The research work entitled “Monophonic Concepts in Graphs” has been carried out under the guidance and supervision of Dr. J. John, Assistant Professor, Department of Mathematics, Government College of Engineering, Tirunelveli, India and the Co-Guidance of Dr. A. Vijayan, Assistant Professor, Research Department of Mathematics, N. M. Christian college, Marthandam, India.

The thesis consists of six chapters

1. Preliminaries
2. The Edge Monophonic Number of a Graph
3. The Forcing Edge Monophonic Number of a Graph
4. The Connected Monophonic Number of a Graph
5. The Connected Edge Monophonic Number of a Graph
6. The Monophonic Domination Number of a Graph

One concept that pervades all of graph theory is that of distance and distance is used in isomorphism testing, graph operations, hamiltonicity problems, extremal problems on connectivity and diameter, and convexity in graphs. The distance $d(u, v)$ between two vertices $u$ and $v$ in a connected graph $G$ is the length of a shortest $u – v$ path in $G$. This gives rise to the concept of geodetic set and geodetic number, edge geodetic set, edge geodetic number, connected geodetic set, connected geodetic number, connected edge geodetic set and connected edge geodetic number of a graph [1, 2, 3, 4, 6, 16, 17, 18, 19, 21, 22]. The geodetic concepts have applications in location theory and convexity theory [10].

A chord of a path $u_0, u_1, u_2, \ldots, u_h$ is an edge $u_iu_j$, with $j \geq i + 2$. An $u – v$ path is called a monophonic path if it is a chordless path. This gives rise to the concept of
monophonic set and monophonic number of a graph [8, 12, 13]. The monophonic concepts have applications in location theory and convexity theory [15].

In this thesis we define and develop various concepts viz. the edge monophonic number of a graph, the forcing edge monophonic number of a graph, the connected monophonic number of a graph, the connected edge monophonic number of a graph and the monophonic domination number of a graph. These concepts have interesting applications in location theory and convexity theory.

By a graph we mean a finite undirected connected graph without loops or multiple edges. In Chapter 1, we collect the basic definitions and theorems, which are needed for the subsequent chapters. For basic definitions and graph theoretic terminologies, we refer to [11].

Let $G = (V, E)$ be a connected graph with at least two vertices. The order and size of $G$ are denoted by $p$ and $q$ respectively. The distance $d(u, v)$ between two vertices $u$ and $v$ in a connected graph $G$ is the length of a shortest $u - v$ path in $G$. An $u - v$ path of length $d(u, v)$ is called an $u - v$ geodesic. It is known that the distance is a metric on the vertex set $V(G)$. A vertex $x$ is said to lie on a $u - v$ geodesic $P$ if $x$ is a vertex of $P$ including the vertices $u$ and $v$.

A geodetic set of $G$ is a set $S \subseteq V(G)$ such that every vertex of $G$ is contained in a geodesic joining a pair of vertices of $S$. The geodetic number $g(G)$ of $G$ is the minimum order of its geodetic sets and any geodetic set of order $g(G)$ is called a geodetic basis of $G$. These concepts were studied by Chatrand et.al. [2, 3]

For two vertices $u$ and $v$ in a connected graph $G$, the monophonic distance $d_m(u, v)$ is the length of the longest $u - v$ monophonic path in $G$. An $u - v$ monophonic
path of length $d_m(u, v)$ is called an $u – v$ monophonic. For a vertex $v$ of $G$, the monophonic eccentricity $e_m(v)$ is the monophonic distance between $v$ and a vertex farthest from $v$. The minimum monophonic eccentricity among the vertices is the monophonic radius, $rad_m(G)$ and the maximum monophonic eccentricity is the monophonic diameter, $diam_m(G)$. These concepts were studied by A. P. Santhakumaran et. al. [20]. A vertex $x$ is said to lie on a $u – v$ monophonic $P$ if $x$ is a vertex of $P$ including the vertices $u$ and $v$. A monophonic set of $G$ is a set $M \subseteq V(G)$ such that every vertex of $G$ is contained in a monophonic path joining some pair of vertices in $M$. The monophonic number $m(G)$ of $G$ is the minimum order of its monophonic sets and any monophonic set of order $m(G)$ is a minimum monophonic set of $G$.

For a subset $D$ of vertices, we call $D$ a dominating set if for each $x \in V(G) – D$, $x$ is adjacent to at least one vertex of $D$. The domination number $\gamma(G)$ of $G$ is the minimum cardinality of a dominating set of $G$ [5, 7]. A set of vertices $M$ in $G$ is called a geodetic dominating set if $M$ is both a geodetic set and a dominating set. The minimum cardinality of a geodetic dominating set of $G$ is its geodetic domination number and is denoted by $\gamma_g(G)$. A geodetic dominating set of size $\gamma_g(G)$ is said to be a $\gamma_g$-set [9].

In Chapter 2, we introduce the concept of the edge monophonic number of a connected graph $G$. For a connected graph $G = (V, E)$, an edge monophonic set of $G$ is a set $M \subseteq V(G)$ such that every edge of $G$ is contained in a monophonic path joining some pair of vertices in $M$. The edge monophonic number $m_1(G)$ of $G$ is the minimum order of its edge monophonic sets and any edge monophonic set of order $m_1(G)$ is a minimum edge monophonic set of $G$. Connected graphs of order $p$ with edge monophonic number $p$ are characterized. Necessary condition for edge monophonic number to be $p – 1$ is given. It is shown that for every two integers $a$ and $b$ such that $2 \leq a \leq b$, there exists a
connected graph $G$ with $m(G) = a$ and $m_1(G) = b$, where $m(G)$ is the monophonic number of $G$.

An edge monophonic set $M$ in a connected graph $G$ is called a minimal edge monophonic set if no proper subset of $M$ is an edge monophonic set of $G$. The upper edge monophonic number $m_1^+(G)$ of $G$ is the maximum cardinality of a minimal edge monophonic set of $G$. Some general properties satisfied by this concept are studied. For a connected graph $G$ of order $p$ with upper edge monophonic number $p$ is characterized. It is shown that for every two positive integers $a$ and $b$, where $2 \leq a \leq b$, there exists a connected graph $G$ with $m_1(G) = a$ and $m_1^+(G) = b$.

In Chapter 3, we introduce the concept of the forcing edge monophonic number of a connected graph $G$. For a connected graph $G = (V, E)$, let a set $M$ be a minimum edge monophonic set of $G$. A subset $T \subseteq M$ is called a forcing subset for $M$ if $M$ is the unique minimum edge monophonic set containing $T$. A forcing subset for $M$ of minimum cardinality is a minimum forcing subset of $M$. The forcing edge monophonic number of $M$, denoted by $f_m(M)$, is the cardinality of minimum forcing subset of $M$. The forcing edge monophonic number of $G$, denoted by $f_m(G)$, is $f_m(G) = \min\{f_m(M)\}$, where the minimum is taken over all minimum edge monophonic sets $M$ in $G$. Some general properties satisfied by this concept are studied. The forcing edge monophonic number of certain classes of graphs are determined. It is known that $m(G) \leq m_1(G)$, where $m(G)$ and $m_1(G)$ respectively the monophonic number and the edge monophonic number of a connected graph $G$. However, there is no relation between $f_m(G)$ and $f_{m_1}(G)$, where $f_m(G)$ is the forcing monophonic number of a connected graph $G$. We give realization results for various possibilities of these four parameters.
In Chapter 4, we introduce the concept of the connected monophonic number of a connected graph $G$. A connected monophonic set of a graph $G$ is a monophonic set $M$ such that the subgraph $\langle M \rangle$ induced by $M$ is connected. The minimum cardinality of a connected monophonic set of $G$ is the connected monophonic number of $G$ and is denoted by $m_c(G)$. Connected graphs of order $p$ with connected monophonic number 2 or $p$ are characterized. It is shown that for any positive integers $2 \leq a < b \leq c$, there exists a connected graph $G$ with $m(G) = a$, $m_c(G) = b$ and $g_c(G) = c$, where $g_c(G)$ is the connected geodetic number of a graph $G$.

A connected monophonic set $M$ in a connected graph $G = (V, E)$ is called a minimal connected monophonic set if no proper subset of $M$ is a connected monophonic set of $G$. The upper connected monophonic number $m^+_c(G)$ is the maximum cardinality of a minimal connected monophonic set of $G$. Connected graphs of order $p$ with upper connected monophonic number 2 or $p$ are characterized. It is shown that for any positive integers $2 \leq a < b \leq c$, there exists a connected graph $G$ with $m(G) = a$, $m_c(G) = b$ and $m^+_c(G) = c$, where $m(G)$ is the monophonic number and $m_c(G)$ is the connected monophonic number of a graph $G$. Let $M$ be a minimum connected monophonic set of $G$. A subset $T \subseteq M$ is called a forcing subset for $M$ if $M$ is the unique minimum connected monophonic set containing $T$. A forcing subset for $M$ of minimum cardinality is a minimum forcing subset of $M$. The forcing connected monophonic number of $M$, denoted by $f_{mc}(M)$, is the cardinality of a minimum forcing subset of $M$. The forcing connected monophonic number of $G$, denoted by $f_{mc}(G)$, is $f_{mc}(G) = \min\{f_{mc}(M)\}$, where the minimum is taken over all minimum connected monophonic set $M$ in $G$. It is shown that for every integers $a$ and $b$ with $a < b$, and $b - 2a - 2 > 0$, there exists a connected graph $G$ such that $f_{mc}(G) = a$ and $m_c(G) = b$. 
In Chapter 5, we introduce the concept of the connected edge monophonic number of a connected graph \( G \). For a connected graph \( G = (V, E) \), an edge monophonic set \( M \subseteq V(G) \) is called a connected edge monophonic set if the subgraph \( G[M] \) induced by \( M \) is connected. The minimum cardinality of a connected edge monophonic set of \( G \) is the connected edge monophonic number of \( G \) and is denoted by \( m_{1c}(G) \). Connected graphs of order \( p \) with connected edge monophonic number 2 or \( p \) are characterized. It is shown that for every two integers \( a, b \) and \( c \) such that \( 2 \leq a < b < c \), there exists a connected graph \( G \) with \( m(G) = a, \ m_1(G) = b \) and \( m_{1c}(G) = c \), where \( m(G) \) is the monophonic number and \( m_1(G) \) is the edge monophonic number of \( G \). A connected edge monophonic set \( M \) in a connected graph \( G = (V, E) \) is called a minimal connected edge monophonic set if no proper subset of \( M \) is a connected edge monophonic set of \( G \). The upper connected edge monophonic number \( m_{1c}^+(G) \) is the maximum cardinality of a minimal edge connected monophonic set of \( G \). Connected graphs of order \( p \) with upper connected edge monophonic number 2 or \( p \) are characterized. It is shown that for any positive integers \( 2 \leq a < b \leq c \), there exists a connected graph \( G \) with \( m_1(G) = a, \ m_{1c}(G) = b \) and \( m_{1c}^+(G) = c \), where \( m_1(G) \) is the edge monophonic number and \( m_{1c}(G) \) is the connected edge monophonic number of a graph \( G \). Let \( M \) be a minimum connected edge monophonic set of \( G \). A subset \( T \subseteq M \) is called a forcing subset for \( M \) if \( M \) is the unique minimum connected edge monophonic set containing \( T \). A forcing subset for \( M \) of minimum cardinality is a minimum forcing subset of \( M \). The forcing connected edge monophonic number of \( M \), denoted by \( f_{m_{1c}}(M) \), is the cardinality of a minimum forcing subset of \( M \). The forcing connected edge monophonic number of \( G \), denoted by \( f_{m_{1c}}(G) \), is \( f_{m_{1c}}(G) = \min\{f_{m_{1c}}(M)\} \), where the minimum is taken over all minimum connected edge monophonic sets \( M \) in \( G \). It is shown that for every integers \( a \)