Chapter 2

Determination of the CKM angle $\gamma$ with $B_c$ decays

It is now known that in the standard model (SM), CP violation arises from the nonzero weak phase in the complex CKM matrix which is responsible for the charged current weak interaction. One of the main ingredients of the SM description of CP violation is the CKM unitarity triangle (UT). When we consider the most relevant unitarity relation describing $B$ decays, we obtain the angles of the UT termed as $\alpha (\phi_2)$, $\beta (\phi_1)$ and $\gamma (\phi_3)$ [15]. Therefore, to have a more precise information of CP violation, it is important that we find new ways and methods to extract the three angles $\alpha$, $\beta$ and $\gamma$ of this triangle. Large CP violation, as was expected, has already been established in $B$ systems in the running $B$ factories at SLAC and KEK. The present status is that we have measured, with the huge data sets available, the angle $\beta$ (actually, $\sin (2\beta)$) with a reasonable accuracy and we expect to have a precision measurement of angle $\beta$ in the years to come, with the help of the golden mode $B_d^0 \rightarrow J/\psi K_S$. Unfortunately, we do not have three golden modes to determine the three angles of the UT. So we have to be contented with the best available modes like $B \rightarrow \pi \pi$ (and some related modes) for the determination of the angle $\alpha$, but these modes are accompanied by a generic problem called penguin contamination. So finally, we are left with the angle $\gamma = \arg (-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$, which was believed to be the most difficult one,
among all the three angles, at the beginning. But, fortunately, in this case, nature has been very kind to provide us with many options to determine the angle $\gamma$ in various avenues.

There have been many attempts in the past to devise methods to determine the CKM angle $\gamma$ as cleanly as possible. One of the methods to determine $\gamma$ is the Gronau-London-Wyler (GLW) method [16], which uses the interference of two amplitudes ($b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$) in $B \rightarrow DK$ modes. In this method, $\gamma$ can be determined by measuring the decay rates $B^- \rightarrow D^0 K^-$, $B^- \rightarrow \bar{D}^0 K^-$ and $B^- \rightarrow D^0_+ K^-$ (where $D^0_+$ is the CP-even eigenstate of neutral $D$ meson system) and their corresponding CP conjugate modes. However, because the mode $B^- \rightarrow \bar{D}^0 K^-$ is both color and CKM suppressed with respect to $B^- \rightarrow D^0 K^-$, the corresponding amplitude triangles are expected to be highly squashed and it is also very difficult to measure the rate of $B^- \rightarrow \bar{D}^0 K^-$. To overcome the problems of the GLW method, Atwood-Dunietz-Soni (ADS) [17] proposed an improved method where they have considered the decay chains $B^- \rightarrow K^- D^0 \rightarrow f$ and $B^- \rightarrow K^- \bar{D}^0 \rightarrow f$, where $f$ is the doubly Cabibbo suppressed (Cabibbo favored) non-CP eigenstate of $D^0 (\bar{D}^0)$. These methods are being explored in the $B$-factory experiments and will also be taken up at the collider experiments along with another method called the Aleksan-Dunietz-Kayser (ADK) method [18], which uses the time-dependent measurement of $B^0_s (\bar{B}^0_s) \rightarrow D^\pm K^\mp$ modes. Because of its importance and, of course, possible options available, there are many methods that exist in the literature. Some of the alternative methods to obtain $\gamma$ are those using $B$ and $B_s$ decays [19]-[27], $B_c$ decays [28] and also $\Lambda_b$ decays [29].

Another method, the Giri-Grossman-Soffer-Zupan (GGSZ) method (otherwise also known as the Dalitz method) [19] has also been proposed (using $B \rightarrow D^0 (\bar{D}^0) K \rightarrow K_S \pi \pi K$), which has many attractive features and has already been explored at both the $B$ factories. It should be noted here that the GGSZ method uses the ingredients of GLW and ADS methods where the
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$D^0(\bar{D}^0)$ decays to multi-particle final states. This method in turn helps us to constrain the angle $\gamma$ directly from the experiments. But at present the error bars are quite large, which are expected to come down in the coming years. It may be worthwhile to emphasize here that one has to measure the angle with all possible clean methods available to arrive at a conclusion and thereby reducing the error in $\gamma$ to a minimum.

2.1 The Method

In the continued effort, we now wish to explore yet another method with the decays $B_c^\pm \rightarrow D_s^0D^0 \rightarrow D_s^\pm (K^+K^-)_D^0$ and $B_c^\pm \rightarrow D_s^0\bar{D}^0 \rightarrow D_s^\pm (K^+K^-)_\bar{D}^0$. It has been shown earlier in [28] that the decay $B_c^\pm \rightarrow D^0(\bar{D}^0)D_s^\pm$ modes can be used to determine the CKM angle $\gamma$ in a better way since the interfering amplitudes in $B_c$ case are roughly of equal sizes, whereas the corresponding ones in GLW method (using $B$ mesons) are not so. In [28], it has been shown that $\gamma$ can be determined from the decay rates $B_c^\pm \rightarrow D^0D_s^\pm$, $B_c^\pm \rightarrow \bar{D}^0D_s^\pm$ and $B_c^\pm \rightarrow D_s^0D_s^\pm$ (where $D_s^0$ are the CP eigenstates of neutral $D$ meson system with CP eigenvalues $\pm 1$, which can be identified by the CP-even and CP-odd decay products of neutral $D$ meson). In this work, another method is proposed where we consider the $B_c^\pm \rightarrow D^0(\bar{D}^0)D_s^\pm$ decay modes, that are followed by $D^0(\bar{D}^0)$ decaying to $K^+K^-$, which is a non-CP eigenstate.

The decay modes $B_c^- \rightarrow D_s^-D^0$ and $B_c^- \rightarrow D_s^-\bar{D}^0$ are described by the quark level transition $b \rightarrow c\bar{u}s$ and $b \rightarrow u\bar{c}s$, respectively and the amplitudes for these processes are given as

$$A(B_c^- \rightarrow D^0D_s^-) = \frac{G_F}{\sqrt{2}} V_{cb}V_{us}^*(C + A),$$

$$A(B_c^- \rightarrow \bar{D}^0D_s^-) = \frac{G_F}{\sqrt{2}} V_{ub}V_{cs}^*(\tilde{C} + \tilde{T}), \quad (2.1)$$

where $C$ and $A$ denote the color suppressed tree and annihilation topologies for $b \rightarrow c$ transition and $\tilde{C}$ and $\tilde{T}$ denote the color suppressed tree and color
allowed tree contributions for $b \to u$ transition. Now let us denote these amplitudes as

$$A_B = \mathcal{A}(B_c^- \to D^0 D_s^-), \quad \bar{A}_B = \mathcal{A}(B_c^- \to \bar{D}^0 D_s^-),$$

and their ratios as

$$\frac{\bar{A}_B}{A_B} = r_B e^{i(\delta_B - \gamma)}, \quad \text{with} \quad r_B = \left| \frac{\bar{A}_B}{A_B} \right| \quad \text{and} \quad \arg \left( \frac{\bar{A}_B}{A_B} \right) = \delta_B - \gamma,$$

where $\delta_B$ and $-\gamma$ are the relative strong and weak phases between the two amplitudes. The ratio of the corresponding CP conjugate processes are obtained by changing the sign of the weak phase $\gamma$. One can then obtain a rough estimate of $r_B$ from dimensional analysis, i.e.,

$$r_B = \left| \frac{V_{ub} V_{cs}^*}{V_{cb} V_{us}^*} \right| \cdot \frac{a_1^{\text{eff}}}{a_2^{\text{eff}}} \approx O(1),$$

where $a_1^{\text{eff}}$ and $a_2^{\text{eff}}$ are the effective QCD coefficients describing the color allowed and color suppressed tree level transitions. For the sake of comparison, we would like to point out here that the corresponding ratio between the $B^- \to D^0(\bar{D}^0)K^-$ amplitudes are given as $|\mathcal{A}(B^- \to \bar{D}^0 K^-)/\mathcal{A}(B^- \to D^0 K^-)| = |(V_{ub} V_{cs}^*)/(V_{cb} V_{us}^*)| \cdot (a_2^{\text{eff}}/a_1^{\text{eff}}) \approx O(0.1)$. The $D^0$ decay amplitudes are denoted as

$$A_D = \mathcal{A}(D^0 \to K^{*+}K^-), \quad \bar{A}_D = \mathcal{A}(\bar{D}^0 \to K^{*+}K^-),$$

and their ratios as

$$\frac{\bar{A}_D}{A_D} = r_D e^{i\delta_D}, \quad \text{with} \quad r_D = \left| \frac{\bar{A}_D}{A_D} \right|.$$

It is interesting to note that the parameters $r_D$ and $\delta_D$ have been measured by CLEO collaboration [30], with values $r_D = 0.52 \pm 0.05 \pm 0.04$ and $\delta_D = 332^\circ \pm 8^\circ \pm 11^\circ$, rendering our study more appealing.
With these definitions the four amplitudes are given as

\[
A_1(B_c^- \to D_s^-(K^{*+}K^-)) = |A_B A_D| \left[ 1 + r_B r_D e^{i(\delta_B + \delta_D - \gamma)} \right],
\]

\[
A_2(B_c^- \to D_s^-(K^{*-}K^+)) = |A_B A_D| \left[ e^{i\delta_D} r_D + r_B e^{i(\delta_B - \delta_D - \gamma)} \right],
\]

\[
A_3(B_c^+ \to D_s^+(K^{*-}K^+)) = |A_B A_D| \left[ 1 + r_B r_D e^{i(\delta_B + \delta_D + \gamma)} \right],
\]

\[
A_4(B_c^+ \to D_s^+(K^{*+}K^-)) = |A_B A_D| \left[ e^{i\delta_D} r_D + r_B e^{i(\delta_B - \delta_D + \gamma)} \right].
\]

From these amplitudes one can obtain the four observables \((R_1, \ldots, R_4)\), with the definition

\[
R_i = \left| A_i(B_c^\pm \to D_s^{\mp}(K^{*\pm}K^\mp)) / A_B A_D \right|^2.
\]

We can now write \(R_1 = 1 + r_B^2 r_D^2 + 2 r_B r_D \cos(\delta_B + \delta_D - \gamma)\) and similarly for \(R_2, R_3\) and \(R_4\).

Here we assume that the amplitudes \(|A_B|\) and \(|A_D|\) are known (so also \(r_B\), which is \(O(1)\)).

Thus, one can obtain an analytical expression for \(\gamma\) as

\[
\sin^2 \gamma = \frac{[R_1 - R_3]^2 - [R_2 - R_4]^2}{4[R_2 - R_{BD}^2][R_4 - R_{BD}^2] - [R_1 - R_{BD}^2][R_3 - R_{BD}^2]},
\]

where \(R_{BD}^2 = (r_B^2 + r_D^2)\) and \(R_{BD}^2 = (1 + r_B^2 r_D^2)\).

We then study the sensitivity of \(\gamma\) in some limiting cases in the method described above.

(a) If the relative strong phase between \(A_B\) and \(A_D\) is zero then Eq.(2.9) can no longer be used to extract the angle \(\gamma\) as both numerator and denominator vanish in this limit. However, still \(\gamma\) can be extracted, in this limit, from either the observables \(R_1\) and \(R_3\) or \(R_2\) and \(R_4\). Now, considering the observables \(R_2\) and \(R_4\), for example, one can obtain an expression for \(\gamma\) as

\[
\tan \gamma = \frac{\cot \delta_D (R_4 - R_2)}{R_2 + R_4 - 2(r_B^2 + r_D^2)}. \tag{2.10}
\]

An analogous expression for \(\gamma\) can also be obtained from \(R_1\) and \(R_3\) with the replacement of \(R_{2,4} \leftrightarrow R_{3,1}\) and \((r_B^2 + r_D^2) \leftrightarrow (1 + r_B^2 r_D^2)\).
If \( r_B = 1 \) and \( \delta_B = 0 \), then the four observables \( (R_1, \cdots, R_4) \) are no longer independent of each other and we have two degenerate sets with \( (R_1 = R_4) \) and \( (R_2 = R_3) \). One can then define two parameters

\[
C_- \equiv \cos(\delta_D - \gamma) = \frac{1}{2 r_B r_D} (R_4 - r_B^2 - r_D^2),
\]
\[
C_+ \equiv \cos(\delta_D + \gamma) = \frac{1}{2 r_B r_D} (R_2 - r_B^2 - r_D^2),
\]

where we have deliberately retained the \( r_B \) term in the above expressions, so that one can still use this method for \( r_B \neq 1 \) case. Thus, one can now obtain the solution for \( \gamma \), in terms of these observables, as

\[
\sin^2 \gamma = \frac{1}{2} \left[ 1 - C_+ C_- \pm \sqrt{(1 - C_-^2)(1 - C_+^2)} \right],
\]

one solution of which will give \( \sin^2 \gamma \) while the other being \( \sin^2 \delta_D \). Since \( \delta_D \) has already been measured, \( \sin^2 \gamma \) could be extracted from these observables, once we know the values of \( R_2, R_4 \) (otherwise \( R_1 \) and \( R_3 \)) and \( r_B \) (it may be noted that the value of \( r_D \) is already known now).

Our method consists of two parts, the first one being the \( B_c^\pm \to D^0(\bar{D}^0)D_s^\pm \), which will be measured at the hadron colliders, such as LHC, whereas the second part consists of the measurement of \( D^0(\bar{D}^0) \to K^{\ast +}K^- \), which can also be measured at the same collider experiments. Moreover, since we already have experiments and there are upcoming dedicated experiments to measure the parameters in the charm-sector, like at CLEO-c and the BEPCII, which will provide us half of the parameters needed in our study, it is meaningful to combine the data from various experiments, mentioned above, to obtain \( \gamma \) with a better accuracy.

The possible effect of \( D^0 - \bar{D}^0 \) mixing for the determination of \( \gamma \) is not taken into account in our analysis since it has been well studied in the literature [19, 31] and found that the effect is very small, unless we are doing a precision measurement of \( \gamma \). To be quantitative, the error could be around \( 1^\circ \), with the present data available, which for all practical purposes is ignored at the moment.
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Now, with $r_D$ already known (so also $\delta_D$), we are left with only two unknowns ($\delta_B$ and $\gamma$). Therefore, we have two unknowns and four observables. We can consider different non-CP eigenstates (like $\rho^+\pi^-$), which will increase the observables by four and unknowns by two ($r'_D$ and $\delta'_D$) for each additional eigenstate. One can also take $B_c^\pm \to D^0(D^0)s^{-\pm}$ mode, thereby further increasing number of observables by four and unknowns by two (say $r'_B$ and $\delta'_B$, in fact it could be just $\delta'_B$). Hence we hope to have enough observables and at best half the number of unknowns (actually, it will always be less than half since new unknown parameters, namely, $r'_D$ and $\delta'_D$ can also be inferred from the $D$ decay data) and we can obtain the value of $\gamma$ without hadronic uncertainties.

Now we estimate the branching ratios for these modes. Using the generalized factorization approximation, the amplitudes are given as

$$A(B_c^- \to D^0D_s^-) = \frac{G_F}{\sqrt{2}}V_{ub}V_{us}^*(a_{eff}^1 X + a_{eff}^2 Y) ,$$

$$A(B_c^- \to \bar{D}^0D_s^+) = \frac{G_F}{\sqrt{2}}V_{ub}V_{us}^*(a_{eff}^1 X_1 + a_{eff}^2 X) ,$$

(2.13)

where $X = if_{D^0}(m_{B_c}^2 - m_{D_s}^2)F_{0B_cD^0}(m_{D^0})$, $X_1 = if_{D_s}(m_{B_c}^2 - m_{D_s}^2)F_{0B_cD^0}(m_{D_s})$ and $Y = if_{B_c}(m_{D_s}^2 - m_{B_c}^2)F_{0BD_s}(m_{B_c})$ are the factorized hadronic matrix elements. For numerical evaluation we use the values of the form factors at zero recoil from [32] as $F_{0B_cD^0}(0) = 0.352$, $F_{0B_cD_s}(0) = 0.37$, the decay constants (in MeV) as $f_{D^0} = 235$, $f_{D_s} = 294$, $f_{B_c} = 360$, the QCD coefficients $a_{eff}^1 = 1.01$, $a_{eff}^2 = 0.23$, particle masses, lifetime of $B_c$ and CKM matrix elements from [33]. We thus obtain the branching ratios as

$$\text{BR}(B_c^- \to D^0D_s^-) = 7.0 \times 10^{-6} , \quad \text{BR}(B_c^- \to \bar{D}^0D_s^+) = 4.5 \times 10^{-5} .$$

(2.14)

Let us now make a crude estimate of the number of reconstructed events that could be observable at LHC per year of running. At LHC, one expects about $10^{10}$ untriggered $B_c$’s per year [34]. For the estimation, we use the branching ratios as $\text{BR}(B_c^- \to D^0D_s^-) = 7.0 \times 10^{-6}$ and $\text{BR}(D^0 \to K^{*-}K^-) = 3.7 \times 10^{-3}$
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[33] and assume that the $D_s$ can be reconstructed efficiently by combining a number of hadronic decay modes. As the LHCb trigger system has a good performance for hadronic modes, we assume an overall efficiency of 30% and hence we expect to get nearly 80 events per year of running at LHC.

2.2 Discussion and Conclusion

We have outlined here that $B_c^+ \rightarrow (D^0)D_s^\pm \rightarrow (K^{*\pm}K^\mp)D_s^\pm$ and $B_c^\pm \rightarrow (\bar{D}^0)D_s^\pm \rightarrow (K^{*\pm}K^\mp)D_s^\pm$ can be used to determine the CKM angle $\gamma$ at the LHC. Since the interfering amplitudes are of equal order (which is not the case with $B \rightarrow DK$ methods) and furthermore neither tagging nor time-dependent studies are required to undertake this strategy and above all the final particles are charged ones (and of course with reduced background), this method may be very well suited for the determination of $\gamma$ without hadronic uncertainties. But one has to pay the price for all the niceties of this method in the sense that the branching ratios are smaller by an order compared to the earlier modes. Nevertheless, we hope that this should not cause any hindrance for the clean determination of the angle $\gamma$ using this method and even if we get lesser number of events, the predictive power will not be diluted.

In conclusion, in this study we have looked into the possibility of extracting the CKM angle $\gamma$ using multibody $B_c$ decays and in view of the fact that LHC is already underway, this method can be found to be very useful to obtain $\gamma$ and to supplement the results from other methods.