Chapter-V

Tools of Time Series Analysis

5.0. Introduction

The discussion in the preceding chapter reveals that there is neither theoretical nor empirical consensus on any definite pattern or consistent relationship between private foreign capital inflows and macroeconomic variables. Similarly, no conclusive generalization can be made about the casual relationship between private foreign capital inflows and economic growth. The present chapter discusses the tools of time series analysis, where various methodologies have discussed according to the objectives. It is necessary to introduce about the time series techniques that are used in this chapter. In this chapter we wish to discuss all these methods in detail. We try to understand if the observed fluctuations in the time-series of some macroeconomic variables viz., interest rate, wholesale price index, money supply, exchange rates, exports, import and foreign exchange reserve as reported theoretically in the earlier chapter, can be explained in relation to the fluctuations in the time series of inflows of foreign capital. Research done over the past decades shows that before indulging in any econometric modeling using time-series data, one should be concerned about the problem of non-Stationarity or unit root problem. Results from a regression exercise involving non-stationary data is observed to be spurious (Granger and Newbold, 1974 and Granger, 1981). Therefore, the following methodology is carried out in the light of the recent developments in the time-series analysis.

The chapter is organized as follows. In section 5.1, we describe the description of variables and data. In section 5.2, we discuss the methodology and time series econometric tests used in the study of this chapter, which includes vector autoregression (VAR), Engel-Granger (1987) two step procedure, Granger’s (1969) casualty test, and other important extensions of these models as used.
5.1. Description of the Variables and Data

The present study constructs variables according to the objectives. To examine the effects of private foreign capital inflows on macroeconomic variables in India, the following variables are used. These are;

Call money rate (CMR) is taken as domestic interest rate. The model incorporates domestic interest rate i.e. call money rate. This is because the significant increase in call money rate attracts huge private foreign capital inflows. Foreign Exchange Reserve (FOREX) directly involves with the private foreign capital inflows. As increase in private foreign capital inflows into the countries helps to increase in foreign exchange reserve. A private foreign capital inflow (FINV) is the major variable in the present study, which classified as FDI and FPI, where as FII the major segment of FPI. Money supply (M3) is one of the major macroeconomic variables. The huge inflows of private foreign capital affect the money supply. Whole sale price Index (WPI) is taken as for inflation rate. The WPI is the proper measurement of inflation rate. Exchange rate (EXR) which is rupees (Rs.) against US $, appreciates or depreciates as the increase or decrease of private foreign capital inflows. Export (EXP) and Import (IMP) is directly relates to private foreign capital inflows. The index of industrial production (IIP) is taken as the proxy of GDP, though the study is based on monthly time series data, the monthly data of GDP is not available.

To examine the effects of volatility of international oil price and international interest rate on private foreign capital inflows, following variables are used. These are; Private foreign capital inflows (FINV), London Inter Bank Offered Rate (LIBOR) is taken as the international interest rate and International Crude oil Price (ICOP). Both of the variables are very important for the study to examine the volatility of capital flows.

The variables have been used for examining the impact of capital flows on economic growth in India are Foreign Direct investment (FDI), Foreign Portfolio Investment (FPI), Foreign Institutional Investment (FII), and Index of Industrial Production (IIP). To examine the impact of volatility of capital flows on exchange rate, variables such as
Nominal Effective Exchange Rate (NEER) and Real Effective Exchange rate (REER) are used.

5.2. Methodology of the Study

Firstly, in order to examine the effects of private foreign capital inflows on macroeconomic variables namely, wholesale price index, exchange rate, money supply, export, import, foreign exchange reserve, rate of interest, index of industrial production, Vector Autoregressive (VAR) method, impulse response function and variance decomposition technique are employed examine the short-term dynamics and casual relationship between variables.

Secondly, to examine the effect of volatility of international oil price and international interest rate on private foreign capital flows, the study makes use of regression analysis generating volatility series through Generalized Autoregressive Conditional Heteroscedasticity (GARCH 1.1) process and simple variance model. Also, to examine the impact of volatility of capital flows on exchange rates in India, regression analysis generating volatility series through Generalized Autoregressive Conditional Heteroscedasticity (GARCH 1.1) process and simple variance model have been used.

Finally, to examine the impact of capital flows on economic growth in India, Engel-Granger two-step cointegration procedure (1987) and pair-wise Granger causality test (1969) are used. However, the non-stationarity nature of most series data and the need for avoiding the problem of spurious nonsense regression calls for the examination of their stationarity property.

Briefly, variables whose mean, variance and auto-covariance (at various lags) change over time are said to constitute a non-stationarity time series or are variables with unit root\(^1\). Conversely, a time series is stationarity, if it mean, variance, autocovariance (at various lags) are time independent.

\(^1\) The term unit root refers to the root of the polynomial in the lag operator.
Let $Y_t$ be a stochastic time series being represented as $Y_1, Y_2, \ldots, Y_t$ and the mean and the variance of the process at time $t$ is given as:

\[
\text{Mean } \mu = E(Y_t) \quad \ldots \ldots (5.1) \\
\text{Variance } (Y_t) = E((Y_t - \mu)^2) = \sigma^2 \quad \ldots \ldots (5.2)
\]

Similarly, the covariance between $Y_t$ and $Y_{t+k}$ is given by

\[
\text{Cov}[Y_t + Y_{t+k}] = \gamma_k = E[(Y_t - \mu)(Y_{t+k} - \mu)] \quad \ldots \ldots (5.3)
\]

Where $k = 1, 2, 3 \ldots t$, $\gamma_k$, the covariance (or autocovariance) at lag $k$, is the covariance between the values $Y_t$ and $Y_{t+k}$, that is between two $Y$ values $K$ periods apart. If $K=0$, we obtain $\gamma_0$, which is the simply the variance of $Y$ ($=\sigma^2$); if $K=1$, $\gamma_1$ is the covariance between two adjacent values of $Y$.

Formally, the series $Y_t$ is said to be stationary, if the following conditions are satisfied for all $m$:

\[
E(Y_{t+m}) = \mu \quad \ldots \ldots \ldots (5.4) \\
E((Y_{t+m} - \mu)^2) = \sigma^2 \text{ and,} \quad \ldots \ldots (5.5) \\
E[(Y_{t+m} - \mu)(Y_{t+m+k} - \mu)] = \gamma_k \quad \ldots \ldots (5.6)
\]

The above conditions are generally known as the conditions of weak stationarity. For most of the applications these are the conditions required in the time series literature.

A stationarity (weak) series will have a well determined mean which will not vary much with the sampling period and will tend to return to its mean value. The simplest example of a non-stationarity process is the random walk hypothesis, which is defined as:

\[
Y_t = Y_{t-1} + u_t \quad \ldots \ldots \ldots \ldots (5.7)
\]

Where, $u_t \sim (0, \sigma^2)$
It may be noted here that the present study employs two tests namely, Dickey Fuller (DF) tests and Augmented Dickey Fuller (ADF) tests for the detection of the unit roots in the concerned variables. Both Engel-Granger and Johansen cointegration technique have been employed to examine the long run dynamic relationship between private foreign capital flows and macroeconomic variables and capital flows and economic growth in India. Detailed discussions about the methods to be employed are discussed below.

5.2.1. Test of Stationarity

Before estimating the VAR model, the unit root tests examine the stationary properties of the variables. In this study two unit root tests, viz. Dickey Fuller (DF) and Augmented Dickey Fuller (ADF) tests have been conducted to examine the stationarity properties of the variables.

5.2.1.1. Dickey Fuller (DF) and Augmented Dickey Fuller (ADF) Tests

Dickey and Fuller (1979) consider three different regression equations that can be used to test the presence of a unit root:

\[ \Delta Y_t = \gamma Y_{t-1} + \varepsilon_t \quad \ldots \quad (5.8) \]
\[ \Delta Y_t = \alpha_0 + \gamma Y_{t-1} + \varepsilon_t \quad \ldots \quad (5.9) \]
\[ \Delta Y_t = \alpha_0 + \gamma Y_{t-1} + \alpha_2 t + \varepsilon_t \quad \ldots \quad (5.10) \]

In the above equations, the difference between the three regressions concerns the presence of the deterministic elements \( \alpha_0, \alpha_2 t \). The first is a pure random walk model, the seconds adds an intercept or drift term, and the third equation includes both a drift and linear time trend. The parameter of interest in all the regression equation is \( \gamma \); if \( \gamma = 0 \), the \( \{Y_t\} \) sequence contains a unit root. The test involves estimating one or more of the equations above using OLS in order to obtain the estimated value of \( \gamma \) and associated standard error. Comparing the resulting t-statistic with the appropriate value reported in the Dickey Fuller tables allows us to determine whether to accept or reject the null hypothesis \( \gamma = 0 \).
In conducting Dickey Fuller test as in Equations 5.8, 5.9 and 5.10, it was assumed that the error term $\varepsilon_t$ was uncorrelated. But when the assumption of uncorrelated error term is $\varepsilon_t$ is relaxed, Dickey and Fuller have developed another test of unit root which is known as the Augmented Dickey Fuller (ADF) test, where the lagged difference terms of the variable are included in the model to make the error term serially independent. This test is conducted by ‘augmenting’ the preceding three equations such as 5.8, 5.9 and 5.10 by adding the lagged values of the independent variable $\Delta Y_t$. The ADF test may be specified as follows:

$$\Delta Y_t = a_0 + a_1 t + \gamma Y_{t-1} + \sum_{i=1}^{k} \beta_i Y_{t-i} + \varepsilon_t \quad \ldots \ldots \ldots (5.11)$$

Where $\varepsilon_t$ is a pure white noise error term and where $\Delta$ is difference operator, $\gamma$ and $\beta$ are the parameters.

In ADF test we still test whether $\gamma = 0$ and the ADF test follows the same asymptotic distribution as the DF statistics, so the same critical values can be used. It is worth while pointing out that the appropriate static to be used depends on the deterministic components included in the regression equation. When there is no intercept and trend, we use $\tau$ statistic; with only the intercept, use the $\tau_0$ statistic; and with both an intercept and trend, use $\tau_t$ statistic. The statistics labeled $\tau$, $\tau_0$ and $\tau_t$ are the appropriate statistics to be used in 5.8, 5.9 and 5.10 respectively. The DF test forms a special case of the ADF test when the summation part in the right hand side of Equation 5.11 is detected or when $K = 0$ [Dickey Fuller (1979)]. For ADF test, the value of $K$ is determined, based on the Akaike Information Criteria (AIC) and Schwarz Information Criteria (SIC).

One advantage of ADF is that it corrects for higher order serial correlation by adding lagged difference term on the right hand side. If the simple unit root test is valid only if the series is an $AR(1)$ process. One of the important assumptions of DF test is that error terms are uncorrelated, homoscedastic as well as identically and independently distributed (iid).
5.3. Vector Autoregression (VAR)

To examine the dynamic relationship between private foreign capital inflows with macroeconomic variable, a vector auto regression model (VAR) is employed. This approach has two major advantages over the extent of empirical research on this issue. First, VAR superficially resembles simultaneous equation modeling in that all the variables are considered to be endogenous. However, each endogenous variable is explained by its lagged or past values and lagged values of the other endogenous variables included in the model. Usually there are no exogenous variable in the model. Thus, by avoiding the imposition of a priori restriction on the model the VAR adds significantly to the flexibility of the model. Second, the VAR methodology can accommodate general dynamic relationship among economic variables. Because most of the relevant empirical analyses utilize a partial equilibrium framework and do not account fully for dynamic interrelations, previous studies relating this topic may yield misleading inferences.

A natural starting place for multivariate models is treating each variable symmetrically. In a two variable case we can let the time path of private foreign capital inflows \( \{P_t\} \) and macroeconomic variables \( \{E_t\} \) sequence and let the time path of macroeconomic variables \( \{E_t\} \) sequence be affected by current and past realizations of the private capital flows \( \{P_t\} \). Consider the simple bivariate system

\[
P_t = b_{10} - b_{12} E_t + \gamma_{11} P_{t-1} + \gamma_{12} E_{t-1} + \varepsilon_{P_t} \quad \ldots (5.12)\]

\[
E_t = b_{20} - b_{21} P_t + \gamma_{21} P_{t-1} + \gamma_{22} E_{t-1} + \varepsilon_{E_t} \quad \ldots (5.13)\]

Where, it is assumed that

(i) private foreign capital inflows \( \{P_t\} \) and macroeconomic variables \( \{E_t\} \),
(ii) both \( \{P_t\} \) and \( \{E_t\} \) are stationary,
(iii) \( \varepsilon_{P_t} \) and \( \varepsilon_{E_t} \) are white-noise disturbances with standard deviations of \( \sigma_P \) and \( \sigma_E \) respectively, and
(iv) \( \{\varepsilon_{P_t}\} \) and \( \{\varepsilon_{E_t}\} \) are uncorrelated white-noise disturbances.
The structure of the system incorporates feedback, since $P_t$ and $E_t$ are allowed to affect each other. For example, $-b_{12}$ is the contemporaneous effect of a unit change of $E_t$ on $P_t$ and $\gamma_{12}$ is the effect of a unit change in $E_{t-1}$ on $P_t$. The terms $\varepsilon_{P_t}$ and $\varepsilon_{E_t}$ are pure innovations (or shocks) in $P_t$ and $E_t$ respectively. If $b_{21}$ is not equal to zero, $\varepsilon_{P_t}$ has an indirect contemporaneous effect on $E_t$ and if $b_{12}$ is not equal to zero, $\varepsilon_{E_t}$ has an indirect contemporaneous effect on $P_t$.

The Equations 5.12 and 5.13 are not reduced form equations since $E_t$ has a contemporaneous effect on $P_t$ and $P_t$ has a contemporaneous effect on $E_t$. Using the matrix algebra, the system of equations can be transformed into a more usable and compact form. Rewriting the system of equations in matrix form we get:

$$Bx_t = \Gamma_0 + \Gamma_1 x_{t-1} + \varepsilon_t$$

... (5.14)

Where,

$$B = \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix}; \quad x_t = \begin{bmatrix} P_t \\ E_t \end{bmatrix}; \quad \Gamma_0 = \begin{bmatrix} b_{10} \\ b_{20} \end{bmatrix}; \quad \Gamma_1 = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{bmatrix} \quad \text{and} \quad \varepsilon_t = \begin{bmatrix} \varepsilon_{P_t} \\ \varepsilon_{E_t} \end{bmatrix}$$

Equation 5.14 represents primitive form of VAR. Pre-multiplication by $B^{-1}$ in Equation 5.14 gives us the Vector Autoregressive (VAR) model in standard form:

$$x_t = A_0 + A_1 x_{t-1} + e_t$$

... (5.15)

Where, $A_0 = B^{-1} \Gamma_0; A_1 = B^{-1} \Gamma_1,$ and $e_t = B^{-1} \varepsilon_t$. The process in Equation 5.15 looks like an autoregressive process but with a difference that $x_t, A_0$ and $e_t$ are now vectors. For notational purposes, we can define $a_{i0}$ as element $i$ of the vector $A_0; a_{ij}$ as the element in row $i$ and column $j$ of the matrix $A_1; \quad e_{it}$ as the element $i$ of the vector $e_t$. Using this notation, the Equation 5.15 can be rewritten in the equivalent form:
\[
P_t = a_{10} + a_{11}P_{t-1} + a_{12}E_{t-1} + e_{1t} \quad \ldots \ (5.16)
\]
\[
E_t = a_{20} + a_{21}P_{t-1} + a_{22}E_{t-1} + e_{2t} \quad \ldots \ (5.17)
\]

It is important to note that the error terms (i.e. \(e_{1t}\) and \(e_{2t}\)) are composites of the two shocks \(\varepsilon_{Pt}\) and \(\varepsilon_{Et}\). Since \(e_t = B^{-1}e_{lt}\), \(e_{1t}\) and \(e_{2t}\) can be computed as:

\[
e_{1t} = (\varepsilon_{P_t} - b_{12}\varepsilon_{E_t})/(1 - b_{12}b_{21}) \quad \ldots \ (5.18)
\]
\[
e_{2t} = (\varepsilon_{E_t} - b_{21}\varepsilon_{P_t})/(1 - b_{12}b_{21}) \quad \ldots \ (5.19)
\]

Since \(\varepsilon_{Pt}\) and \(\varepsilon_{Et}\) are white noise processes, it follows that both \(e_{1t}\) and \(e_{2t}\) have zero means, constant variances and are individually serially uncorrelated. But the critical point to be noted is that the covariance between \(e_{1t}\) and \(e_{2t}\) will not be zero so that two shocks will be correlated. In the special case, where \(b_{12} = b_{21} = 0\) (i.e. if there are no contemporaneous effects of \(P_t\) on \(E_t\) and \(E_t\) on \(P_t\)), the shocks will be uncorrelated. It is useful to determine the variance and covariance matrix of the \(e_{1t}\) and \(e_{2t}\) shocks as:

\[
\Sigma = \begin{bmatrix}
\text{var}(e_{1t}) & \text{cov}(e_{1t}, e_{2t}) \\
\text{cov}(e_{1t}, e_{2t}) & \text{var}(e_{2t})
\end{bmatrix}
\]

Since all elements of \(\Sigma\) are times independent, we can use the more compact form:

\[
\Sigma = \begin{bmatrix}
\sigma_1^2 & \sigma_{12} \\
\sigma_{21} & \sigma_2^2
\end{bmatrix}
\]

Where, \(\text{var}(e_{it}) = \sigma_i^2\) and \(\sigma_{12} = \sigma_{21} = \text{cov}(e_{1t}, e_{2t})\).

Now we may discuss the different steps that are associated with computation of VAR model
5.3.1. Choice of Lag Length

In order to check lag length at first, the longest plausible length or longest feasible length is chosen given degrees of freedom consideration. For example, using quarterly data, lag length 12 is chosen. Second the VAR is estimated and variance and covariance matrixes of residuals are formed. Variance and covariance matrixes of residuals from 12-lag model can be called $\Sigma_{12}$. Now suppose, we want determine if 8 lag is appropriate. The restriction of model from 12 to 8 lags would reduce the number of estimated parameters by $4n$ in each equation.

5.3.2. Selection of Variables in the System

Now, we discuss some of the important steps, which are involved in VAR estimation. To begin with, the selection of appropriate variable to be included in the model is very important. There is no specific method for selection of the variable. The choice is purely based on the underlying economic theory. Testing the Stationarity of the variables is the next step. In time series literature, unit root tests are used to check whether a variable or series included in the model is stationary or not. For the VAR estimation, it is essential that all the variables included in the system should be stationary either at level or at first differences.

The last and vital step of VAR estimation is the selection of appropriate lag length of each variable in the system. The selection of the appropriate lag length is the biggest practical challenge in VAR modeling. It may be possible to use different lag length for each variable in the equation. Such type of VAR is called as NEAR VAR and can be estimated through seemingly unrelated regression (SUR). But for the sake of simplicity the same lag length is used for all equations. Various lag selection criteria are used to select the optimum lag length of the model. These are likelihood ratio (LR), final prediction error (FPE), Akaike information criteria (AIC), Schwarz information criteria (SIC) and Hannan–Quinn information criteria (HQ). Having set the lag length, the final step is to estimate the model.
The model is estimated through ordinary least squares (OLS). The most important thing is that the individual coefficients in estimated VAR models are often difficult to interpret directly. To overcome this problem, we use innovation accounting techniques, which include impulse response function and variance decomposition.

The variables to be included in the VAR are selected according to the relevant economic model. Otherwise no explicit attempt is made to ‘pare down’ the number of parameters estimates. Suppose a multivariate VAR is given as follows:

\[ X_t = A_0 + A_1 X_{t-1} + A_2 X_{t-2} + \ldots \ldots + A_p X_{t-p} + e_t \]

Where, \( X_t \) = the \((n \times 1)\) vector containing each of the \(n\) variables included in the VAR
\( A_0 \) = an \((n \times 1)\) vector of intercept terms.
\( A_i \) = an \((n \times n)\) matrix of coefficient.
\( e_t \) = an \((n \times 1)\) vector of error terms.

In the above example, matrix \( A_0 \) contains \(n\) intercept term and each matrix \( A_i \) contains \(n^2\) coefficients, hence \(n+pn^2\) terms need to be estimated. Unquestionably, a VAR will be over parameterized by which many of these coefficient estimates can be properly exclude.

5.3.3. Exogenity in VAR Model

A necessary condition for the exogenity of \( S_t \) is that current and past values of \( E_t \) do not affect \( S_t \). The sequence \( \{S_t\} \) may not exogenous to \( \{E_t\} \) even though \( \{E_t\} \) does not Granger cause \( \{S_t\} \). Because pure shocks to \( \{E_t\} \), i.e. \( \varepsilon_{E_t} \), may affect the value of \( \{S_t\} \), though \( \{E_t\} \) sequence does not Granger cause the \( \{S_t\} \) sequence.

A block exogenity test is useful to determining whether to incorporate a variable into a VAR. Given the above distinction between causality and exogenity, the multivariate generalization of the Granger-Causality test should be called a ‘block causality’ test. In any event, the issue is to determine whether the lags of one variable, say \( W_t \), Granger
cause any of the variables in the system. In the three variables case, \( W_t, S_t \) and \( E_t \), the test is whether lags of \( W_t \) in the \( S_t \) and \( E_t \) equations to be equal to zero. This cross equation restriction is properly tested using the likely hood ratio test given as follows:

\[
(T-c) \left( \log |\Sigma_r| - \log |\Sigma_u| \right)
\]

Where, \( \Sigma_u \) and \( \Sigma_r \) are the variance and covariance matrixes of the unrestricted and restricted respectively.

### 5.3.4. Impulse Response Function (IRF)

The impulse response function (IRF) shows the dynamic responses of all variables in the system to a shock or innovation in each variable. For computing IRFs, it is essential that the variables in the system are ordered and that the system is represented by a moving average process. The vector moving average (VMA) representation of Equation 5.18 expresses the variables \( P_t \) and \( E_t \) in terms of current and past values of the two shocks \( \varepsilon_{P_t} \) and \( \varepsilon_{E_t} \).

Writing Equations 5.16 and 5.17 in matrix form, we get:

\[
\begin{bmatrix}
P_t \\
E_t
\end{bmatrix} =
\begin{bmatrix}
a_{10} & a_{11} & a_{12} \\
a_{20} & a_{21} & a_{22}
\end{bmatrix} +
\begin{bmatrix}
P_{t-1} \\
E_{t-1}
\end{bmatrix} +
\begin{bmatrix}
e_{P_t} \\
e_{E_t}
\end{bmatrix}
\]

... (5.20)

Now, recalling the VAR model in standard form, i.e. Equation 5.18, we have:

\[ x_t = A_0 + A_1 x_{t-1} + e_t \]

If we iterate back-wards and assume that stability condition is met, then the particular solution for \( x_t \) is:

\[ x_t = \mu + \sum_{i=0}^{\infty} \phi_i e_{t-i} \]

... (5.21)

Where \( \mu = [\bar{P}, \bar{E}] \)

Using Equation 5.21 we can rewrite Equation 5.20 as:
Equation 5.22 expresses $P_t$ and $E_t$ in terms of the \{e_{1t}\} and \{e_{2t}\} sequences. However, it is possible to rewrite Equation 5.22 in terms of \{\varepsilon_{Pt}\} and \{\varepsilon_{Et}\} sequences. From equation 5.18 and 5.19, the vector of error terms can be written as:

$$
\begin{bmatrix}
\varepsilon_{1t} \\
\varepsilon_{2t}
\end{bmatrix} = \begin{bmatrix} 1/b_{21} & b_{12} \\ b_{21} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{Pt} \\
\varepsilon_{Et}
\end{bmatrix} \quad \ldots (5.23)
$$

Now, Equations 5.22 and 5.23 can be combined to form:

$$
\begin{bmatrix}
P_t \\
E_t
\end{bmatrix} = \begin{bmatrix} \overline{P} \\
\overline{E}
\end{bmatrix} + (1/b_{21} - b_{12}) \sum_{i=0}^{\infty} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} 1 \\
1/b_{21}
\end{bmatrix} \begin{bmatrix} \varepsilon_{Pt} \\
\varepsilon_{Et}
\end{bmatrix} \quad \ldots (5.24)
$$

To simplify the above notation, now define the $2 \times 2$ matrix $\phi_i$ with elements $\phi_{jk}(i)$ such that:

$$
\phi_i = [A_i^T/(1-b_{12}b_{21})] \begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix}
$$

Hence, the moving average representation of Equations 5.23 and 5.24 can be written in terms of \{\varepsilon_{Pt}\} and \{\varepsilon_{Et}\} sequences:

$$
\begin{bmatrix}
P_t \\
E_t
\end{bmatrix} = \begin{bmatrix} \overline{P} \\
\overline{E}
\end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} \phi_{11}(i) & \phi_{12}(i) \\ \phi_{21}(i) & \phi_{22}(i) \end{bmatrix} \begin{bmatrix} \varepsilon_{St-i} \\
\varepsilon_{Et-i}
\end{bmatrix}
$$

Or, more compactly:

$$
x_t = \mu + \sum_{i=0}^{\infty} \phi_i \varepsilon_{t-i} \quad \ldots (5.25)
$$

The moving average representation is especially useful to examine the interaction between \{P_t\} and \{E_t\} sequences. The coefficients of $\phi_i$ can be used to generate the effects of $\varepsilon_{Pt}$ and $\varepsilon_{Et}$ shocks on the entire time paths of the \{P_t\} and \{E_t\} sequences. The four elements $\phi_{jk}(0)$ are called as impact multipliers. For example, coefficient $\phi_{12}(0)$ is the
The instantaneous impact of a one unit change in $\varepsilon_{Et}$ on $P_t$. Similarly, the elements $\phi_{11}(I)$ and $\phi_{12}(I)$ are the one-period response of unit changes in $\varepsilon_{Pt-1}$ and $\varepsilon_{Et-1}$ on $P_t$ respectively.

The four sets of coefficients $\phi_{11}(i)$, $\phi_{12}(i)$, $\phi_{21}(i)$ and $\phi_{22}(i)$ are called impulse response functions. Plotting the impulse response functions [i.e. plotting the coefficients of $\phi_{jk}(i)$ against $i$] is a practical way to visually represent the behavior of the $\{P_t\}$ and $\{E_t\}$ series in response to the various shocks. With knowledge of knowing all the parameters of the primitive system of Equations (5.12) and (5.13), it is possible to trace out the time paths of the effects of pure $\varepsilon_{Pt}$ or $\varepsilon_{Et}$ shocks. However, this methodology is not applicable if the estimated VAR is under or over identified. Here, in this example, the estimated VAR is under-identified, because primitive VAR system contains 10 parameters whereas VAR in standard form contains only 9 parameters. So, an additional restriction on the VAR system must be imposed in order to identify the impulse responses. One possible identification restriction is to use the Choleski decomposition. For example, it is possible to contain the system such that the contemporaneous value of $P_t$ does not have a contemporaneous effect on $E_t$. Finally, this restriction is represented by setting $b_{21} = 0$ in the primitive system. In terms of Equation (5.23), the error terms can be decomposed as follows:

$$e_{1t} = \varepsilon_{Pt} + b_{12}\varepsilon_{ET} \quad ........ (5.26)$$

$$e_{2t} = \varepsilon_{Et} \quad ........ (5.27)$$

Equation 5.27 shows all the observed errors from the $\{e_{2t}\}$ sequence are attributed to $\varepsilon_{Et}$ shocks. Given the calculated $\{\varepsilon_{Et}\}$ sequence, knowledge of the values of the $\{e_{1t}\}$ sequence and the correlation coefficient between $e_{1t}$ and $e_{2t}$ allows for the calculation of the $\{\varepsilon_{Pt}\}$ sequences using equation 5.26. Although this decomposition contains the system such that a $\varepsilon_{Pt}$ shock has no direct effect on $E_t$, there is an indirect effect in that lagged values of $P_t$ affect the contemporaneous value of $E_t$. The key point is that the decomposition forces potentially important asymmetry on the system, because $\varepsilon_{Et}$ has contemporaneous effects on both $P_t$ and $E_t$. For this reason Equations 5.26 and 5.27
imply an ordering of variables. An $\epsilon_E$ shock directly affects $e_{1t}$ and $e_{2t}$, but an $\epsilon_Pt$ shock does not affect $e_{2t}$. Hence, $E_t$ is ‘prior’ to $P_t$. Alternatively, by putting $b_{12} = 0$, the errors can be decomposed as:

$$e_{1t} = \epsilon_p t$$

$$e_{2t} = b_{21} \epsilon_{pt} + \epsilon_{Et}$$

It is crucial to note that the importance of the ordering depends on the magnitude of the correlation coefficient between $e_{1t}$ and $e_{2t}$. For example, if the correlation coefficient is equal to zero, the ordering is immaterial. Finally, Equations 5.26 and 5.27 can be replaced with $e_{1t} = \epsilon_{pt}$ and $e_{2t} = \epsilon_{Et}$. On the other hand, if the correlation coefficient is unity (so that two shocks are equivalent), it is inappropriate to attribute the shock to a single source.

5.3.5. Variance Decomposition

Variance decomposition is used to detect the causal relation among the variables. It explains the extent to which a variable is explained by the shocks in all the variables in the system. The forecast error variance decomposition explains the proportion of the movement’s private foreign capital inflows in a sequence due to its own shock versus shocks to the other macroeconomic variable. The VAR in standard form, i.e. Equation 3.4A is written as follows:

$$x_t = A_0 + A_1 x_{t-1} + e_t$$

Now, suppose the coefficient $A_0$ and $A_1$ is known and we want to forecast the various values of $x_{t+1}$ conditional to the observed value of $x_t$. Updating the above equation by one period (i.e. $x_{t+1} = A_0 + A_1 x_t + e_{t+1}$), the conditional expectation of $x_{t+1}$ is:

$$E_t x_{t+1} = A_0 + A_1 x_t$$

Here, one-step ahead forecast error is $x_{t+1} - E_t x_{t+1} = e_{t+1}$. Similarly, the two-step ahead forecast error of $x_{t+2}$ is:

$$E_t x_{t+2} = (1 + A_1) A_0 + A_1^2 x_t$$
The two-step ahead forecast error is $e_{t+2} + A_1 e_{t+1}$. More generally the n-step ahead forecast is:

$$E_t x_{t+n} = [1 + A_1 + A_1^2 + \ldots + A_1^{n-1}] A_0 + A_1^n x_t$$

And, the associated forecast error is:

$$E_t e_{t+n} + A_1 e_{t+n-1} + A_1^2 e_{t+n-2} + \ldots + A_1^{n-1} e_{t+1}$$

It is possible to write the forecast errors in terms of the $\varepsilon_{Pt}$ and $\varepsilon_{Et}$ shocks. The forecast error variance decomposition tells the proportion of their movements in a sequence due to its own shock versus shocks to the other variable. If $\varepsilon_{yt}$ shocks explain none of the forecast error variances of $\varepsilon_{Pt}$ at all forecast horizons, it can be said that $\{P_t\}$ sequence is exogenous. In such a circumstance, the $\{P_t\}$ sequence would evolve independently of the $\varepsilon_{Et}$ shocks and of $\{E_t\}$ sequence. On the other hand, if $\varepsilon_{Et}$ shocks explain all of the forecast error variances in $\{P_t\}$ sequence at all forecast horizons, then $\{P_t\}$ would be entirely endogenous.

5.4. Cointegration Technique

In order to avoid the problem of spurious regression which arise due to the non-stationary nature of the data in time series analysis, cointegration technique came to the rescue. Thus, when the variables contain a unit root, modern time series techniques of cointegration are used to establish long run equilibrium relationship between the private foreign capital flows with macroeconomic variables including economic growth. In general, cointegration is defined as the long run equilibrium relationship among the set of non-stationarity variables provided their linear combination is found to be stationary. A principal feature of cointegrated variables is that their time paths are influenced by any extent of any deviations from long run equilibrium relationship. After all, if the system is to return to equilibrium, the movement of at least some of the variables must respond to the magnitude of disequilibrium. Our study uses two methods of testing for cointegration namely, Johansen-Juselius multivariate cointegration technique to test for cointegration and long-run equilibrium relationship among the macroeconomic variables including
economic growth and Engel-Granger (1987) two step procedures which are discussed below.

The cointegration method is applied to a wide variety of economic models. Cointegration tests examine the possible existence of a long-run equilibrium relationship between two or more variables, which must be integrated of the same order. Any equilibrium relationship among a set of non-stationary variables implies that their stochastic trends must be linked. After all, the equilibrium relationship means that the variables cannot move independently of each other. This linkage among the stochastic trends necessitates that the variables be cointegrated. Since the trends of cointegrated variables are linked, the dynamic paths of such variables must bear some relation to the current deviation from the equilibrium relationship. Thus, the conventional wisdom of differencing all non-stationary variables used in a regression analysis was incorrect.

There are two main approaches to test for cointegration. They are Engle and Granger (1987) two step procedure and the Johansen and Juselius (1990) procedure. Though the Engle-Granger (1987) cointegration procedure is easy to implement, but it is not free from limitations. The estimation of the long run equilibrium regression requires that the researcher place one variable on the left hand side and use the others as regressors. In practice, it is possible to find that one regression indicates the variables are cointegrated whereas reversing the order indicates no cointegration. This is a very undesirable feature of the procedure since the test for cointegration should be invariant to the choice of the variable selected for normalization. Moreover, in tests using three or more variables, we know that there may be more than one cointegrating vector. The method has no systematic procedure for the separate estimation of the multiple cointegrating vectors. Another limitation of the Engle-Granger procedure is that it relies on a two-step estimator. Hence, if any error introduced by the researcher in first step is carried into second step.
5.4.1. **Engle-Granger Two Step Procedure**

Engel-Granger (1987) procedure is employed to detect the presence of long run equilibrium relationship between two or more variables in a single equations system. The equilibrium relationship means that the variables can not move independently of each other. However, Engel Granger procedure necessitates that the variables must be integrated of same order. To check the order of integration among the variables, various test such as DF and ADF tests are to be employed, which are discussed earlier. If a series is differentiated, ‘d’ times before it gets stationary, then it is said to be integrated of order ‘d’ and denoted I(d).

Engel and Granger’s (1987) formal analysis begins by considering a set of economic variables in long run equilibrium when \( \beta_1 x_{1t} + \beta_2 x_{2t} + \ldots + \beta_n x_{nt} = 0 \). If we let \( \beta \) and \( x_t \) denote the vectors \((\beta_1, \beta_2, \ldots, \beta_n)\) and \((x_{1t}, x_{2t}, \ldots, x_{nt})\), the system is in long run equilibrium when \( \beta x_t = 0 \). The deviation from long run equilibrium is called the equilibrium error i.e. \( e_t = \beta x_t \). If the equilibrium is meaningful, it may be the case that the equilibrium error process is stationary. As per the Engel Granger’s methodology, the component of vector \( x_t = (x_{1t}, x_{2t}, \ldots, x_{nt}) \) are said to be cointegrated of order \( d, b \), denoted by \( x_t \sim CI (d, b) \) if:

1. All the components of \( x_t \) are integrated of order \( d \).
2. There exists a vector \( \beta = (\beta_1, \beta_2, \ldots, \beta_n) \) such that linear combination \( \beta x_t = \beta_1 x_{1t} + \beta_2 x_{2t} + \ldots + \beta_n x_{nt} \) is integrated of order \( (d-b) \), where \( b > 0 \). The vector \( \beta \) is called cointegrating vector.

The detailed procedure of Engel Granger’s two step procedure is as follows:

Consider two variables; say capital flows denoted by \( S_t \) and economic growth denoted by \( E_t \), which are integrated of order 1; then the Engel and Granger procedure to check for cointegration involves the following two step.

Step 1: In order to estimate the long run equilibrium relationship between capital flows \( (S_t) \) and economic growth \( (E_t) \), it is only necessary to estimate the static model:
\[ S_t = \beta_0 + \beta_1 E_t + \epsilon_t \quad \ldots \quad (5.28) \]

Estimating Equation 5.28 using OLS achieves a consistent estimate of the long run steady state relationship between the variables in model, and, all dynamic can be ignored asymptotically. This arises because of what is termed as the super consistency property of OLS estimators when series are cointegrated.

Step 2: In order to determine if the variables are actually cointegrated, the estimated residuals are generated from Equation 5.28. If these deviations are found to be stationary, then \( S_t \) and \( E_t \) sequences are cointegrated of order \((1, 1)\). It would be convenient if we could perform DF test on these residuals to determine their order of integration. Let us consider the following regression:

\[ \Delta e_t = \alpha_1 e_{t-1} + \epsilon_t \quad \ldots \quad (5.29) \]

Since the \( \{e_t\} \) sequence is residuals from a OLS regression, there is no need to be include an intercept term; the associated t statistic of ‘\( \alpha_1 \)’ coefficient can be used to check for stationarity of residuals. However, it may be noted here that when there are two variables tested for cointegration, the usual DF table can be used, but when there involves more than two variables the appropriate critical values are provided by Engel and Yoo (1987). If we can not reject the null hypothesis of non-stationarity \( (H_0: \alpha_1 = 0) \), we can conclude that the residual series contains a unit root. Thus, given that both \( S_t \) and \( E_t \) were found to be \( I(1) \) and that the residuals sequence is stationary, we can conclude that the series are cointegrated of order \((1, 1)\).

Now, if we variables are cointegrated, the residuals from equilibrium regression can be used to estimate error correction model (ECM). When \( S_t \) and \( E_t \) are cointegrated of order \((1, 1)\) the error correction is represented as:

\[ \Delta S_t = \alpha_1 + \alpha_{11} \Delta E_{t-1} + \sum_{i=1}^{m} \alpha_{11}(i) \Delta S_{t-1} + \sum_{i=1}^{m} \alpha_{12}(i) \Delta E_{t-1} + \epsilon_t \quad \ldots \quad (5.30) \]
\[ \Delta E_t = \alpha_2 + \alpha_E \hat{E}_{t-1} + \sum_{i=1}^{m} \alpha_{21}(i) \Delta S_{t-i} \sum_{i=1}^{m} \alpha_{22}(i) \Delta E_{t-i} + \epsilon_{E_t} \]  

\[ \Delta S_t = \text{Change in Private foreign capital flows} \]
\[ \Delta E_t = \text{change in economic growth} \]

The ECM involves the estimation of the above equation system and the speed of adjustment is given by the coefficients \( \alpha_S \) and \( \alpha_E \) which have important implications for the dynamics of the system. Thus, for any given value of \( e_{t-1} \), a large value of \( \alpha_E \) is associated with the large values of \( \Delta E_t \). On the other hand, \( \alpha_E \) is zero, the change in \( E_t \) does not at all respond to the deviation from long run equilibrium in period \( t-1 \). Thus, \( \alpha_S \) and \( \alpha_E \) quantify the extent to which the short run deviation from long run equilibrium is adjusted in the next period.

5.5. Granger Causality Test

The short run dynamic relationship between the capital flows and economic growth may be examined by using the concept of Granger’s (1969) causality test. Granger’s causality [proposed by Granger (1969) and popularized by Sims (1972)] may be defined as the forecasting relationship between two variables. In short, Granger causality test states that if \( S \) & \( E \) are two time series variables and, if past values of a variable \( S \) significantly contribute to forecast the value of the other variable \( E \), then \( S \) is said to be Granger causing \( E \) and vice versa. The test involves the following two regression equations:

\[ S_t = \gamma_0 + \sum_{i=1}^{n} \alpha_i E_{t-i} + \sum_{j=1}^{n} \beta_j S_{t-j} + u_{1t} \]  

\[ E_t = \gamma_1 + \sum_{i=1}^{m} \lambda_i X_{t-i} + \sum_{j=1}^{m} \delta_j + E_{t-j} + u_{2t} \]  

Where, \( S_t \) and \( E_t \) are the are capital inflows and economic growth to be tested, and \( u_{1t} \) and \( u_{2t} \) are mutually uncorrelated white noise errors, and \( t \) denotes the time period. Equation
5.32 postulates that current $S$ is related to past values of $S$ as well as of past $E$. Similarly, Equation 5.33 postulates that $E$ is related to past values of $E$ as well as related to past values of $S$. Three possible conclusions can be adduced from such analysis viz., unidirectional causality, bi-directional causality and that they are independent of each other.

1. Unidirectional causality from $E$ to $S$ is indicated if the estimated coefficients on the lagged $E$ in equation (5.32) are statistically different from zero as a group (i.e. $\sum_{i=1}^{n} \alpha_i \neq 0$) and set of estimated coefficients on the lagged $E$ in (equation (5.33)) is not statistically different from zero (i.e. $\sum_{j=1}^{n} \delta_j = 0$).

2. Unidirectional causality from $S$ to $E$ exists if the set of lagged $E$ coefficients in Equation 5.32 is not statistically different from zero (i.e. $\sum_{i=1}^{n} \alpha_i = 0$) and the set of the lagged $S$ coefficients in Equations 5.33 is statistically different from zero (i.e. $\sum_{j=1}^{n} \delta_j \neq 0$).

3. Feedback or bilateral causality is suggested when the sets of $E$ and $S$ coefficients are statistically and significantly different from zero in both regression.

4. Finally, independence is suggested when the sets of $E$ and $S$ coefficients are not statistically significant in both regressions.

There are two important steps involved with the Granger’s causality test. First, stationary data is required for Equation 5.32 and 5.33. Second, in addition to the need for testing the stationary property of the data, the Granger methodology somewhat sensitive to the lag length used in equations 5.32 and 5.33. It is better to use more rather than fewer lag length since the theory is couched in terms of the relevant past information. The chosen lag length must be matched with the actual lag length. If it is lesser than actual lag length, the omission of relevant lags can be cause bias and if it is more than the relevant lag length causes the equations to be insufficient. To deal with this problem, it developed a
systematic autoregressive method for choosing appropriate lag length. Therefore, the appropriate lag length is one where Akaike’s Final Prediction Error (FPE) is lowest. Akaike Information Criteria (AIC), or Schwarz Information Criteria (SIC), or Likelihood Ratio (LR) Criterion or Hannan-Quinn information Criterion (HQIC) is also useful for choosing the lag length.

5.6. Generalized ARCH (GARCH) Models

In order to examine the effect of volatility of international oil price and international interest rate on private foreign capital inflows and also examines the impact of volatility of capital flows on exchange rate in India. The ARCH and GARCH methods employed. The detailed about the methods of ARCH and GARCH are discussed below:

The above discussed ARCH model may call for long lag structure to model the underlying volatility in the market. Keeping this view in mind, a more parsimonious and broad class of model was developed by Bollerslev (1986) and Taylor (1986). The simplest form of GARCH model is the GARCH (1, 1) model which is written as

\[
\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2
\]

or

\[
h_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2
\]  

\[\text{……..(5.34)}\]

Where, \(\alpha_0 > 0; \alpha_1 \geq 0; \beta_1 \geq 0\)

The stationary condition for GARCH (1,1) is \(\alpha_1 + \beta_1 < 1\).

GARCH model says that the conditional variance of \(u\) at time \(t\) depends not only on the squared error terms in the previous time period [i.e. ARCH (1)] but also the conditional variance in the previous time period. The GARCH model essentially generalizes the purely autoregressive moving average model. The weight on pass squared residuals is assumed to decline geometrically at a rate to be estimated from the data.

The conditional volatility equation represented by (5.34) comprises of three terms, viz., (a) the mean, \(\alpha\) (b) news about volatility from the previous period, measured as the lag of the squared residual from the mean equation, \(u_{t-1}^2\); \(\alpha\) and (c) the last periods forecast
error variance $\delta^2_{t-1}$. The GARCH specification suggests that an agent predicts this period’s variance by forming a weighted average of a long term average (the constant), the forecast variance from the last period and information about the volatility observed in the previous period. In the GARCH (1, 1) model, the effect of a return shock on current volatility declines geometrically over time. This model gives weights to the conditional variance ($\delta^2_t$), the previous error variance ($\delta^2_{t-1}$) and the news about volatility of the previous period ($u^2_{t-1}$).

The GARCH specification allows us to model the variance of exchange rate changes as time dependent. This is in contrast with the usual assumption made when estimating a moving average process in which it is assumed that the error term has a constant variance. Overall, this specification permits us to exploit pattern and persistence in the behavior of volatility. The time dependent specification has the additional property that it explains the heavy tailed nature of the distribution of the exchange rate changes. By modeling volatility explicitly, GARCH model directly relates risk in the foreign exchange market to trade performance. Another advantage of the GARCH approach lies in producing more efficient estimates since heteroskedasticity of the error is handled properly.

5.6.1. GARCH (p,q) Model

The GARCH model can be extended to a GARCH (p,q) model in which p is the lagged term of the squared error term and q is lagged conditional variance. This may be represented as;

$$h_i = \alpha_0 + \alpha_1 u^2_{t-1} + \alpha_2 u^2_{t-2} + \ldots + \alpha_q u^2_{t-q} + \beta_1 \sigma^2_{t-1} + \beta_2 \sigma^2_{t-2} + \ldots + \beta_p \sigma^2_{t-p} \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots
$$

Where, $\alpha > 0, \alpha_i \geq 0, \beta_j \geq 0$

In both ARCH and GARCH models, restrictions are to be placed on the parameters to keep the conditional volatility positive. This also implies that any shock is always an
indication of increase in conditional volatility forever. In order to check the presence of ARCH effects on the data, we have applied Lagrange Multiplier (LM) tests.

Since the empirical results are influenced by the measurement of capital flow volatility, the selection of a particular measure as a crucial for any studies in this regard. In this study the volatility is measured by simple variance model and GARCH method to identify whether the results vary across the volatility measures. More recently, it has been noted that only asset prices are characterized by many large changes but they seem also to be characterized by “volatility clustering”. The autoregressive conditional heteroskedasticity (ARCH) model by Engle (1982) and Generalized ARCH (GARCH) act as the most popular models to model volatility which capture empirical regularities as outlined above. The recent approach of measuring volatility is the GARCH approach. We discuss those methods elaborately in methodology chapter-v. In case of GARCH specification the variance of capital flows changes is time dependent. The time dependent specification has the additional property that explains the heavily tailed nature of the distribution of flow of capital. By modeling volatility explicitly, the GARCH approach directly relates risk at the volatility of international oil price and international interest rate. Both international oil prices (ICOP) and international interest rate (LIBOR) are used in the study to find the effect of their volatility effect on private foreign capital flows.