CHAPTER VIII

ACCEPTANCE SAMPLING PLANS
WHERE ACCEPTANCE CRITERION IS
THE WARRANTED LIFE TIME
ACCEPTANCE SAMPLING PLANS WHERE ACCEPTANCE CRITERION IS THE WARRANTED LIFE TIME

8.1 INTRODUCTION

The electronic industry has heavily prioritized enhancing the quality, lifetime and conforming rate (conforming to specifications) of electronic components, and thus, maximizing profitability. Various methods and approaches have been developed in the literature for assessing quality performance. One approach is to use the notion of process capability in acceptance plans discussed in previous chapter. The notion of aging for engineering systems has been the subject of considerable attention for many research workers. References may be made to various life testing plans available in literature and to that of the criterion for aging [Bryson and Siddiqui (1969)]. We come across many situations in real life where there exist a particular age $t_0$ (say, $t_0 > 0$) at (from) which deterioration of unit sets in. For example, the performance of an electric product may deteriorate after $t_0$ hours of service, making extensive overall maintenance of the equipment after $t_0$ hours. Therefore, there is a requirement of a minimum warrantee life of an electronic product for its acceptability.

In this chapter, an attempt has been made to develop an acceptance sampling plan where acceptance criterion is the minimum warrantee period. An item is assumed to be of satisfactory quality if its warrantee life is greater
than or equal to some specific value. Similarly, an item is unsatisfactory, if its warrantee life is less than the specified one. The proposed plan is based on the W-statistic defined and studied by and Sen and Srivastva (2001). The OC function of the plan has been derived by considering the asymptotic distribution of W-Statistic. The acceptance constant of the plan have been derived by taking single point on the OC curve for a known value of sample size n. Lastly, numerical examples have been included to illustrate the mathematical findings. For simplicity, the lifetime distribution of the product is assumed to be exponentially distributed.

8.2 W-STATISTIC AND ITS PROPERTIES

Let life time x is exponentially distributed with probability density function

\[ f(x; \theta) = \frac{1}{\theta} \exp(-x/\theta), \quad x > 0, \quad \theta > 0 \]

where \( \theta \) is mean life.

Suppose n units are put on life test and let \( X_1, X_2, X_3, \ldots, X_n \) be the observed failure times, respectively.

Define the kernels \( \phi_1 \) and \( \phi_2 \) of degree three and two respectively as

\[ \phi_1(X_1, X_2, X_3) = \mathbb{I}(X_1 > t_0)(X_2 - X_3)\mathbb{I}(X_2 > X_3) \quad (8.2.1) \]
and \( \phi_2 (X_1, X_2) = [X_1 - (X_2 + t_0)] I[X_1 > X_2 + t_0] \) \((8.2.2)\)

where \( t_0 \) is the minimum warrantee life and \( I \), indicator function, is defined as \( I(a > b) = 1 \) and \( I(a \leq b) = 0 \).

Now, using equation (8.2.1) and (8.2.2), the W-function [Sen and Srivastava (2001)] may be defined as follows:

\[
w = v_1 - v_2
\]

(8.2.3)

where,

\[
v_1 = \left( \frac{1}{n p_3} \right) \sum_1 \phi_1(X_{i1}, X_{i2}, X_{i3})
\]

\[
v_2 = \left( \frac{1}{n p_2} \right) \sum_2 \phi_2(X_{i1}, X_{i2})
\]

\[\sum_1 \] is the summation over all the \( np_3 \) permutations of 3 integers \( \{i_1, i_2, i_3\} \) chosen from the set of \( n \) integers \( \{1, 2, \ldots, n\} \) and \[\sum_2 \] is the summation over all the \( np_2 \) permutations of 2 integers \( \{i_1, i_2\} \) chosen from the set of \( n \) integers \( \{1, 2, \ldots, n\} \).

Sen and Srivastava (2001) have shown that \( \sqrt{n} w \) is asymptotically normally distributed with mean zero and variance \( \sigma^2 \). Where

\[
\sigma^2 = \frac{7e^{-t_0/\theta} + 4e^{-3t_0/\theta} - 11e^{-2t_0/\theta}}{12(t_0/\theta)^2 n}
\]

(8.2.4)
That is, \( w \sim N(0, \sigma^2/n) \)

Now, we define warrantee index for warrantee time \( t \) as
\[
W_I = \frac{t}{\theta}
\]

where \( t \) is variable and \( \theta \) is fixed (known). It may be noted that an item is satisfactory (or conforming) if its warrantee index is greater than or equal to \( t/\theta \), otherwise, it is unsatisfactory.

8.3 THE PROPOSED PLAN AND OC FUNCTION

The hypothesis to be tested is as follows:

\[
H_0 : \frac{t}{\theta} = \frac{t_0}{\theta} \quad \text{against} \quad H_1 : \frac{t}{\theta} = \frac{t_1}{\theta}
\]

or

\[
H_0 : t = t_0 \quad \text{against} \quad H_1 : t = t_1 \quad (\theta \text{ is fixed})
\]

The proposed plan proceeds as follows:

Accept the lot, if \( (w/t \leq c) \)

Reject the lot, otherwise.
The producer’s risk and consumer’s risk are found as
\[ \alpha = P[w / t > c / t = t_0] \] \hspace{1cm} (8.3.1)
and \[ \beta = P[w / t < c / t = t_0] \] \hspace{1cm} (8.3.2)

Under H_0, \( w / t \) is asymptotically normally distributed with mean zero and variance \( \sigma^2 \),

where \[ \sigma^2 = \frac{7e^{-t_0/\theta} + 4e^{-3t_0/\theta} - 11e^{-2t_0/\theta}}{12(t_0 / \theta)^2 n} \] \hspace{1cm} (8.3.3)

The OC function of the plan is given by

\[ P_A = P(w / t \leq c) \]
\[ P_A = P[z \leq \frac{c \lambda}{\sigma}] \] \hspace{1cm} (8.3.4)

Thus, at \( t = t_0 \), we have \( \lambda = t / t_0 = 1 \) and consequently,

\[ P_A = P[z \leq \frac{c}{\sigma}] \]
\[ = \phi \left( \frac{c}{\sigma} \right) \] \hspace{1cm} (8.3.5)
8.4 ILLUSTRATION AND DISCUSSION OF THE RESULT

In order to facilitate the users of the new criteria, the value of the acceptance constant c has been computed from (8.3.5) for different combinations of n, t/θ and α = .05.

Now, to illustrate the application of the various results obtained in the previous sections, we shall consider the following plan:

- n = 30, t₀ = 100, θ = 1000, and α = .05

The value of $W.I = t₀/θ = 0.1$. The value of c corresponding to $t₀ /θ = .041$ is found from Table (8.1) as $c = 0.4677$.

The OC curve of the plan has been shown in Table (8.2) for different value of n. It is observed that from the table that probability of acceptance increases with increasing sample size n. The development of acceptance sampling where the acceptance criterion is the warrantee life has been successfully carried out. The calculation of W-Statistic has been illustrated by Sen and Srivastava (2001).
TABLE (8.1)

ACCEPTANCE CONSTANT $c$ FOR

$\alpha = .05$, $\lambda = 1$

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<th>$n$</th>
<th>$t/\theta=1$</th>
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<th>0.50</th>
<th>0.20</th>
<th>0.10</th>
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TABLE (8.2)
THE OC FUNCTION FOR DIFFERENT VALUES OF $t/\theta$ AND $n$

$c = 0.2, \quad \lambda = 1$

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<th>$t/\theta$</th>
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<th>$n=90$</th>
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<td>0.9999</td>
<td>1.000</td>
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<td>0.9994</td>
<td>0.9999</td>
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<td>0.9991</td>
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