Chapter 5

Signature of new physics in $B \to \phi \pi$ decay

Over the years, there has been profound interest in the search for physics beyond the SM. The observed discrepancy between the measured $S_{\phi K_S}$ and $S_{\psi K_S}$ [88] already gave an indication of the possible existence of NP in the $B \to \phi K_S$ decay amplitude and this has, in one way, motivated many to carry out an intensive search for NP. Although the presence of NP in the $b$-sector is not yet firmly established, but there exists several smoking gun signals [89] which will be verified in the upcoming LHCb experiment or super $B$ factories. As stated earlier, one of the ways of searching for new physics is by studying the rare decay modes arising at the one-loop level, which are induced by flavor changing neutral current (FCNC) transitions. Thus the study of the same will provide us with an excellent testing ground for NP. Therefore, it is interesting to examine as many different rare decay channels as possible to have an indication of new physics.

In this study, we explore the effect of the extra fourth generation of quarks and FCNC mediated $Z(Z')$ boson(s) on the rare decay mode $B^- \to \phi \pi^-$, which is a pure penguin induced process, mediated by the quark level transition $b \to d\bar{s}s$. The interesting feature of this process is that it is dominated by the electroweak penguin contributions as the QCD penguins are Okubo-Zweig-Iizuka (OZI) suppressed and therefore expected to be highly
suppressed in the SM. It therefore serves as a suitable place to search for new physics. At present, only the upper limit of its branching ratio is known [90]

\[
\text{BR}(B^- \to \phi \pi^-) < 0.24 \times 10^{-6}.
\] (5.1)

This decay mode has been analyzed both in the SM [91] and in various extensions of it [92] where it has been found that in some of these new physics models, the branching ratio can be enhanced significantly from its corresponding SM value.

5.1 The standard model result

In order to discuss the effect of the fourth quark generation and FCNC mediated $Z(Z')$ boson, we would first like to present the SM result using the QCD factorization [93]. As the decay mode $B^- \to \phi \pi^-$ proceeds through the quark level transition $b \to d \bar{s}s$ and is a pure penguin induced process occurring at the one-loop level, the relevant effective Hamiltonian describing this process is given by

\[
\mathcal{H}^{\text{SM}}_{\text{eff}} = \frac{G_F \sqrt{2}}{2} V_{pb} V^*_{pd} \sum_{i=3}^{10} C_i(\mu) O_i,
\] (5.2)

where $p = u, c$, $C_i(\mu)$'s are the Wilson coefficients evaluated at the $b$-quark mass scale and $O_i$'s are the QCD and electroweak penguin operators.

In QCD factorization [93], the decay amplitude can be represented in the form

\[
A(B^- (p_B) \to \phi(\epsilon, p_1) \pi^-(p_2)) = -i \frac{G_F}{\sqrt{2}} 2m_\phi f_\phi (\epsilon^* \cdot p_B) F_{B\pi}^+ (0)
\times \sum_{p=u,c} \lambda_p (\alpha^p_3 - \frac{1}{2} \alpha^p_{3,\text{EW}}),
\] (5.3)

where $\lambda_p = V_{pb} V^*_{pd}$, the QCD coefficients $\alpha^p_{3(3,\text{EW})}$ are related to the Wilson coefficients as defined in [93] and $F_{B\pi}^+$ is the form factor describing $B \to \pi$ transition. It should be noted that the QCD coefficients contributing to
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$B^- \to \phi \pi^-$ are independent of $p = u, c$, (i.e., the virtual particles in the loop). Therefore, one can also represent the above amplitude using CKM unitarity $\lambda_u + \lambda_c + \lambda_t = 0$, as

$$A(B^- \to \phi \pi^-) = i \frac{G_F}{\sqrt{2}} 2m_\phi f_\phi (\epsilon^* \cdot p_B) F_+^{B\pi}(0) \lambda_t (\alpha_3 - \frac{1}{2} \alpha_{3,EW}) ,$$

(5.4)

where we have now omitted the superscripts on $\alpha$’s. The above amplitude can be simplified by replacing $2m_\phi \epsilon^* \cdot p_B \to m_B^2$. The branching ratio thus can be obtained using the formula

$$\text{BR}(B^- \to \phi \pi^-) = \frac{\tau_B}{16\pi m_B} |A(B^- \to \phi \pi^-)|^2 ,$$

(5.5)

where $\tau_B$ is the lifetime of $B^-$ meson. Another possible observable in this decay mode is the direct CP violation parameter, defined as

$$A_{CP} = \frac{\Gamma(B^+ \to \phi \pi^+) - \Gamma(B^- \to \phi \pi^-)}{\Gamma(B^+ \to \phi \pi^+) + \Gamma(B^- \to \phi \pi^-)} .$$

(5.6)

In order to have nonzero direct CP violation, it is necessary that the corresponding decay amplitude should contain at least two interfering contributions with different strong and weak phases. Since in the SM this decay mode does not have two such different contributions in its amplitude, the direct CP violation turns out to be identically zero.

For the numerical evaluation, we use the input parameters as given in the S4 scenario of QCD factorization approach [93]. The particle masses and lifetime of the $B$ meson are taken from [33]. The value of the form factor at zero recoil is taken as $F_+^{B\pi}(0) = 0.28$. The value of the CKM matrix elements used are $|V_{ub}| = 3.96 \times 10^{-3}$, $|V_{ud}| = 0.97383$, $|V_{cb}| = 42.21 \times 10^{-3}$, $|V_{cd}| = 0.2271$ and $\gamma$ the phase associated with $V_{ub}$ as 70°. With these values as input parameters, the branching ratio obtained in the SM is

$$\text{BR}^{SM}(B^- \to \phi \pi^-) = 4.45 \times 10^{-9} ,$$

(5.7)

which is quite below the experimental upper limit as given in Eq. (5.1).
5.2 In the presence of new physics

Now in the presence of NP, the transition amplitude (5.4) receives an additional contribution and can be symbolically represented as

\[ A^T(B^- \rightarrow \phi\pi^-) = A^{SM} + A^{NP} = A^{SM}(1 + r \, e^{i\delta} \, e^{-i(\beta - \phi)}) , \] (5.8)

where \( \beta \) is the weak phase of the SM amplitude i.e., we have used \( V_{td} = |V_{td}| e^{-i\beta} \) with the value \( \beta = 0.375 \), \( \phi \) is the weak phase associated with the NP amplitude and \( \delta \) is the relative strong phase between these two amplitudes. It should be noted that the strong phases are generated by the final state interactions (FSI) and at the quark level they arise through absorptive parts of the perturbative penguin diagrams. Furthermore, \( r \) denotes the magnitude of the ratio of NP to SM amplitude. Thus, we obtain the CP averaged branching ratio \( \langle \text{BR} \rangle \equiv \frac{\text{BR}(B^- \rightarrow \phi\pi^-) + \text{BR}(B^+ \rightarrow \phi\pi^+)}{2} \) including the new physics contribution as

\[ \langle \text{BR} \rangle = \text{BR}^{SM}(1 + r^2 + 2r \cos(\beta - \phi) \cos \delta) , \] (5.9)

where \( \text{BR}^{SM} \) is the SM branching ratio. It can be seen from the above equation that if \( r \) is sizable, the branching ratio could be significantly enhanced from its SM value in the presence of new physics. The direct CP violation parameter (5.6) in the presence of NP becomes

\[ A_{CP} = \frac{2r \sin \delta \sin(\phi - \beta)}{1 + r^2 + 2r \cos \delta \cos(\phi - \beta)} . \] (5.10)

5.2.1 Effect of a fourth quark generation

We now consider the effect of a sequential fourth generation of quarks [94]. This model is an extension of the SM with the addition of a fourth quark generation. It retains all the features of the SM except that it brings into existence the new members denoted by \( (t', b') \). The fourth up-type quark \( (t') \) like \( u, c, t \) quarks contributes in the \( b \rightarrow d \) transition at the loop level
and hence will modify the SM result. The effect of the fourth generation of quarks in various $B$ decays is extensively studied in the literature [95, 96].

Due to the additional fourth generation, there will be mixing among the new $b'$ quark and the three down-type quarks of the SM and the resulting mixing matrix will be a $4 \times 4$ matrix. Accordingly, the unitarity condition becomes $\lambda_u + \lambda_c + \lambda_t + \lambda_{t'} = 0$ and thus the effective Hamiltonian modifies as

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} \left[ V_{tb} V^*_{td} \sum C_i O_i + V_{t'b} V^*_{t'd} \sum C'_{ti} O_i \right] , \quad (5.11)$$

where $C'_{ti}$ are the new Wilson coefficients arising due to the $t'$ quark in the loop. The values of these Wilson coefficients at the $M_W$ scale can be obtained from the corresponding contributions from the $t$ quark by replacing the mass of $t$ quark in the Inami Lim functions [14] by $t'$ mass (here we neglect the renormalization group (RG) evolution of these coefficients from $t'$ mass scale to the weak scale $M_W$). These values can then be evolved to the $m_b$ scale using RG equation [10], as

$$\vec{C}(m_b) = U_5(m_b, M_W, \alpha) \vec{C}(M_W) , \quad (5.12)$$

where $\vec{C}$ is the $10 \times 1$ column vector of the Wilson coefficients and $U_5$ is the five flavor $10 \times 10$ evolution matrix. The explicit forms of $\vec{C}(M_W)$ and $U_5(m_b, M_W, \alpha)$ are given in [10]. We briefly present the method here. The renormalization group equation for the Wilson coefficients $\vec{C}$ is given as

$$\frac{d}{d \ln \mu} \vec{C} = \gamma T(g) \vec{C}(\mu) , \quad (5.13)$$

which can be solved with the help of the $U$ matrix

$$\vec{C}(\mu) = U(\mu, M_W) \vec{C}(M_W) , \quad (5.14)$$

where $\gamma T(g)$ is the transpose of the anomalous dimension matrix $\gamma(g)$ and $g$ is the QCD coupling. With the help of $dg/d \ln \mu = \beta(g)$, $U$ obeys the same
signature of new physics in $B \to \phi \pi$ decay equation as $\tilde{C}(\mu)$. We expand $\gamma(g)$ to the first two terms in the perturbative expansion

$$\gamma(\alpha_s) = \gamma(0)\frac{\alpha_s}{4\pi} + \gamma(1)\left(\frac{\alpha_s}{4\pi}\right)^2.$$  

To this order the evolution matrix $U(\mu, m)$ is given by

$$U(\mu, m) = \left(1 + \frac{\alpha_s(\mu)}{4\pi}J\right)U(0)(\mu, m)\left(1 - \frac{\alpha_s(m)}{4\pi}J\right),$$

where $U(0)$ is the evolution matrix in leading logarithmic approximation and the matrix $J$ expresses the next-to-leading corrections. We have

$$\gamma_{D}^{(0)} = V^{-1}\gamma_{D}^{(0)}T,$$

where $V$ diagonalizes $\gamma_{D}^{(0)}T$, i.e., $\gamma_{D}^{(0)} = V^{-1}\gamma_{D}^{(0)}TV$ and $\vec{\gamma}_{D}^{(0)}$ is the vector containing the diagonal elements of the diagonal matrix $\gamma_{D}^{(0)}$. In terms of $G = V^{-1}\gamma^{(1)}T$ and a matrix $H$ whose elements are

$$H_{ij} = \delta_{ij}\gamma_{i}^{(0)}\frac{\beta_1}{2\beta_0} - \frac{G_{ij}}{2\beta_0 + \gamma_{i}^{(0)} - \gamma_{j}^{(0)}},$$

the matrix $J$ is given by $J = VH^{-1}$.

Now there are also additional contributions to the RG evolution from QED and therefore the matrix $U(m_1, m_2)$ is substituted by $U(m_1, m_2, \alpha)$ which is the full $10 \times 10$ QCD-QED RG evolution matrix. The explicit expressions for the coefficients $C(M_W)$ including $O(\alpha)$ corrections can be found in [10]. The $10 \times 10$ anomalous dimension matrix $\gamma(g^2, \alpha)$ which includes QCD and QED contributions now becomes

$$\gamma(g^2, \alpha) = \gamma_s(g^2) + \frac{\alpha}{4\pi}\Gamma(g^2),$$

where the term due to QED corrections has the following expansion

$$\Gamma(g^2) = \gamma_e^{(0)} + \frac{\alpha_s}{4\pi}\gamma_e^{(1)} + \ldots$$
The RG evolution matrix now becomes

\[ U(m_1, m_2, \alpha) = U(m_1, m_2) + \frac{\alpha}{4\pi} R(m_1, m_2) , \]

where \( U(m_1, m_2) \) represents pure QCD evolution and \( R(m_1, m_2) \) is the additional evolution in the presence of electromagnetic interaction. The expression for \( R(m_1, m_2) \) is

\[ R(m_1, m_2) \equiv -\frac{2\pi}{\beta_0} V \left( K^{(0)}(m_1, m_2) + \frac{1}{4\pi} \sum_{i=1}^{3} K_i^{(1)}(m_1, m_2) \right) V^{-1} . \]

The matrix kernels in the above equation are defined by

\[ (K^{(0)}(m_1, m_2))_{ij} = \frac{M^{(0)}_{ij}}{\alpha_i - \alpha_j - 1} \left[ \left( \frac{\alpha_s(m_2)}{\alpha_s(m_1)} \right)^{\alpha_j} - \left( \frac{\alpha_s(m_2)}{\alpha_s(m_1)} \right)^{\alpha_i} \right] , \]

\[ (K_i^{(1)}(m_1, m_2))_{ij} = \begin{cases} \frac{M_i^{(1)}}{\alpha_i - \alpha_j} \left[ \left( \frac{\alpha_s(m_2)}{\alpha_s(m_1)} \right)^{\alpha_j} - \left( \frac{\alpha_s(m_2)}{\alpha_s(m_1)} \right)^{\alpha_i} \right] & i \neq j , \\ \frac{M_i^{(1)}}{\alpha_i - \alpha_j} \ln \frac{\alpha_s(m_1)}{\alpha_s(m_2)} & i = j , \end{cases} \]

\[ K_2^{(1)}(m_1, m_2) = -\alpha_s(m_2) K^{(0)}(m_1, m_2) H , \]

\[ K_3^{(1)}(m_1, m_2) = \alpha_s(m_1) H K^{(0)}(m_1, m_2) \]

with

\[ M^{(0)} = V^{-1} \gamma_e^{(0)T} V , \]

\[ M^{(1)} = V^{-1} \left( \gamma_e^{(1)T} - \frac{\beta_1}{\beta_0} \gamma_e^{(0)T} + \left[ \gamma_e^{(0)T}, J \right] \right) V , \]

where the explicit expressions for the 10 x 10 leading order and next-to-leading order anomalous dimension matrices \( \gamma^{(0)}_s, \gamma^{(0)}_e, \gamma^{(1)}_s \) and \( \gamma^{(1)}_e \) are given in [10].

Thus, on using this RG evolution, we obtain the new Wilson coefficients \( C_i^\prime \) at the \( m_b \) scale and we present their values in Table 5.1, for a representative set of values for \( m_t = 400 \text{ GeV} \).
Table 5.1: Values of the new Wilson coefficients at $m_b$ scale where $C_i^{\text{new}}$ represents $C_i$ for the fourth quark generation model and $\hat{C}_i$ for the FCNC mediated $Z$ boson model. The phase $\phi' = (\phi - \beta)$ is the relative weak phase between the NP and SM amplitudes.

| Wilson Coefficients | 4-Generation ($m_\nu = 400$ GeV) | Z boson model | $Z'$ model ($\xi_{L,R} = |\xi_{L,R}| e^{i\phi'}$) |
|---------------------|---------------------------------|---------------|----------------------------------|
| $C_3^{\text{new}}(m_b)$ | 0.0195 | 0.19 $\kappa e^{i\phi'}$ | 0.05 $\xi_L - 0.01 \xi_R$ |
| $C_4^{\text{new}}(m_b)$ | -0.0373 | -0.066 $\kappa e^{i\phi'}$ | -0.14 $\xi_L + 0.008 \xi_R$ |
| $C_5^{\text{new}}(m_b)$ | 0.0101 | 0.009 $\kappa e^{i\phi'}$ | 0.029 $\xi_L + 0.017 \xi_R$ |
| $C_6^{\text{new}}(m_b)$ | -0.0435 | -0.031 $\kappa e^{i\phi'}$ | -0.162 $\xi_L + 0.01 \xi_R$ |
| $C_7^{\text{new}}(m_b)$ | 0.0044 | 0.145 $\kappa e^{i\phi'}$ | 0.036 $\xi_L - 3.65 \xi_R$ |
| $C_8^{\text{new}}(m_b)$ | 0.002 | 0.053 $\kappa e^{i\phi'}$ | 0.01 $\xi_L - 1.33 \xi_R$ |
| $C_9^{\text{new}}(m_b)$ | -0.029 | -0.566 $\kappa e^{i\phi'}$ | -4.41 $\xi_L + 0.04 \xi_R$ |
| $C_{10}^{\text{new}}(m_b)$ | 0.0062 | 0.127 $\kappa e^{i\phi'}$ | 0.99 $\xi_L - 0.005 \xi_R$ |

After obtaining the values of the new Wilson coefficients at the $b$ quark mass scale, one can directly write the decay amplitude due to the fourth generation of quarks analogous to (5.4) as

$$A^{\text{NP}} = \frac{G_F}{\sqrt{2}} 2m_\phi f_\phi (e^* \cdot p_B) F_+(0) \lambda_\nu \left( \alpha_3' - \frac{1}{2} \alpha_{3,\text{EW}}' \right),$$

where $\alpha_{3,\text{EW}}'$s are the new contributions arising from the $t'$ quark contribution. We parameterize the new CKM elements as $\lambda_\nu = r_d e^{i\phi}$, where $\phi$ is the new weak phase associated with $\lambda_i$. Furthermore, since the unitarity condition has now become modified, the elements of the $3 \times 3$ upper submatrix of the $4 \times 4$ quark mixing matrix will be different from the corresponding values of SM CKM matrix elements. Since $V_{tb}$ and $V_{td}$ are not precisely known (i.e., not directly extracted from the experimental data, but fitted using the unitarity constraint) we use the lower limits from [33] i.e., $|V_{tb}| = 0.78$ and $|V_{td}| = 7.4 \times 10^{-3}$.

In order to study the effect of the fourth generation, we need to know the values of the new parameters $(m_\nu', r_d, \phi)$. Based on an integrated luminosity of $2.3/fb^{-1}$, CDF collaboration [97] gives the lower bound on $m_{\nu'}$ as $m_{\nu'} > 284$ GeV. In [98], it has been shown that the observed pattern of deviations
in the CP symmetries of $B$ system can be explained in the fourth quark generation model if $m_{t'} > 700$ GeV. Therefore, in our analysis we consider three representative values for $m_{t'}$, i.e., $m_{t'} = 400, 600$ and 800 GeV. The value of $r_d$ can be obtained from the measured mass difference $\Delta M_{B_d}$ of $B^0 - \bar{B}^0$ system and the corresponding expression for $\Delta M_{B_d}$ in the presence of fourth quark generation can be found in Ref. [96]. Thus, we obtain the values $r_d$ for different $m_{t'}$, consistent with the unitarity condition of $4 \times 4$ matrix as: $r_d \sim -3.8 \times 10^{-3}$ ($m_{t'} = 400$ GeV), $r_d \sim -2.7 \times 10^{-3}$ ($m_{t'} = 600$ GeV) and $r_d \sim -2.1 \times 10^{-3}$ ($m_{t'} = 800$ GeV). Using these values, in Figure 5.1 and Figure 5.2 we show the variation of the branching ratio and the direct CP asymmetry, respectively, with the new weak phase $\phi$ for three different values of $m_{t'}$. From Figure 5.1, one can see that the branching ratio is significantly enhanced from its SM value and this enhancement is more pronounced for large $m_{t'}$. It should also be noted that nonzero direct CP violation in this mode could be possible in the presence of an additional generation of quarks.

![Figure 5.1: Variation of CP averaged branching ratio (5.9) (in units of $10^{-8}$) with the new weak phase $\phi$, where the solid, dashed and dot-dashed lines correspond to $m_{t'} = 400, 600$ and 800 GeV, respectively. The horizontal line represents the SM value.](image-url)
Figure 5.2: Variation of direct CP asymmetry (5.10) (in %) with the new weak phase $\phi$, where the solid, dashed and dot-dashed lines correspond to $m_\nu = 400, 600$ and $800$ GeV, respectively.

### 5.2.2 Effect of the FCNC mediated $Z$ boson

Now we consider another extension of the SM, where the fermion sector is enlarged by an extra down-type singlet quark. Isosinglet quarks appear in many extensions of the SM like the low energy limit of the $E_6$ GUT models [99]. The mixing of this singlet down-type quark with the three SM down-type quarks provides a framework to study the deviations of the unitarity constraint of the $3 \times 3$ CKM matrix. The mixing also induces tree level flavor changing neutral currents, which can thus substantially modify the SM results. In this model, the $Z$ mediated FCNC interaction is given by [100]

$$\mathcal{L} = \frac{g}{2\cos\theta_W} [\bar{d}_{La} U_{\alpha\beta} \gamma^\mu d_{L\beta}] Z_\mu ,$$

(5.29)

with

$$U_{\alpha\beta} = \sum_{i=u,c,t} V^\dagger_{ai} V_{i\beta} = \delta_{\alpha\beta} - V^*_{4\alpha} V_{4\beta} ,$$

(5.30)

where $\alpha, \beta$ are generation indices and $U$ is the neutral current mixing matrix for the down quark sector. The non-vanishing component of $U_{\alpha\beta}$ will lead to the presence of FCNC transitions at the tree level. The implications of the
FCNC mediated $Z$ boson effect has been extensively studied in the context of $b$ physics [101, 102, 103].

Because of the new interactions the effective Hamiltonian describing $b \to d\bar{s}s$ process is given as [102]

$$
\mathcal{H}_{\text{eff}}^Z = -\frac{G_F}{\sqrt{2}} V_{tb} V_{td}^* [\tilde{C}_3 O_3 + \tilde{C}_7 O_7 + \tilde{C}_9 O_9],
$$

where the four-quark operators $O_3$, $O_7$ and $O_9$ have the same structure as the SM QCD and electroweak penguin operators and the new Wilson coefficients $\tilde{C}_i$'s at the $M_Z$ scale are given by

$$
\begin{align*}
\tilde{C}_3(M_Z) &= \frac{1}{6} \frac{U_{bd}}{V_{tb} V_{td}^*}, \\
\tilde{C}_7(M_Z) &= \frac{2}{3} \frac{U_{bd}}{V_{tb} V_{td}^*} \sin^2 \theta_W, \\
\tilde{C}_9(M_Z) &= \frac{2}{3} \frac{U_{bd}}{V_{tb} V_{td}^*} (1 - \sin^2 \theta_W).
\end{align*}
$$

These new Wilson coefficients will be evolved from the $M_Z$ scale to the $m_b$ scale using the renormalization group equation [10] as described earlier. Because of the RG evolution, these three Wilson coefficients generate a new set of Wilson coefficients $\tilde{C}_i (i = 3, \cdots, 10)$ at the low energy regime (i.e., at the $m_b$ scale) as presented in Table 5.1. Thus, one can write the new amplitude due to the tree level FCNC mediated $Z$ boson effect in a straightforward manner from Eq. (5.4) by replacing $\alpha_{3(3,EW)}$ by $\tilde{\alpha}_{3(3,EW)}$, where $\tilde{\alpha}$'s are related to the new Wilson coefficients $\tilde{C}_i (m_b)$'s. In order to see the effect of this FCNC mediated $Z$ boson effect, we have to know the value of the parameter $Z - b - d$ coupling parameter which can be explicitly written as $U_{bd} = |U_{bd}| e^{i\phi}$ and the allowed range of $|U_{bd}|$ is found to be $(2 \times 10^{-4} \leq |U_{bd}| \leq 1.2 \times 10^{-3})$ [103]. In Figure 5.3 and Figure 5.4, respectively, we present the variation of the CP averaged branching ratio (5.9) with $|U_{bd}|$ and $\phi$ and the direct CP asymmetry parameter $A_{CP}$ with $\phi$, where we have used $\sin^2 \theta_W = 0.231$. From the figures, it can be seen that the branching ratio could be significantly enhanced and large CP violation could be possible in this model.
5.2.3 Effect of the FCNC mediated $Z'$ boson

Now we consider the effect due to an extra $U(1)'$ gauge boson $Z'$. The existence of an extra $Z'$ boson is a feature of many models addressing physics beyond the SM, e.g., models based on extended gauge groups characterized by additional $U(1)$ factors [104]. In particular, they often occur in grand unified theories (GUTs), superstring theories and theories with large extra dimensions. The new physics models which contain exotic fermions also predict the existence of an additional gauge boson. Flavor mixing can be induced at the tree level in the up-type and/or down-type quark sector after diagonalizing their mass matrices. FCNCs due to $Z'$ exchange can be induced by mixing among the SM quarks and the exotic quark which have different $Z'$ quantum numbers. The search for the extra $Z'$ boson occupies an important place in the experimental programs of the Fermilab Tevatron and CERN LHC [105]. At such hadron colliders, heavy neutral gauge bosons with mass upto around 5 TeV can be produced and detected via two fermion decays $pp(p\bar{p}) \rightarrow Z' \rightarrow l^+l^-$ ($l = e, \mu$).

Here we consider the model in which the interaction between the $Z'$ boson and fermions are flavor nonuniversal for left-handed couplings and flavor
Figure 5.4: Variation of direct CP asymmetry (5.10) (in %) with the new weak phase $\phi$ where the dashed and solid lines correspond to $|U_{bd}| = 10^{-4}$ and $5 \times 10^{-4}$.

diagonal for right-handed couplings. The model can be also be found in Ref. [106, 107], where it has been shown that such a model can successfully explain the deviations of $S_{\phi K}$ and $S_{\eta' K}$ from $S_{\psi K}$ and also can explain the $B \rightarrow \pi K$ puzzle. We briefly present the method here. The Lagrangian for the neutral current interaction with the $Z'$ in the gauge basis is

$$\mathcal{L}_{Z'} = -g' J'_\mu Z'^\mu, \quad (5.33)$$

where $g'$ is the gauge coupling associated with the $U(1)'$ group at the $M_W$ scale. The renormalization group running between $M_W$ and $M_{Z'}$ is neglected here. The $Z'$ boson is assumed to have no mixing with the SM $Z$ boson. The chiral current is

$$J'_\mu = \sum_{i,j} \bar{\psi}_i^I \gamma_\mu [(\epsilon_{\psi_L})_{ij} P_L + (\epsilon_{\psi_R})_{ij} P_R] \psi^I_j, \quad (5.34)$$

where the sum extends over the flavors of fermion fields, the chirality projection operators are $P_{L,R} \equiv (1 \mp \gamma_5)/2$, the superscript $I$ refers to the gauge interaction eigenstates and $\epsilon_{\psi_L}$ ($\epsilon_{\psi_R}$) denote the left-handed (right-handed) chiral couplings. $\epsilon_{\psi_L}$ and $\epsilon_{\psi_R}$ are hermitian under the requirement of a real
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Lagrangian. The fermion Yukawa coupling matrices $Y_\psi$ in the weak basis can be diagonalized as

$$Y_\psi^D = V_{\psi R} Y_\psi V_{\psi L}^\dagger \quad (5.35)$$

using the bi-unitary matrices $V_{\psi L,R}$ in $\psi_{L,R} = V_{\psi L,R} \psi_{L,R}^I$, where $\psi_{L,R}^I \equiv P_{L,R} \psi^I$ and $\psi_{L,R}$ are the mass eigenstate fields. The usual CKM matrix is then given by

$$V_{\text{CKM}} = V_{u L} V_{d L}^\dagger. \quad (5.36)$$

The chiral $Z'$ coupling matrices in the physical basis of up-type and down-type quarks are, respectively,

$$B_{u X} \equiv V_{u X} \epsilon_{u X} V_{u X}^\dagger, \quad B_{d X} \equiv V_{d X} \epsilon_{d X} V_{d X}^\dagger, \quad (X = L, R) \quad (5.37)$$

where $B_{u(d)}^{X}$ are hermitian. As long as the $\epsilon$ matrices are not proportional to the identity matrix, the $B^X$ matrices will have nonzero off-diagonal elements that induce FCNC interactions at tree level. The assumption of flavor diagonal right-handed couplings demands $B_{u(d)}^{R} \propto I$. However, the flavor changing left-handed couplings will give new contributions to the SM operators.

The effective Hamiltonian describing the transition $b \rightarrow d \bar{s} s$ mediated by the $Z'$ boson is therefore given by [106]

$$H_{\text{eff}}^{Z'} = - \frac{4 G_F}{\sqrt{2}} V_{tb} V_{td}^* \left[ \left( \frac{g' M_Z}{g_1 M_{Z'}} \right)^2 \frac{B_{d b}^{L}}{V_{tb} V_{td}^*} (B_{s s}^{L} O_9 + B_{s s}^{R} O_7) \right], \quad (5.38)$$

where $g_1 = e/(\sin \theta_W \cos \theta_W)$ and $B_{i j}^{L(R)}$ denote the left (right)-handed effective $Z'$ couplings of the quarks $i$ and $j$ at the weak scale. The diagonal elements are real due to the hermiticity of the effective Hamiltonian but the off-diagonal elements may contain an effective weak phase. Therefore, both the terms in (5.38) will have the same weak phase due to $B_{d b}^{L}$. We can parameterize these coefficients as

$$\xi_L = \left( \frac{g' M_Z}{g_1 M_{Z'}} \right)^2 \left( \frac{B_{d b}^{L} B_{s s}^{L}}{V_{tb} V_{td}^*} \right) = |\xi_L| e^{i \phi'}, \quad \xi_R = \left( \frac{g' M_Z}{g_1 M_{Z'}} \right)^2 \left( \frac{B_{d b}^{L} B_{s s}^{R}}{V_{tb} V_{td}^*} \right) = |\xi_R| e^{i \phi'}, \quad (5.39)$$
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where $\phi' = \phi - \beta$ ($\phi$ is the weak phase associated with $B_{db}^L$).

In order to see the effect of the $Z'$ boson, we have to know the values of the coefficients $\xi_L$ and $\xi_R$ or equivalently $B_{db}^L$ and $B_{ss}^{L,R}$. Assuming only left-handed couplings are present, the bound on FCNC $Z'$ coupling ($B_{db}^L$) from $B^0 - \bar{B}^0$ mass difference has been obtained in Ref. [108] as

$$y|\text{Re}(B_{db}^L)| < 5 \times 10^{-8}, \quad y|\text{Im}(B_{db}^L)| < 5 \times 10^{-8}, \quad (5.40)$$

where $y = (g'M_Z/g_1M_{Z'})^2$. Generally one expects $g'/g_1 \sim 1$, if both the $U(1)$ gauge groups have the same origin from some grand unified theories, $M_Z/M_{Z'} \sim 0.1$ for a TeV scale neutral $Z'$ boson, which yields $y \sim 10^{-2}$. However in Ref. [108], assuming a small mixing between $Z$ and $Z'$ bosons, the value of $y$ is taken as $y \sim 10^{-3}$. Using $y \sim 10^{-2}$, one can obtain a more stringent bound $|B_{db}^L| < 10^{-3}$. It has been shown in [107] that the mass difference of $B_s - \bar{B}_s$ mixing can be explained if $|B_{sb}^L| \sim |V_{tb}V_{ts}^*|$. Similarly, the CP asymmetry anomaly in $B \to \phi K, \pi K$ can be resolved if $|B_{sb}^L B_{ss}^{L,R}| \sim |V_{tb}V_{ts}^*|$. From these two relations, one can obtain $|B_{ss}^L| \sim 1$. Thus, it is expected that $\xi_{L,R} \sim 10^{-3}$. However, in this analysis we vary their values within the range $(0.01 - 0.001)$.

Figure 5.5: Variation of the CP averaged branching ratio (5.9) (in units of $10^{-8}$) with $\xi$ (in units of $10^{-3}$) and the new weak phase $\phi$. 

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After having an idea about the magnitudes of these new coefficients which are at the $M_Z$ scale, we now evolve them to the $b$ scale using the renormalization group equation [10] as described earlier. The new Wilson coefficients at the $m_b$ scale are presented in Table 5.1. Using the values of these coefficients at $b$ scale, we can analogously obtain the new contribution to the transition amplitude as done in the case of $Z$ boson. Now using $|\xi_L| = |\xi_R| = \xi$, in Figure 5.5 and Figure 5.6, respectively, we show the variation of the CP averaged branching ratio with $\xi$ and the new weak phase $\phi$ and the direct CP violation with $\phi$. In this case also one can have a significant enhancement in the branching ratio for large $\xi$, or in other words for a lighter $Z'$ boson. Furthermore, the observation of this mode could in turn help us to constrain the $Z'$ mass.

### 5.3 Conclusion

To conclude, we have studied the $B^+ \rightarrow \phi \pi^+$ decay mode in the standard model and in some beyond the standard model scenarios. This is a pure penguin rare decay process and proceeds through the quark level transition
$b \to d\bar{s}s$, which occurs at the one-loop level and is therefore expected to be highly suppressed in the SM. The SM prediction of its branching ratio is $\sim \mathcal{O}(10^{-9})$ which is below the experimental upper limit of $\mathcal{O}(10^{-7})$. We have analysed this decay mode in the fourth quark generation model and in the FCNC mediated $Z$ and $Z'$ models. In the fourth quark generation model, we find that the branching ratio enhances from its SM value with increasing $m_{t'}$ and it can have a value of $\sim \mathcal{O}(10^{-8})$. In the $Z$ and $Z'$ models, the branching ratio can be significantly enhanced for sizable new physics couplings $|U_{bd}|$ and $\xi$, respectively. In these cases it can reach up to $\mathcal{O}(10^{-7})$ level but still within the experimental upper limit. Furthermore, it is found that large direct CP violation could be possible in this decay mode in the presence of the mentioned new physics models. Thus, if this mode could be observed in the upcoming LHCb experiment, it will provide a clear signal of new physics and also can be used to constrain the parameter space of various new physics models. However, it should be noted that it would not be possible to distinguish between these new physics models considering this mode alone.