A CONTROL CHART MODEL FOR SPECIFIED WARRANTEE LIFE TIME

7.1 INTRODUCTION

The control chart techniques have been applied with considerable success in the study and the statistical control of manufacturing processes. Shewhart's (1931) 3-sigma control chart procedure have been found useful for attainment of a statistical stability of industrial production processes. The variable control chart models have been developed by several authors to control mean value of the quality characteristic and/or its variability. The median control chart developed by Farrel (1953) are of particular importance for in-plant median control. Roberts (1958, 59) studied in detail the moving average and geometric moving average control chart procedures which is useful for a continuing series of individual observations. Acceptance control chart proposed by Freund (1957) are well adopted to trouble shooting in process control. The attribute control charts are studied to control fraction of non-conforming/defective and number of defects observed in attribute data. The economic design of control charts have been extensively studied by several authors. Some references may be made to Duncan
Now-a-days, the electronics industries have heavily prioritized enhancing the quality and life time of electronic goods or its components. We come across many situations in real life where there is requirement of a minimum warrantee life time say \( t_0 \) \((t_0 > 0)\) at (from) which the unit is regarded as satisfactory (or conforming). Most of the control chart in literature have been investigated either for attribute data using binomial model or for variable data assuming normality of the quality characteristic under study. However, no attention has been paid to the development of control charts for life time quality characteristics. In this chapter, an attempt has been made to develop a control chart model for life time quality characteristics with specified warrantee life of the product. An item is characterised to be of satisfactory quality when its life time is more than the specified warrantee life. On the other hand, an item is considered to be unsatisfactory, if its life time is less than the warrantee life. For simplicity, the life time of an item is assumed to be exponentially distributed. Shewhart’s (1931) control chart techniques have been utilized to derive control limits of the proposed control chart. The expression for OC function of the chart have been
derived using W-statistic proposed by Sen and Srivastava (2001). Lastly, numerical examples have been included to illustrate the mathematical findings.

7.2 PRELIMINARIES

In order to avoid excessive references, we first produce a general model for the control chart. Let $w$ be a sample statistic that measure some quality characteristic of interest. Suppose that the mean and standard deviation of $w$ are $\mu_w$ and $\sigma_w$ respectively. The center line (CL), upper control limit (UCL) and lower control limit (LCL) are obtained as follows:

\[
UCL = \mu_w + k \sigma_w \\
CL = \mu_w \\
LCL = \mu_w - k \sigma_w
\]

where $k$ is the “distance” of the control limits from the central line, expressed in standard deviation units. A typical control chart is shown in Fig. (7.1).
The OC curve of the control chart plots the probability of type II error \( \beta \) against the true mean of the statistic under consideration. It provides a measure of sensitivity of the control chart, that is, its ability to detect a shift in the nominal value to some other value. The probability of \( \beta \) risk is obtained.

\[
\beta = P \left[ \frac{LCL < w < UCL}{\mu_w = \text{Shift in other value}} \right]
\]

More specifically, OC curve gives the probability of a single sample falling within the control limits when the process works actually above or below the central line in the chart. It shows the risk of saying that the process is in-control at the designated level because a point falls within the control limits, when actually the process is operating at a different level. A full discussion on OC function and
additional references can be found in different text books [Mont-gomery (1997), Duncan (1986)].

7.3 MODEL FORMULATION

Many current results in life testing are based on the assumption that the life $x$ is described by an exponential distribution with probability density function $f(x: \theta)$ of the form:

$$f(x: \theta) = \frac{1}{\theta} \exp[-x/\theta] \quad x > 0, \theta > 0$$

(7.3.1)

where $x$ is life measured in appropriate units and $\theta$ is the mean life. Epstein (1958) emphasized that the lives of electron tubes or the time intervals between successive breakdowns of electronic systems are, to a first approximation, random variables having the exponential density function. A justification for the assumption of an exponential probability density function has been discussed in detail by Epstein (1953a,53b) and Davis (1952). A brief account of application of exponential distribution in reliability and life testing has been given by Epstein (1958). Suppose that life time of an electronics component is exponential with mean life $\theta$. Draw a sample of $n$ units from the production process and put these units on test. Let $X_1, X_2, \ldots, X_n$ be failure times of $n$ units respectively.

Define the Kernels $\phi_1$ and $\phi_2$ of degree one and two as follows:
\[ \phi_1(\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3) = I(X > t_0) (X_2 - X_3) I(X_2 > X_3) \]

and \[ \phi_2(\mathbf{X}_1, \mathbf{X}_2) = [X_1 - (X_2 + t_0)] I(X_1 > X_2 + t_0) \]

where \( t_0 \) is the minimum warranted time and indicator function \[ I(a > b) = 1 \text{ and } I(a \leq b) = 0. \]

In order to develop the control limits of the proposed control chart, we derive the \( W \)-statistic [see Sen and Srivastava(2001)] as follows:

\[ W = V_1 - V_2 \]

where

\[ V_1 = \frac{1}{np_3} \sum_{i=1}^{np_3} \phi_1(X_{i_1}, X_{i_2}, X_{i_3}) \]

\[ V_2 = \frac{1}{np_2} \sum_{i_1}^{np_2} \phi_2(X_{i_1}, X_{i_2}) \]

\( \sum \) is the summation over all the \( np_3 \) permutations of 3 integers \( \{i_1, i_2, i_3\} \) chosen from the set of \( n \) integers \( \{1, 2, \ldots, n\} \) and \( \sum \) is summation over all the \( np_2 \) permutations of 2 integers \( \{i_1, i_2\} \) chosen from the set of \( n \) integers \( \{1, 2, \ldots, n\} \).
Further, Sen and Srivastava (2001) have shown $\sqrt{n}W$ is asymptotically normally with mean zero and variance $\sigma^2$.

where

$$\sigma^2 = \frac{7e^{-t_0/\theta} + 4e^{-3t_0/\theta} - 11e^{-2t_0/\theta}}{12(1/\theta)^2}$$

Thus, $W \sim N \left(0, \sigma^2 / n\right)$

Now, we define warrantee index as follows:

$$W.I. = \frac{t_0}{\theta}$$

where $t_0$ is the specified warrantee life and $\theta$ is mean (known) of the exponential distribution. Suppose that the in-control value of W.I. is $\frac{t_0}{\theta}$. Here, it may be noted that an item is satisfactory (or conforming) if its W.I. is greater than or equal to $\frac{t_0}{\theta}$, otherwise it is unsatisfactory. It can be observed that when process is under control, $W / t_0$ is normally distribution with mean zero and variance

$$\sigma_0^2 = \frac{7e^{-t_0/\theta} + 4e^{-3t_0/\theta} - 11e^{-2t_0/\theta}}{12\left(t_0 / \theta\right)^2}$$

The control limits of the proposed charts are obtained as

$$UCL = \frac{k\sigma}{\sqrt{n}} \quad \text{(7.3.1)}$$

$$CL = 0$$
The 3-σ limits of the proposed model as found with \( k = 3 \).

If any value of \( \left( \frac{w}{t} \right) \) falls outside the control limits, this means that warrantees life reduced from specified value of \( t_0 \) (say) to another value \( t_1 = \lambda \cdot t_0 \) \((0 < \lambda < 1)\). The OC function is derived [Montgomery(1997)] as follows:

\[
\beta = P \left\{ \frac{w}{t_0} \leq \frac{-k \sigma}{\sqrt{n}} \leq \frac{UCL}{t_1} = \lambda \cdot t_0 \right\}
\]

\[
= P \left\{ \frac{-k \sigma}{\sqrt{n}} \leq \frac{w}{t_0} \leq \frac{k \sigma}{\lambda \sqrt{n}} / t_1 = \lambda \cdot t_0 \right\}
\]

\[
= P \left\{ \frac{-k}{\lambda \sqrt{n}} \leq Z \leq \frac{k}{\lambda \sqrt{n}} \right\}
\]

\[
= \Phi \left[ \frac{k}{\lambda \sqrt{n}} \right] - \Phi \left[ \frac{-k}{\lambda \sqrt{n}} \right]
\]

where \( \Phi \) denotes the standard normal cumulative distribution function, discriminating ratio \( \lambda = \frac{t / \theta}{t_0 / \theta} = t / t_0 \). It is important to note that \( \lambda = 1 \) when observed product life is equal to the specified warranteeed life i.e. when process is under control.
7.4 ILLUSTRATION AND CONCLUSION

For studying the behaviour of OC function of the proposed control chart, we consider control limit given in (7.3.1) with \( k = 3 \). The OC curve have been drawn in Fig. (7.2) for \( \beta \) against the discriminating ratio \( \lambda = t / t_0 \). It is observed from the figure that the value of \( \beta \) decreases with increasing \( \lambda \). Thus, the probability of not detecting the reducing warranted life decreases when \( \lambda \) increases i.e. when \( \text{ratio} \) of observed product life and specified warranted life increases. Figure (7.2) further shows that how the OC curve for a chart varies with sample size lone. As \( n \) increases, the curve is tightened up, and the effect is to lower the OC curve. As \( n \) decreases, the curve become more lax, and the effect is to raise the OC curve.

Fig. (7.3) presents a relationship between \( t / \theta \) and \( \beta \) with \( n = 30 \) and \( k = 3 \). It is found from the figure that \( \beta \)-risk decreases with the increasing \( t / \theta \). In otherwords, probability of not detecting the reducing warranted life decreases with increasing ratio of product life to the mean life time.
Fig. (7.2) : The OC curves for different values of $n$ ($\beta$ versus $\lambda$) $K = 3$, $\lambda = t/t_0$
Fig. (7.3): OC Curves for different values of $t_0 / \theta (\beta \text{ versus } t / \theta) n = 30, k = 3$. 