CHAPTER – 3

IMPACT OF STRESS EXPONENT ON

STEADY STATE CREEP IN A ROTATING

COMPOSITE DISK

3.1 GENERAL

In most of the applications of rotating disk, it operates under elevated temperature and is simultaneously subjected to high stresses caused by centrifugal loading. As a result, the disk undergoes creep deformations, which affect performance of the system (Laskaj et al., 1999; Farshi et al., 2004; Gupta et al., 2005; Gupta et al., 2007).

Singh and Ray (2000; 2001; 2002; 2003a; b; 2004) analyzed steady state creep response of a rotating disk made of aluminum/aluminum alloy based metal matrix composites by using Norton’s power law creep. The selection of Norton’s power law to describe steady state creep behavior of aluminum or its alloy based composites has been strongly objected due to high and often variable values of apparent stress exponent and activation energies observed in these composites (Tjong and Ma, 2000). In order to rationalize the strong stress and temperature dependence of creep rate as observed in discontinuously reinforced aluminum/aluminum alloy matrix composites, the concept of an effective stress has been adopted widely (Gupta et al., 2005). The value of true stress exponent
appearing in creep law based on effective stress has usually been selected as 3, 5 and 8. Gupta et al (2004a; b; 2005; 2007) analyzed steady state creep response of the rotating disk by using substructure-invariant model with a stress exponent of 8. A detailed analysis of the published creep data for aluminum-based composites suggests that the substructure-invariant model is untenable because it leads to consistently higher value of activation energies for creep than anticipated for lattice self-diffusion in aluminum. It is suggested that the creep behavior of metal matrix composites could be analyzed in a better way by assuming the flow to be controlled by creep of matrix material so that the true stress exponent, $n$, is ~3 or ~5 (Cadek et al, 1995; Li and Langdon, 1999). In this context, it is decided to investigate the impact of stress exponent ($n$), on the creep response of a rotating composite disk. In this segment of the study, we have analyzed steady state creep in a rotating disk made of Al-SiCp (‘p’ refers to particle shape of SiC) while describing its creep behavior by threshold stress based law, with the values of $n$ as 3, 5 and 8. The results obtained are compared to bring out the impact of stress exponent ($n$) on the creep stresses and creep rates in the rotating composite disk.

3.2 CREEP LAW AND ESTIMATION OF CREEP PARAMETERS

In the present investigation, the material of disk (i.e. Al-SiCp) is assumed to undergo steady state creep following a well documented threshold stress based creep law given by:

$$
\dot{\varepsilon} = A \left( \frac{\sigma - \sigma_0}{E} \right)^n \exp \left( \frac{-Q}{RT} \right)
$$

(3.1)
where the symbols $\dot{\varepsilon}, \bar{\sigma}, \sigma_0, A, n, Q, E, R$ and $T$ are already described in section 2.3.6.

The true stress exponent ($n$) appearing in Eqn. (3.1) is usually selected as 3, 5 and 8, which correspond to three well-documented creep cases for metals and alloys: (i) $n = 3$ for creep controlled by viscous glide processes of dislocation, (ii) $n = 5$ for creep controlled by high temperature dislocation climb (lattice diffusion), and (iii) $n = 8$ for lattice diffusion-controlled creep with a constant structure (Tjong and Ma, 2000).

The creep law given by Eqn. (3.1) may alternatively be written as:

$$\dot{\varepsilon} = [M(\bar{\sigma} - \sigma_0)]^n$$

(3.2)

where $M = \frac{1}{E} \left( A \exp \frac{-Q}{RT} \right)^{\frac{1}{n}}$

The creep parameters $M$ and $\sigma_0$ appearing in Eqn. (3.2), describing steady state creep behavior, are dependent on the type of material and operating temperature ($T$). In a composite, the dispersoid size ($P$) and content ($V$) are the primary material variables deciding the values of these parameters. In the present study, the values of parameters $M$ and $\sigma_0$ are extracted from the available creep results reported for Al-SiC$_p$ under uniaxial loading (Pandey et al., 1992). The values of $P$, $V$ and $T$ in this investigation are assumed respectively as 1.7 $\mu$m, 10 vol% and 623 K. The values of $M$ and $\sigma_0$ for the above mentioned combination of $P$, $V$ and $T$, are extracted from the published creep results for Al-SiC$_p$ (Pandey et al., 1992) by representing them on $\dot{\varepsilon}^{1/n}$ versus $\sigma$ plot (Fig. 3.1). Following the linear extrapolation technique (Lagneborg and Bergman, 1976), the values of
parameters $M$ and $\sigma_0$ have been estimated from the slope and intercepts of the best fitted lines shown in Fig. 3.1 and are reported in Table 3.1.

<table>
<thead>
<tr>
<th>Stress Exponent ($n$)</th>
<th>Creep parameter ($M$)</th>
<th>Threshold Stress ($\sigma_0$) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.00078 ($s^{-1/3}$/MPa)</td>
<td>22.84</td>
</tr>
<tr>
<td>5</td>
<td>0.00435 ($s^{-1/5}$/MPa)</td>
<td>19.83</td>
</tr>
<tr>
<td>8</td>
<td>0.00963 ($s^{-1/8}$/MPa)</td>
<td>15.25</td>
</tr>
</tbody>
</table>

3.3 ANALYSIS OF CREEP IN ROTATING COMPOSITE DISK

Let us assume a constant thickness disk made of Al–SiC$_p$ with the inner and outer radii ‘$a$’ and ‘$b$’ respectively, and rotating at an angular speed $\omega$ (radian/sec). For the purpose of analysis the following assumptions are made:

(i) Material of the disk is incompressible, isotropic and has uniform distribution of SiC$_p$ in aluminum matrix.

(ii) Stresses at any point in the disk remain constant with time i.e. steady state condition is assumed.

(iii) Elastic deformations in the disk are small and therefore neglected as compared to creep deformations.

(iv) The disk thickness is very small compared to its diameter, therefore, the axial stress ($\sigma_z$) in the disk is assumed to be zero.
The generalized constitutive equations for creep in an isotropic composite under biaxial state of stress (i.e. $\sigma_z = 0$) takes the following form when reference frame is along the principal directions $r, \theta$ and $z$ (Gupta et al, 2004a),

$$\dot{\varepsilon}_r = \frac{\varepsilon}{2\sigma}[2\sigma_r - \sigma_\theta]$$

$$\dot{\varepsilon}_\theta = \frac{\varepsilon}{2\sigma}[2\sigma_\theta - \sigma_r]$$

$$\dot{\varepsilon}_z = \frac{\varepsilon}{2\sigma}[-\sigma_r - \sigma_\theta]$$

where $\dot{\varepsilon}_r, \dot{\varepsilon}_\theta, \dot{\varepsilon}_z$ and $\sigma_r, \sigma_\theta, \sigma_z$ are the strain rates and stresses respectively along $r, \theta$ and $z$ directions, as indicated by the respective subscripts.

It is assumed that the material of disk is isotropic and yields according to von–Mises criterion. Therefore, the effective stress ($\bar{\sigma}$) in the disk under biaxial state of stress is given by (Gupta et al, 2005),

$$\bar{\sigma} = \frac{1}{\sqrt{2}}\left[\sigma^2_\theta + \sigma^2_r + (\sigma_r - \sigma_\theta)^2\right]^{1/2}$$

Substituting the values of $\dot{\varepsilon}$ from Eqn. (3.2) and $\bar{\sigma}$ from Eqn. (3.4) into first equation amongst set of Eqs. (3.3), we get,

$$\dot{\varepsilon}_r = \frac{d\dot{u}_r}{dr} = \frac{(2x - 1)}{2(x^2 - x + 1)^{1/2}} [M(\bar{\sigma} - \sigma_0)]^n$$

where $x(=\sigma_r/\sigma_\theta)$ is the ratio of radial and tangential stresses and $\dot{u}_r (=du/dt)$ is the radial deformation rate.

Similarly, the second equation amongst set of Eqs. (3.3) may be written as,

$$\dot{\varepsilon}_\theta = \frac{\dot{u}_r}{r} = \frac{(2 - x)}{2(x^2 - x + 1)^{1/2}} [M(\bar{\sigma} - \sigma_0)]^n$$
Dividing Eqn. (3.5) by Eqn. (3.6) and integrating the resulting equation between limits \(a\) to \(r\),

\[
\dot{u}_r = \dot{u}_a \exp \left[ \int_a^r \frac{\phi(r)}{r} \, dr \right]
\]  

(3.7)

where \(\dot{u}_a\) is the radial deformation rate at the inner radius and \(\phi(r) = \frac{(2x-1)}{(2-x)}\).

Dividing Eqn. (3.7) by \(r\) and equating it to Eqn. (3.6), we get,

\[
\frac{\dot{u}_a}{r} \exp \left[ \int_a^r \frac{\phi(r)}{r} \, dr \right] \frac{r}{2(2-x-x+1)^{1/2}} = \frac{(2-x)M(\sigma - \sigma_0)^n}{2(x^2-x+1)^{1/2}}
\]

On simplifying, one obtains the tangential stress,

\[
\sigma_\theta = \frac{\dot{u}_a^{1/n} \psi_1(r)}{M} + \psi_2(r)
\]  

(3.8)

where,

\[
\psi_1(r) = \frac{\psi(r)^{1/n}}{(x^2-x+1)^{1/2}}; \quad \psi_2(r) = \frac{\sigma_\theta}{(x^2-x+1)^{1/2}}
\]  

(3.9)

and,

\[
\psi(r) = \frac{2(x^2-x+1)^{1/2}}{r(2-x)} \exp \left[ \int_a^r \frac{\phi(r)}{r} \, dr \right]
\]  

(3.10)

Considering the equilibrium of forces acting on an element of the disk (Fig. 3.2), one obtains the following force equilibrium equation (Gupta et al, 2004a),

\[
\frac{d}{dr}(r\sigma_r) - \sigma_\theta + \rho \omega^2 r^2 = 0
\]  

(3.11)

where \(\rho\) is the density of composite disk.
Assuming that the disk is connected to the shaft by means of splines where small axial movement is allowed, a free-free condition applies. Therefore, the boundary conditions are,

\[ \sigma_r = 0 \quad \text{at} \quad r = a \quad \text{and} \quad \sigma_r = 0 \quad \text{at} \quad r = b \]  

(3.12)

Integrating Eqn. (3.11) from \( a \) to \( b \) under the imposed boundary conditions given in Eqn. (3.12), we get,

\[
\int_{a}^{b} \sigma_\theta \, dr = \frac{\rho \omega^2 (b^3 - a^3)}{3} \tag{3.13}
\]

Substituting the value of \( \sigma_\theta \) from Eqn. (3.8) into above equation, one gets,

\[
\frac{u_a^{1/2}}{M} \left( b - a \right) \sigma_{\theta_{avg}} - \int_{a}^{b} \psi_2 (r) \, dr
\]

\[
\int_{a}^{b} \psi_1 (r) \, dr
\]

\[ \frac{u_a^{1/2}}{M} = \left( \frac{1}{b - a} \int_{a}^{b} \sigma_\theta \, dr \right) \]  

(3.14)

where, \( \sigma_{\theta_{avg}} = \left( \frac{1}{b - a} \int_{a}^{b} \sigma_\theta \, dr \right) \) is the average tangential stress in the composite disk.

The tangential stress \( (\sigma_\theta) \) in the disk is obtained after substituting the value of \( \frac{u_a^{1/2}}{M} \) from Eqn. (3.14) into Eqn. (3.8) as,

\[
\sigma_\theta = \left( \frac{b - a}{M} \right) \sigma_{\theta_{avg}} - \int_{a}^{b} \psi_2 (r) \, dr
\]

\[ + \int_{a}^{b} \psi_1 (r) \, dr \]  

(3.15)

Integrating Eqn. (3.11) from \( a \) to \( r \), we get,

\[
\sigma_r = \frac{1}{r} \int_{a}^{r} \sigma_\theta \, dr - \frac{\rho \omega^2 (r^3 - a^3)}{3r} \]  

(3.16)
The tangential and radial stresses at any point within the composite disk are estimated respectively from Eqs. (3.15) and (3.16). Using these values, the strain rates \( \dot{e}_r \) and \( \dot{e}_\theta \) are calculated respectively from Eqs. (3.5) and (3.6).

### 3.4 NUMERICAL COMPUTATIONS

Following the procedure described in previous section, the stresses and strain rates in the disk are evaluated by an iterative numerical scheme illustrated in Fig. 3.3. To find the first approximation of \( x \), i.e. \([x]_1\), to be used in Eqn. (3.10) in the first iteration, it is assumed that \( \sigma_\theta = \sigma_{\theta v} \) in Eqn. (3.16) and on integration one gets the first approximation of \( \sigma_r \), i.e. \([\sigma_r]_1\). The subscript has been used to denote the iteration number. Dividing \([\sigma_r]_1\) by \( \sigma_{\theta v} \), one gets \([x]_1\), which is substituted in Eqn. (3.10) for \( x \) to obtain \([\psi]_1\), to be used in the first iteration. Using \([\psi]_1\) in Eqn. (3.9), \([\psi_1]_1\) and \([\psi_2]_1\) are estimated, which are substituted in Eqn. (3.14) to obtain \([\hat{u}_a]^{1/\alpha} \). Using \([\psi_1]_1\), \([\psi_2]_1\) and \([\hat{u}_a]^{1/\alpha} \) in Eqn. (3.15), \([\sigma_\theta]_1\) is obtained. Substituting \([\sigma_\theta]_1\) for \([\sigma_\theta]_1\) in Eqn. (3.16), the second approximation of \( \sigma_r \), i.e. \([\sigma_r]_2\), is obtained by numerical integration, which is used to obtain the second approximation of \( x \), i.e. \([x]_2\). The iteration is continued till the process converges and yields the values of stresses at different points of the radius grid. For rapid convergence 75% of the value of \( \sigma_\theta \) obtained in the current iteration has been mixed with 25% of the value of \( \sigma_\theta \) obtained in the previous iteration for use in the next iteration. Therefore, the strain rates in the disk are estimated from Eqs. (3.5) and (3.6).
3.5 RESULTS AND DISCUSSION

A computer code based on the mathematical formulation presented in section 3.3 has been developed to obtain the distribution of steady state stresses and strain rates in the rotating disk for different values of true stress exponent (i.e. \( n = 3, 5 \) and 8).

3.5.1 Validation

Before discussing the results obtained in this study, it is necessary to check the validity of analysis carried out and the software developed. To achieve this goal, the radial and tangential creep strains have been computed for a rotating steel disk by following the current analysis scheme and compared with the available experimental results for steel disk (Wahl et al., 1954), the parameters and operating conditions for which are mentioned in Table 3.2.

In order to evaluate the creep parameters \( M \) and \( \sigma_0 \), reported in Table 3.2, for steel disk, Eqn. (3.2) has been integrated to obtain,

\[
\bar{\varepsilon} = \left[ M (\sigma - \sigma_0) \right]^n \int_0^t f(t) dt
\]

where \( \bar{\varepsilon} \) is the effective strain and \( f(t) \) is a function of time \( t \).

Table 3.2: Parameters and operating conditions for steel disk

<table>
<thead>
<tr>
<th>Parameters for steel disk:</th>
<th>Density ((\rho)) = 7,823.18 kg/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Disk radii: ( a = 31.75 , mm ), ( b = 152.4 , mm )</td>
<td></td>
</tr>
<tr>
<td>Creep parameters: ( M = 4.72 \times 10^{-4} , s^{-1/8} / MPa )</td>
<td></td>
</tr>
<tr>
<td>( \sigma_0 = -54.05 , MPa )</td>
<td></td>
</tr>
<tr>
<td>Operating conditions:</td>
<td>Disk rpm = 15,000</td>
</tr>
<tr>
<td></td>
<td>Operating temperature = 810.78 K</td>
</tr>
<tr>
<td></td>
<td>Creep duration = 180 hrs</td>
</tr>
</tbody>
</table>
Similar to the study of Wahl et al (1954), the function $f(t)$ is taken as unity. Wahl et al (1954) in their experimental study on steel disk observed that the average strain at the end of 180 hrs, corresponding to the mean true stress $(\bar{\sigma}) = 25,150$ psi, is 0.0109 in/in and at a mean stress $(\bar{\sigma}) = 29,450$ psi, the strain is 0.029 in/in. Using above data in Eqn. (3.17), the creep parameters $M$ and $\sigma_0$ are obtained as $3.22 \times 10^{-6} \text{ s}^{-1} / \text{psi} = 4.72 \times 10^{-4} \text{ s}^{-1/8} / \text{MPa}$ and -7,898.47 psi ($= -54.05 \text{ MPa}$) respectively.

The parameters reported in Table 3.2 have been used in the computer program developed in this study to calculate the tangential and radial creep strains in the steel disk. A good agreement is observed in Fig. 3.4 between the results obtained by the procedure outlined in this investigation and the experimental results reported by Wahl et al (1954), thereby, validating the analysis presented in section 3.3.

### 3.5.2 Error in Estimation of Creep Parameters

The error associated with the values of creep parameters $M$ and $\sigma_0$, reported in Table 3.1 for different values of stress exponent, has been estimated. For this purpose, the creep parameters reported in Table 3.1 have been substituted in the constitutive creep model given by Eqn. (3.2), to estimate the strain rates corresponding to the observed values of experimental stress reported by Pandey et al (1992) for Al-SiC$_P$ for the selected combination of material parameters and temperature ($P = 1.7 \mu m$, $V = 10 \text{ vol\%}$ and $T = 623 K$).

The estimated strain rates, corresponding to different values of stress exponent, have been subtracted from the experimental strain rates observed by Pandey et al (1992), to estimate the error associated with the prediction of strain.
rates. Fig. 3.5 shows the error in strain rate corresponding to different stress levels for different values of stress exponent. It is observed that the error in strain rate increases as the level of stress increases from 24.28 MPa (level 1) to 31.17 MPa (level 5). At lower stress levels (levels 1 and 2), the error corresponding to different values of stress exponents is almost the same and is practically insignificant. However, at higher stress levels (levels 3 and 4), the error is the lowest for $n = 5$ and the highest for $n = 8$. At these stress levels the estimated values of strain rate are relatively lower than that observed experimentally (i.e. negative error). However, corresponding to the highest stress level of 31.17 MPa (level 5), the error for $n = 8$ is the lowest and the highest for $n = 3$. Therefore, it appears that except at higher stress level (level 5), the steady state creep in Al-SiC$_p$ could be described in a better way by selecting the value of stress exponent $n = 5$ rather than $n = 3$ or 8.

### 3.5.3 Effect of Stress Exponent on Creep Behavior of Rotating Disk

Figs. 3.6 to 3.10 show creep response of the composite disk for different values of stress exponent. The radial stress in the disk increases from zero at the inner radius, reaches a maximum, before decreasing to zero again at the inner radius, under the imposed boundary conditions (Eqn. 3.12). The radial stress, Fig. 3.6, estimated using $n = 3$ is relatively higher over the entire disk radius as compared to those estimated by assuming $n = 5$ or 8. Whereas the radial stress estimated using $n = 8$ is relatively lower than those estimated by selecting $n = 3$ or 8. The maximum difference observed in radial stress, corresponding to $n = 3$ and $n = 8$, is about 2.5 MPa at a radius of 63.92 mm. Fig. 3.7 shows the variation of tangential stress with radial distance. As one moves from the inner to outer radius
of disk, the tangential stress increases, reaches a maximum, before decreasing again towards the outer radius. Near the inner radius, the tangential stress corresponding to \( n = 3 \) is relatively higher and is relatively lower for \( n = 8 \), similar to that observed for radial stress in Fig. 3.6. However, towards the outer radius, the trend of variation is just the reverse to those observed at the inner radius. The values of tangential stress in the disk, estimated using \( n = 5 \), lie in between to those estimated for \( n = 3 \) and \( n = 8 \). In the middle of the disk, around a radius of 84 mm, the tangential stress estimated using various values of stress exponent remains the same. The maximum variation observed in tangential stress, corresponding to \( n = 3 \) and 8, is around 9 MPa at the inner radius of the disk. The effective stress (Fig. 3.8) in the disk decreases on moving from the inner to outer radius of the disk. The variation of effective stress for different values of stress exponent (Fig. 3.8) is similar to those observed for tangential stress in Fig. 3.7.

The radial strain rate (compressive) in the disk (Fig. 3.9) decreases on moving from the inner to outer radius, becomes minimum in the middle of the disk, following by increase with further increase in radius. The radial strain rate is significantly affected by varying the value of stress exponent. Throughout the disk, the radial strain rate corresponding to \( n = 8 \) is the highest and the lowest for \( n = 3 \), with intermediate values corresponding to \( n = 5 \). The radial strain rate corresponding to \( n = 8 \) is higher by about two order of magnitude than that estimated by assuming \( n = 3 \). The tangential strain rate in the disk decreases, as one moves from the inner to outer radius of the disk (Fig. 3.10). The effect of stress exponent on tangential strain rate is similar to those observed for radial strain rate in Fig. 3.9.
Fig. 3.1: Variation of $\dot{\varepsilon}^{1/n}$ versus $\sigma$ in Al-SiCp for different values of stress exponent
Fig. 3.2: Free body diagram of an element of the rotating disk
LEGENDS:
ITER = Iteration no.
\[ E_{RR} = \frac{[\sigma_0]_{\text{ITER}} - [\sigma_0]_{\text{ITER-1}}}{[\sigma]_{\text{ITER-1}}} \]
\( h = \) Limiting Value of ERR (=0.01)
ITM = Maximum no. of iteration (= 50)

**Fig. 3.3**: Numerical scheme of computation
Fig. 3.4: Comparison of theoretical (current analysis) and experimental (Wahl et al, 1954) strains in steel disk
Fig. 3.5: Error in strain rate corresponding to different stress levels for various values of stress exponent.
Fig. 3.6: Effect of stress exponent on radial stress
Fig. 3.7: Effect of stress exponent on tangential stress
Fig. 3.8: Effect of stress exponent on effective stress
Fig. 3.9: Effect of stress exponent on radial strain rate
Fig. 3.10: Effect of stress exponent on tangential strain rate