions provided an extremely well-organized structure for signal or image data.

The wavelet transform function offers suitable design. Beginning with selection, spatial-frequency tiling and a variety of wavelet threshold strategies result optimized output for a processing application, data individuality and feature of importance. Fast accomplishment of
wavelet transforms by means of a filter-bank structure allows real time processing capacity. As an alternative of trying to restore image processing techniques, wavelet transforms recommend a well-organized demonstration of the signal, finely tuned to its built-in property. By combining such demonstration with simple processing techniques in the transform, multi-scale examination can achieve remarkable performance for several image processing problems.

Wavelet-based compression is an individual category of transform-based compression. Universal transform-based compression system is shown in Figure 3.1. For wavelet-based compression, a wavelet transform and its inverse are used for the transform and inverse transform, correspondingly (James S Walker 2002).

![Diagram of Compression and Decompression Process]

(a) Compression process

(b) Decompression process

Figure 3.1 Transform based compression

The compression schema starts by transforming the image from data space to wavelet space. This process can be done in several ways. By applying the data into bi-dimensional transform matrix and in the
resulting image the coefficients were obtained and grouped into four zones shown in Figure 3.2, where H symbolizes high frequency data and L symbolizes low frequency data

![Figure 3.2 Discrete wavelet transform frequency quadrants](image)

The resulting image of LL quadrant of the DWT is considered as input image for next step. Regularly for image compression purposes 4 or 5 steps will suffice. Figure 3.3 shows the result of Discrete Wavelet Transform (1 level and 5 levels)

![Figure 3.3 Discrete wavelet transform (1 level and 5 levels)](image)
DWT of 5 levels data is also obtainable in a mesh form to visualize different improved intensities of the coefficients. The majority of the DWT intensities are situated in the upper of LL quadrant.

3.3 DISCRETE WAVELET TRANSFORM IN IMAGE COMPRESSION

Discrete Wavelet Transform (DWT) is a type of linear transformation function which operates on a data vector whose length is an integer power of two and correspondingly transforming into a numerically dissimilar vector having the equal length. It is a tool that divides the data into dissimilar frequency components, and then studies every component with resolution coordinated to its scale. DWT is calculated with a cascade of filterings followed by a factor 2 subsampling (Figure 3.4) (Mallat & Zhong 1989).

![DWT Tree](image)

**Figure 3.4** DWT tree

H and L defines the high and low-pass filters correspondingly, ↓ 2 denotes subsampling. Outputs of these filters are known by Equations (3.1) and (3.2).

\[
a_{j+1}[p] = \sum_{n=-\infty}^{\infty} h[n-2p]a_j[n] \quad (3.1)
\]
\[ d_{j+1}[p] = \sum_{n=-\infty}^{\infty} h[n-2p] a_j[n] \] (3.2)

Elements \( a_j \) are used for next step (scale) of the transform and basics \( d_j \), called wavelet coefficients, establish output of the transform. \( l[n] \) and \( h[n] \) are coefficients of low and high-pass filters correspondingly. One can assume that on scale \( j+1 \) there is simply half from numeral of \( a \) and \( d \) elements on scale \( j \). This causes that DWT can be complete until simply two \( a \) elements stay behind in the analyzed signal and it’s called as scaling function coefficients.

DWT algorithm for two-dimensional pictures is comparable. The DWT is performed initially for all images in rows and subsequently for all columns as shown in Figure 3.5.

![Wavelet decomposition for two-dimensional pictures](image)

**Figure 3.5**  Wavelet decomposition for two-dimensional pictures

The major feature of DWT is multiscale illustration of function. By using the wavelets, specified function can be analyzed at variety levels of resolution. The DWT is also additive and can be orthogonal.
Wavelets appear to be efficient for analysis of textures recorded with diverse resolution. It is very significant problem in NMR imaging, since high-resolution images necessitate long time of attainment. This causes a raise of artifacts caused by patient actions, which be supposed to be avoided. There is a probability that the proposed approach will offer a tool for speedy, small resolution NMR medical diagnostic.

3.3.1 Significant Properties of Wavelets

The key transform used in denoising is by using wavelets Tim Park (2011). To assist stimulate and to employ wavelets in universal and specially for de-noising, describe several of the properties which have made wavelets so useful in several different areas. Wavelet analysis has more advantages than Fourier analysis. This in several ways comes down to the detail that Fourier analysis describes global properties of the data and, as such, is not a good illustration of local properties.

3.3.2 Significance of Wavelets in Image Compression

Loss of several information is inevitable for image compression as discussed in previous section. Amongst all of the mentioned above lossy compression methods, vector quantization needs numerous computational resources for large vectors; fractal compression is a time consuming task for coding; analytical coding has inferior compression ratio and poorer reconstructed image superiority than transformation based coding. So, transform based compression methods are well suited methods for image compression.

For transform based compression, JPEG image compression schemes based on DCT (Discrete Cosine Transform) have more
advantages such as easy, enhanced performance results, and for implementation. Since the input image is blocked separately, correlation among individual block boundaries cannot be eliminated. This results in acceptable and annoying blocking artifacts mainly at low bit rates.

Over the past years, wavelet transform function has been used for signal processing mainly, in image compression. Wavelet-based schemes achieve greater results than other coding schemes based on DCT. Since there is no need of blocking the input image, their base function includes variable length.

3.4 AN OVERVIEW OF CURVLET TRANSFORM

Wavelet transform is regarded as a better selection than Fourier transform due to its ability to restrict in frequency and time at the same time Kiruthika & Thirumaraiselvi (2012). This approach is extremely useful while examining time-varying phenomenon in normal images (Burrus et al 1998). The fine data content in images is regularly found in the high frequencies whereas the coarse information content is present in the low frequencies. WT exploits its multi-resolution capability to decompose the image into several frequency bands. The WT suffers from the following issues:

2-D line singularities: Piecewise smooth signals are equivalent to images having one dimentional Singularities. In 2-D image, the smooth areas are partitioned by edges, whereas edges are discontinuous across. They are classically smooth curves.

Lack of shift invariance: This occurs due to the down sampling procedure at every stage. When the input signal is shifted a small, the
wavelet coefficients amplitude differs mostly (Kingsbury 2001).

Lack of directional selectivity: As the DWT filters are real and distinguishable the DWT cannot discriminate amongst the opposite diagonal directions (Kingsbury 2001).

The above said issues can be overcome by the development of well-organized transformations. In order to study local line or curve singularities, ridgelet transform can be applied to the sub-images (Siva Nagi Reddy et al 2012). The second generation of curvelet transforms deal with the image boundaries by mirror expansion. The second-generation curvelet transform have been observed to be extremely effectual for numerous applications in image processing. So, this thesis work uses approach uses curvelet transformation.

Curvelets are a non-adaptive method for multi-scale object demonstration. Being an expansion of the wavelet concepts, they are appropriately used in related fields, specifically in image processing and scientific computing.

Wavelets specify the by means of a basis along with the representation of location and spatial frequency. Directional wavelet transforms using basis functions are restricted in orientation of 2D and 3D signals. Curvelet transform varies from traditional directional wavelet transforms based on the degree of localization in orientation. Specifically fine-scale base functions are lengthy ridges; the shape of these function ranges from $j$ is $2^1$ by $2^{j/2}$ so the fine-scale basis are thin ridges with an exactly single-minded direction.
Curvelets are suitable for representation of basic images which are smooth and different from singularities all along with smooth curves, where the smooth curves enclose bordered curvature with images having less length scale. This type of property used for cartoons, geometrical diagrams, and multimedia application such as text files. Zooming the images with the edges they enclose appear, gradually more instantly. Curvelet methods use this property, by defining the upper resolution curvelets in the direction of skinnier to lower resolution curvelets values. However, normal image do not follow this property; they contain details at each and every scale. So, for the natural images, it is preferable to make use of some sort of directional wavelet transform for which wavelets have the similar characteristic ratio at every scale. While the image is of the correct kind, curvelets give an illustration that is significantly sparser than other wavelet transforms. This can be quantified by taking into consideration, the best estimation of a geometrical test image that can be represented using n wavelets and analyzing the estimation error as a function of n. For a Fourier transform, the error decreases only as $\left(\frac{1}{n^{1/2}}\right)$. For a wide range of wavelet transforms, together with both directional and non-directional variants, the error decreases as $O\left(\frac{1}{n}\right)$. The additional assumption with curvelet transform allows it to achieve the value of $O\left(\frac{\left(\log(n)\right)^3}{n^2}\right)$.

Efficient numerical algorithms are used for computing the curvelet transform of discrete data. Computational cost of a curvelet transform is approximately 10 - 20 times higher than the Fast Fourier Transform (FFT) and has the similar complexity of $O(n^2\log(n))$ for an
image of dimension $n \times n$.

The curvelet transform of a given image is described as a set of coefficients defined as follows

$$c(j,l\mid k) = \int f(w) U_{j,l}(S_{j,l}^k w)e^{i-b,w}\,dw$$  

(3.3)

where $c(j,l\mid k)$ is the curvelet coefficient of the image with scale $j$, wedge location $l$, and coordinates $k = (k_1, k_2) \in Z^2$. The distribution of scales and locations results of the image as is shown in Figure 3.6. This pseudopolar dyadic tiling of frequency domain successfully used with wavelets and other harmonic analysis algorithms has been studied by Ana Georgina Flesia & Da\'vid L Donoho (2003).

![Pseudopolar tiling of the frequency domain](image)

**Figure 3.6** Pseudopolar tiling of the frequency domain
f(w) is the FFT value at w. \( S_\phi = \begin{pmatrix} 1 & 0 \\ -\tan(\phi) & 1 \end{pmatrix} \) is the same matrix.

\[
\Theta = \left[ -\frac{\pi}{4}, \frac{\pi}{4} \right]. \ U_{j,l} \text{ is a localizing smoothing window. } b = (k_1 2^{-j}, k_1 2^{-j})
\]

The following steps show the details of coefficients values for computation:

1. The 2D FFT of the desired image, is found

2. For each scale \( j \) and scale \( l \), the FFT values in each wedge is smoothened by the smoothing window \( U_{j,l} \). This prevents discontinuities and is essential for prefect recovery. To prevent loss of energy, The functions \( U_{j,l} \) also have to be constructed so that they overlap in a way that makes the squared sum of \( U_{j,l} \) at each coordinate equal to one:

\[
\sum_{j} \sum_{l} |U(j,l)|^2 = 1 \quad (3.4)
\]

3. Then the inverse FFT for every wedge is calculated. Because the wedges do not correspond to rectangles, the inverse FFT is calculated using the periodicity of the Fourier transform. FFT values of the given wedge are described using the periodicity in frequency domain. The FFT is then computed for the values in the rectangle surrounding the origin.

The inverse transform is computed by reversing the above steps as given below:

1. Compute the 2D FFT for each scale \( j \) and location \( l \).
2. Multiply each one scale/location by the correspondent smoothing window. The smoothing window needs to be ‘wrapped’ to correspond to the novel distribution of frequency values are computed by step 3 in the forward transform.

3. The frequency values are mapped as group ‘unwrapped’ to their unique scale/wedge location. Then perform 2D inverse FFT, to acquire the original image back again.

Substitute to the wrapping approach, in computing the FFT for curvelet wedges, is to turn the coordinate system by aligning the orientation of each wedge locations. In this case,

$$c(j,l|k) = \int f(w) U_{j,l}(S_{j,l}^{-1}w)e^{b,w} dw$$ (3.5)

These types of curvelet based implementation are done through online. The results presented in this thesis work are relevant to either one of them. Due to high computational complexity and to avoid interpolation artifacts in this work wrapper based implementation is done from Figure 3.7, it is clear that the coarsest level ($j = 1$) is not directional. Its effect is similar to a low pass filter. The corresponding coefficient values represent the smoothest areas in the image. Outer finest level is developed using either directional or non-directional related curvelets. Directional curvelets are high computational cost with enhanced better results. In this work focus is made on directional curvelets. The ratio of the new size to the old one is fixed at 1.3. The number of scales $J$ is determined by the size of the input image. The number of wedges per scale rises from the coarsest scale to the finest that is equal to $2^j$ where $j = 1, 2, ..., J$. Although, that values of curvelet coefficients are entirely estimated by the frequency content, tiling of the frequency space takes the similar dyadic
approach with frequently fixed parameters. Using these adaption divisions to match to the frequency content in the image will generate better performance results.

3.5 QUANTIZATION

Quantization is a lossy compression technique in image processing. It is achieved by compressing the series of values to a single quantum value. The number of distinct symbols in a known stream is reduced and the stream becomes further compressible. Precise applications contain DCT data quantization in JPEG and DWT data quantization in JPEG 2000. The quantizer has an important impact on the amount of compression obtained and loss, subject to a lossy compression system. Discrete sources are a topic of interest in their own right and also provide as the internal layer for encoding analog source sequences and waveform sources (Figure 3.7). Source coding for analog values is typically called as quantization. This is also the middle layer for waveform encoding/decoding.

Figure 3.7  Encoding and decoding

Figure 3.7 shows the analog sequence sources, Encoding and decoding of discrete sources, and waveform sources. Quantization is the
middle layer process and should be understood before entering into the outer layer, which deals with waveform sources.

In quantization process, the input of the quantizer is considered as the analog random variable sequence \( (U_1, U_2... U_N) \). It is similar to that for modeling the input to a discrete source encoder as a sequence of random symbols. Also, it is also popular to treat extremely rare inputs in a different way from very ordinary inputs, and a probability density is a perfect approach for this. Quantizer measures the incoming sequence into a sequence of discrete random variable (rv) \( V_1, V_2, \ldots \), where the objective of the random variable with \( 'm' \) sequence with little distortion is defined as \( U_{mm} \). It is assumed that encoder/decoder at the internal layer of Figure 3.7 is uniquely decodable with random variable sequence and output the discrete encoder. It will be passed all the way through the middle layer with input sequence. The result of the quantizer layer is called a lookup table with a fixed set of real numbers for each distinct random variable \( V_m \) and these are regularly mapped into real numbers 1 to M. Thus on the output side a look-up function is essential to convert back to the arithmetical value \( V_m \). Increasing \( 'm' \), characteristically reduces the distortion, but actually cannot reduce it. Mean square error (MSE) is defined as \( \text{MSE} = \mathbb{E}[(U-\hat{V}(U)/2] \) where \( U \) is the analog random variable and it is quantized to discrete random variable \( V \). Mean-squared distortion is used to calculate distortion of image. When studying the conversion of waveforms into sequences in the subsequent section, it will be seen mean-squared distortion is used for converting the distortion for the sequence into the distortion for the waveform, which is one of the simplest and most general type in lossy compression.
3.5.1 Vector Quantization

Image compression can be categorized into two types namely as Lossy Compression and Lossless Compression. In lossless compression, the unique image can be entirely recovered from the compressed image. Lossless Compression is useful for applications with precise requirements such as medical imaging, whereas lossy compression is mainly suitable for natural images similar to photos in applications wherever minor loss is acceptable to achieve a significant reduce in bit rate.

Vector Quantization (VQ) is a type of quantization technique in signal processing which uses the probability density functions (pdf) by the circulation of prototype vectors. It was originally used for data compression. It works by partitioning a large set of vectors into groups having approximately the similar number of vectors closest to them. Each group of vector is represented as centroid value in k-means and other clustering algorithm also. Probability density function is powerful, particularly for identifying the density level of large and high-dimensioned data. Since data points are represented by the index of their closest centroid, normal data contain low error and exceptional data high error. Hence it is appropriate to consider this for lossy data compression. It can also be used for lossy data correction and density estimation.

3.5.2 Image Compression based on Vector Quantization

Vector Quantization (VQ) coding is an image compressing scheme (Chin-chen et al 2013). In this method, an index table for the original image is generated. The receiver can restructure the image simply by the index table and the pre-training VQ codebook. VQ is unique of lossy compression techniques. For a given image which consists of
256×256 pixels, the superiority of reconstructing is an image is getting 27 dB to 30 dB where the codebook size is 256. In order to get better performance of the quantization, in this work dead zone quantization technique is used.

3.6 ENCODING APPROACHES

In lossless compression techniques, the input image can be easily recovered from the compressed image. These are also called noiseless image, because it is performed without adding noise value in image. It is also well-known as entropy coding because it uses statistics/decomposition approaches to remove/reduce redundancy. Lossless compression is used only for a small number of applications with stringent necessities such as medical imaging.

3.6.1 Set Partitioning In Hierarchical Trees (SPIHT)

There is a parent-child association among the wavelet coefficients. Every coefficient at a known scale is associated to a set of coefficients at the subsequently finer scale of similar orientation. The wavelet coefficient in the coarse range is called parent. Every parent has four children at the next finer range of similar direction. It is based on the theory of parent-child association among the wavelet coefficients. Encoding step consists of two quantization passes namely sorting pass and the refinement pass. The algorithm maintains the data structure with three linked lists the LSP, LIP and the LIS.

The algorithm maintains three linked list to keep important and unimportant pixels and their corresponding sets:

- LSP: list of significant pixels
LIP: list of insignificant pixels

LIS: list of insignificant sets

These three lists are used to keep track of the original image throughout encoding process. Throughout sorting pass, new important entries in LIP and LIS are identified and their corresponding signs are coded. In every refinement pass, every coefficient in LSP apart from the ones added in the previous sorting pass is refined. The image is reconstructed by the quantization procedure; the quantization step halves the threshold every time. The encoding procedure stops up when a target bit rate or threshold or quality condition is reached.

Roots of the all spatial orientation trees are

\( O(i,j) \): Set of offspring of the coefficient

\( (i,j) = \{(2i,2j),(2i,2j+1),(2i+1,2j),(2i+1,2j+1)\} \), except \((i,j)\) is in LL; when coefficient \((i,j)\) is in LL subband,

\( O(i,j) \) is defined as: \( O(i,j) = \{(i,j+wLL), (i+hLL,j), (i+hLL,j+wLL)\} \), where \( wLL \) and \( hLL \) is the width and height of the LL subband, respectively.

\( D(i,j) \): Set of all descendants of the coefficient \((i,j)\), \( L(i,j) \): \( D(i,j) - O(i,j) \)

A significant function \( S_n(\tau) \) which decides the significance of the set of coordinates, \( \tau \), with respect to the threshold \( 2n \) is defined by:

\[
S_n(\tau) = \begin{cases} 
1, & \max(i,j) \leq \left\lfloor \frac{2^n - 1}{2} \right\rfloor \\
0, & \text{else} 
\end{cases}
\]  

(3.6)
At the initialization stage, LSP is set to empty list. LIP values are initialized with all coefficients in the maximum level of the wavelet LL subband. LIS values are initialized with all the coefficients in the maximum level of the wavelet LL subband with descendents. During the sorting pass, the algorithm primary traverses during the LIP, testing the magnitude of its fundamentals beside the present threshold and representing their significance by 0 or 1. Each time when a coefficient is found important, its sign is converted and it is moved to LSP. The algorithm then examines the LIS and performs a magnitude on all coefficients of set. If a particular tree/set is established to be important, it is partitioned into its subsets and tested for significance. Else a single bit is added to the bit stream to point out an unimportant set. After every sorting pass is completed SPIHT outputs modification bits at the present level of bit which have been moved to LSP list. The results of SPIHT output the refinement bits that decrease highest error. This process continues by decreasing current threshold by factor of two until desired bit rate is achieved. SPIHT is an embedded coding technique. In embedded coding algorithms, encoding of the similar signal at lower bit rate is embedded at the start of the bit stream for the target bit rate. Effectively, bits are ordered in importance. This type of coding is particularly helpful for continuous transmission by means of an embedded code; where an encoder can conclude the encoding procedure at any situation.

SPIHT algorithm is based on the following concepts (Said & Perlman 1996, Kim & Pearlman 1997)

1. Ordered bit plane progressive transmission.

2. Set partitioning sorting algorithm.
3. Spatial orientation trees.

SPIHT keeps each state with three lists namely LIP, LSP and LIS. Each list stores pixel values. Insignificant pixels are stored by LIP, significant pixel values are stored by LSP and insignificant pixel values are stored by LIS. At the start, LSP is empty, LIP keeps all coefficients values in lower sub band, and LIS keeps the complete tree roots which are at the lower sub band. SPHIT algorithm can be performed in two pass. The primary pass is the sorting pass. It initially browses the LIP and move towards entire considerable coefficients to LSP and outputs its sign. Then it browses LIS executing the information and subsequent the partitioning sorting algorithms.

The subsequent pass is the refining pass. It searches the coefficients in LSP and results a single bit only based on the present threshold. After the two passes are completed, the threshold is separated by 2 and the encoder process these two passes over again. This process is recursively applied until the output bits reach the preferred number.

3.6.2 Use of Dead Zone Quantization with Curvelet

After the curvelet transform, all the subbands are quantized in lossy compression to reduce the precision of the subbands to assist in achieving compression. Quantization of curvelet subbands is one of the major sources of data/information loss in the encoder. Quantization is done by uniform scalar quantization with dead-zone (Soo-Chang Pei & Ching-Min Cheng 1995). In dead-zone scalar quantizer the step size is defined as \( \Delta b \), the width of the dead-zone is defined as \( 2\Delta b \) as shown in Figure 3.8. It supports split quantization step sizes for every subband. The quantization step size (\( \Delta b \)) for a subband (b) is determined based on the active series of
the subband values. The method of uniform scalar quantization through a dead-zone is,

\[ q_b(i,j) = \text{sign}(y_b(i,j)) \left\lfloor \frac{y_b(i,j)}{\Delta_b} \right\rfloor \]  

(3.7)

Where \( y_b(i,j) \) represents a curvelet coefficient in subband band. \( \Delta_b \) denotes the quantization step size for the subband \( b \). All the resulting quantized curvelet coefficients \( q_b(i,j) \) are signed integers.

All the evaluations up to the quantization step are carried out in two complement form. After the quantization process completed the quantized curvelet coefficients are changes into sign-magnitude illustration earlier than the encoding procedure. Due to the advantages found in dead zone quantization, it is decided to use dead zone quantization with all transforms.

### 3.6.3 Encoding Process

The quantized data includes unnecessary information. The storage space is exhausted if saved. Modified encoding process
overcomes this problem. It is an illustration of a lossless data compression approach that offers a way of eliminating the unnecessary quantized information without any loss of information.

A. The Zero Shifting

The method of zero shifting is a straightforward and easy-to-implement technique which conserves the embedded features of the SPIHT coding. By downward scaling of the pixel values by $2(N-1)$, $N$ being the number of bits of the original pixel, the spatial field magnitude becomes bipolar ranging from $(-2(N-1))$ to $(+2(N-1)-1)$, nearly equal to the half of the maximum absolute spatial value.

In the transform field, the wavelet lifting system results the high pass subband by measuring the weighted differences between of pixel values after completion of prediction. Likewise, the low pass subband is decreased by computing the weighted average of pixel values after the completion of updating step. This low pass subband is decomposed iteratively for every level of decomposition ending with one lowest frequency band, the rest being high frequency bands.

SPIHT coding is one of the types of bit plane coding techniques. In this method, the largest number bit plane is determined by the maximum absolute assessment of the altered coefficients. As the maximum coefficient reduces using the method of zero shifting, it takes less time for both encoding and decoding. Further, the lower section of the complete image in the spatial domain values from $0 \leq p_{i,j} \leq 2N-1$ are shifted to a higher exact range in the bipolar sense. The lower image values get detailed information pertaining to the limits, ends, etc. These coefficients become important at a larger threshold value and hence
are obtained in the bit level surface progressive transmission of the SPIHT bit stream. Since SPIHT algorithm forever encodes a sign bit for significant coefficients, no other additional bit is needed for being bipolar. By stopping the decoding process still at a lower bit rate, the quality improves, together individually and independently, due to the inclusion of some point of information at that rate.

**B. Modified SPIHT**

The spatial orientation tree in 2D SPIHT and their parent offspring dependency of tree is as shown in Figure 3.9 which includes three levels of decomposition. Every node of the tree is represented by its coordinates in the algorithm. The tree is defined in such a way that each node has no leaves or four leaves at the equivalent spatial location. It is used for next finer level stage (Said & Perlman 1996).

**Figure 3.9 Spatial orientation tree in 2D SPIHT**

In original SPIHT, the pixels in the coarsest level e.g., LL subband of the 3rd level of Figure 3.9, tree roots and tree roots are grouped into blocks of 2 X 2 neighboring pixels. In the Initial stage of the process each and every coordinate of coefficients along with the rough level are
placed into the LIP listing and the coordinates of coefficients having only
descendants are placed into LIS list as D-type sets. Proposed work can be
done by making modification in initial stage of the process of the SPIHT
algorithm with the principle of null shifting value as follows: in the
decomposition level the coarsest level of LL subband which consists of
only one pixel level and set LIS list with all coordinates of the coarsest
level which is also single pixel level, else initialize the LIS list through LL
subband at the same time as of the LIP list with every coordinates.
Consequently, sorting the values in LIS list, SPIHT algorithm first tests the
band values that corresponding to LH, HL, and HH for lowest subband.
For each root in LL, it compares the utmost value among the other three
bands at the same time in spatial orientation and sorts it in the output bit
stream. Once the process is completed the set separation rule of SPIHT
(Wei Li & Zhen Peng Pang 2010) separates the left out descendants into
LIS list either D-type set or L-type set. Original decoder duplicates the
encoder result of execution. Encoder data gets the liberty to encode the
complete bit level without any bit restrictions. Experiments were carried
out with different images spatially with different features. It shows that
proposed system provides an improved encoding result than the original
result and with same PSNR values. In order to decode the encoded data, it
is initiated with binary tree in root node and traverses the path to leaf node
until the incoming bit length ends. A symbol is decoded every time while
the leaf node is reached. This procedure is repeated until all the bit length
in the received input has been decoded.

3.7 IMPORTANCE OF CURVELETS OVER WAVELETS

Curvelets will be improved than the wavelets in following
scenarios:
i. Curvelet transform has optimal illustration of objects with edges with minimum detail.

ii. Curvelet transform is best for image reconstruction tool in severely ill-posed issues.

iii. Curvelet transform is very good for sparse demonstration of wave propagators.

The curvelets provide best optimal sparseness for curve that occurs in smooth images. Sparseness is computed by the rate of decompose of the m-term approximation of the algorithm. By means of a sparse illustration and with enhanced compression potential, and it facilitates for enhancing denoising performance as further sparseness increases the amount of smooth areas in the image. It has been experimented by Candès et al (2005) and showed that orthogonal systems contains best m-term approximations that decay in $L^2$ with rate $O(m^{-2})$. On images with $C^2$ boundaries, non-optimal systems have the rates: Fourier Approximation:

$$\|f - f_m\|_{L^2}^2 \approx O(m^{-1})$$  \hspace{1cm} (3.8)

Wavelet Approximation

$$\|f - f_m\|_{L^2}^2 \approx O(m^{-1})$$  \hspace{1cm} (3.9)

Curvelet Approximation

$$\|f - f_m\|_{L^2}^2 \approx O((\log m)^3(m^{-2}))$$  \hspace{1cm} (3.10)

As it is experimental from the m-term approximation, the
Curvelet Transform provides the closest m-term approximation to the lesser bound. Thus, in images with a large numeral of $C^2$ curves, curvelet algorithm would be very effectual.

3.7.1 Continuous Curvelet Transform

Continuous curvelet transform has two major categories. The primary continuous curvelet transform used a composite series of steps comprising of the ridgelet assessment of an image with random transform (Candes & Donoho 1999). The accuracy result of this algorithm was tremendously slow. So, the algorithm was modified in 2003 (Candes & Donoho 2003).

The utilization of the ridgelet transform was taken into consideration, thus minimizing the quantity of redundancy in the transform and ever-increasing the speed considerably. In 1994, Kashin & Temlyakov considered this new approach of curvelets as tight frames. By means of tight frames, a single curvelet has frequency support values in a parabolic-wedge region. A sequence of curvelets $\gamma_{j,l,k}$ are tight frames which presents definite value for $A$ such that:

$$A\|f\|L^2 = \sum_{j,l,k} |\langle f, \gamma_{j,l,k} \rangle|^2 = \forall f \in L^2$$

(3.11)

where every curvelet in the space domain is defined as follows

$$\gamma_{j,l,k} = 2^{j/2} \gamma(D_j R_\theta x - k_\theta)$$

(3.12)

With $D_j$ = Parabolic Scaling matrix, $R_\theta$ = Rotation matrix, $k_\theta$ = translation parameter, $\gamma$ = the “mother” curvelet by means of the property of tight frames, the inverse of the curvelet transform is simply
established as Equation (3.13).

$$f = \sum_{j,l,k} <f, \gamma'_{j,l,k}> \gamma'_{j,l,k}$$  \hspace{1cm} (3.13)

The entire curvelets come under down into one of the three categories.

i. Curvelet type A is defined as length-wise support maintains, which do not intersect a discontinuity and curvelet coefficient magnitude will be zero which is shown in Figure 3.10(a).

ii. Curvelet type B is defined as the length-wise support maintains, It intersects a point of discontinuity and with magnitude of curvelet coefficient will be closer to zero, without critical angle as shown in Figure 3.10(b).

iii. Curvelet type C is defined as the length-wise support maintain, intersects with a discontinuity and is tangent to that discontinuity. The magnitude of curvelet coefficient will be a large amount larger than zero as shown in Figure 3.10(c).

Figure 3.10 Curvelet type A, curvelet type B and curvelet type C

After the curvelet transform, the correlation coefficients of the curvelet transform are quantized with dead zone quantization technique.
3.7.2 Proposed Image Compression Algorithm

The proposed image compression technique is extremely efficient by overcoming the issues of general wavelet transform based coding compression algorithm which are applicable to the images with linear curves. The steps involved in the proposed approach are:

i. Represent the image data as intensity values of pixels in the spatial co-ordinates.

ii. Apply curvelet transform on the image matrix and get the curvelet coefficients of the image.

iii. Quantize the available coefficients using dead zone quantization algorithm.

iv. Use modified SPIHT coding on the bit stream.

Thus, the proposed image compression approach is based on Curvelet Transformation with dead zone quantization and SPIHT, modified SPIHT encoding and hybrid algorithm.

3.8 PERFORMANCE EVALUATION METRICS

The performance metrics taken for evaluation of these approaches are:

3.8.1 Root Mean Square Error (RMSE) and Peak Signal to Noise Ratio (PSNR)

Root Mean Square Error (RMSE) metric is extensively used for it is easiness to compute, having obvious physical interpretation and
mathematically suitable. RMSE is computed by averaging the squared intensity difference of reconstructed image \( \hat{x} \), the original image, \( x \). The RMSE is calculated as Equation (3.14).

\[
\text{RMSE} = \sqrt{\frac{2}{MN}[x(i,j) - \hat{x}(i,j)]^2}
\]  

(3.14)

where \( x(i,j) \) is the original image, \( \hat{x}(i,j) \) approximated version (which is actually the decompressed image) and \( M,N \) are the dimensions of the images. \( M \times N \) is the size of the image and assuming the grey scale image of 8 bits per pixel (bpp), PSNR is defined as Equation (3.15).

\[
\text{PSNR} = 10 \log_{10} \left[ \frac{255^2}{\text{MSE}} \right]
\]  

(3.15)

3.8.2 Compression Ratio

Compression ratio is used to enumerate the minimization in the size of image representation, produced by an image compression algorithm. The data compression ratio is equivalent to the physical compression ratio used to evaluate physical compression of substances and is defined as the ratio between the uncompressed image size and the compressed image size.

3.8.3 Correlation

Correlation is used to measure the similarity of two images. If it is close to 1, then two images are similar. Normalized cross-correlation can be used to determine how to register or align the images by translating one of them. It is calculated by Equation (3.16).
\[
\frac{\sum_{(i,j) \in W} l_1(i,j) l_1(x+i, y+j)}{2\sqrt{\sum_{(i,j) \in W} l_2^2(i,j) \sum_{(i,j) \in W} l_2^2(x+i, y+j)}}
\]  
(3.16)

Table 3.1 Comparison of various parameters among SPIHT, modified SPIHT and hybrid coding for wavelet transform and curvelet transform

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Wavelet</th>
<th>Curvelet</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>SPIHT</td>
<td>MSPIHT</td>
</tr>
<tr>
<td>PSNR</td>
<td>25.6479</td>
<td>29.4976</td>
</tr>
<tr>
<td>CR</td>
<td>70.3364</td>
<td>82.3716</td>
</tr>
<tr>
<td>Correlation</td>
<td>0.9811</td>
<td>0.9859</td>
</tr>
</tbody>
</table>

Table 3.1 shows the performance comparison among SPIHT, MSPIHT and Hybrid of MSPIHT with Huffman coding with dead zone quantization. It is shows that Modified SPIHT approach gives quality results when compared with SPIHT approach and it is slightly better with Hybrid coding. For SPIHT coding, improvement is PSNR is 32.7%. The reduction in error is 61.92%.

3.9 SUMMARY

This chapter clearly compares the compression results of wavelet image compression approach with the compression of the same with curvelet transform. The metrics parameters of PSNR, RMSE, compression ratio and correlation are analyzed. Due to the drawbacks of wavelets, proposed approach uses a curvelet transformation with dead zone.
quantization approaches. Moreover, for the encoding purpose, SPIHT, modified SPIHT and hybrid are used.