PREFACE

The thesis entitled "Studies in Graph Labeling, Divisor Cordial Labeling and Other Labelings" embodies the work done by R. Varatharajan under the guidance of Dr. S. Naveenethakrishnan.

The thesis consists of five chapters.

1. Preliminaries

2. Divisor Cordial Graphs

3. Special Classes of Divisor Cordial Graphs

4. Cycle Related Divisor Cordial Graphs

5. Divisor Cordial Books

By a graph $G = (V, E)$, we mean a finite, undirected graph with neither loops nor multiple edges. For graph theoretic terminology we refer to Harary [11]. For graph labeling, Gallian [8] is referred to.

In Chapter 1, we collect some basic definitions and theorems on graphs which are needed for the subsequent chapters. Also, we state some number theoretic concepts which will be used to label the various graphs [6].
Graph labeling [8] is a strong communication between Number theory [6] and structure of graphs [11]. By combining the divisibility concept in Number theory and Cordial labeling concept in Graph labeling, we introduce a new concept called divisor cordial labeling.

A vertex labeling [8] of a graph $G$ is an assignment $f$ of labels to the vertices of $G$ that induces for each edge $uv$ a label depending on the vertex label $f(u)$ and $f(v)$. The two best known labeling methods are called graceful and harmonious labelings. Cordial labeling is a variation of both graceful and harmonious labelings [4].

Let $G = (V, E)$ be a graph. A mapping $f : V(G) \rightarrow \{0, 1\}$ is called binary vertex labeling of $G$ and $f(v)$ is called the label of the vertex $v$ of $G$ under $f$.

For an edge $e = uv$, the induced edge labeling $f^* : E(G) \rightarrow \{0, 1\}$ is given by $f^*(e) = |f(u) - f(v)|$. Let $v_f(0)$ and $v_f(1)$ be the number of vertices of $G$ having labels 0 and 1 respectively under $f$ and $e_f(0)$ and $e_f(1)$ be the number of edges having labels 0 and 1 respectively under $f^*$.

The concept of cordial labeling was introduced by Cahit [4].

A binary vertex labeling of a graph $G$ is called a cordial labeling if $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$. A graph $G$ is cordial if it admits cordial labeling.
Cahit proved that some standard graphs are cordial in [5].

Sundaram, Ponraj and Somasundaram [15] have introduced the notion of prime cordial labeling [15].

A prime cordial labeling of a graph $G$ with vertex set $V$ is a bijection $f$ from $V$ to $\{1, 2, \ldots, |V|\}$ such that if each edge $uv$ assigned the label 1 if $\text{gcd}(f(u), f(v)) = 1$ and 0 if $\text{gcd}(f(u), f(v)) > 1$, then the number of edges labeled with 1 and the number of edges labeled with 0 differ by at most 1.

In [15], they have proved that some graphs are prime cordial.

Motivated by the concept of prime cordial labeling, we introduce a new special type of cordial labeling called divisor cordial labeling as follows.

Let $G=(V, E)$ be a simple graph and $f : V \rightarrow \{1, 2, \ldots |V|\}$ be a bijection. For each edge $uv$, assign the label 1 if either $f(u) \mid f(v)$ or $f(v) \mid f(u)$ and the label 0 if $f(u) \nmid f(v)$. $f$ is called a divisor cordial labeling if $|e_{f}(0) - e_{f}(1)| \leq 1$.

A graph with a divisor cordial labeling is called a divisor cordial graph.

We have established the divisor cordiality of standard graphs, some kinds of trees, cycle related graphs and some book graphs in each chapter. Also we identify the vertices of divisor cordial graphs
to the vertices of some other graphs and proved that the resulting graph is divisor cordial.

In chapter 2, we defined divisor cordial graphs and proved the standard graphs such as path, cycle, wheel, star and some complete bipartite graphs $K_{2,n}$ and $K_{3,n}$ are divisor cordial. We also proved that $K_n$ is not divisor cordial for $n \geq 7$. Also we proved subdivision of a star and bistar are divisor cordial.

We defined the following concepts related to divisor cordial graphs.

1. Divisor dominated cordial graph

2. Non-divisor dominated cordial graph

3. Strictly divisor dominated cordial graph

4. Strictly non-divisor dominated cordial graph

Various graphs are classified according to the above category.

We proved that the following results related to the identification of vertices of two graphs.

1. Let $G$ be any divisor cordial graph of even size. Then the graph $G \ast K_{1,n}$ obtained by identifying the central vertex of $K_{1,n}$ with that labeled 2 in $G$ is also divisor cordial.
2. Let $G$ be any divisor cordial graph odd size. If $n$ is even, then the graph $G * K_{1,n}$ obtained by identifying the central vertex of $K_{1,n}$ with that labeled with 2 in $G$ is also divisor cordial.

We proved that the graphs $G * K_{1,n}$, $G * K_{2,n}$ and $G * K_{3,n}$, are divisor cordial.

The content of this chapter was published in [24].

In chapter 3, we give the guarantee for existence of divisor cordial graphs by the following theorem.

*Given a positive integer $n$, there is a divisor cordial graph $G$ which has $n$ vertices.*

Also, we proved that the following special classes of graphs are divisor cordial.

(i) Corona (ii) Full binary trees.

Next, we present divisor cordial labeling for the new graph [23] denoted by $< K_{1,n}^{(1)}, K_{1,n}^{(2)} >$ and obtained by joining apex vertices of two stars to a new vertex. We extend this result for three copies of stars $< K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)} >$.

The content of this chapter was published in [25].

In chapter 4, we proved that the following cycle related graphs are divisor cordial.

(i) Dragon (ii) Wheel with two centres (iii) Fan (iv) Double fan
(v) *Shell graph* and (vi) *One point union of cycles*.

Also, we proved that the identification of vertices of a divisor cordial graphs to the vertices of above cycle related graphs are also divisor cordial.

The content of this chapter was communicated to [26].

In chapter 5, we proved that some special classes of book graphs such as books with triangular, rectangular (or quadrilateral), pentagonal and hexagonal pages are divisor cordial. Also, we proved the identification of vertices of a divisor cordial graphs to the vertices of these book graphs are also divisor cordial.