5.1 INTRODUCTION

This chapter emphasises the common-cause failure models in evaluating the availability measure of the system including both time dependent availability, steady-state availability. Also we discussed the coefficient of efficiency of renewal with respect to a single server. This can be derived using the reliability and availability measures of the system. In this chapter we developed the estimate of availability measure under the influence of common-cause failures, in three categories, namely

(i) Chance common-cause shocks.
(ii) Non-lethal common-cause shocks.
(iii) Lethal common-cause shocks.
The measure of availability is very important in the case of maintained systems, which tells the proportion of time the system under consideration is working satisfactorily. Thus, first we derive the availability of the system when the system is influenced by common-cause shocks occurring with certain chance.

5.2 CCS MODEL - AVAILABILITY ANALYSIS

The assumptions governing in the CCS Model was explained in Section (4.2) of Chapter-IV. The availability of the system in concept slightly differs to that of reliability in the sense that, we allow repair even after the system is found in failed condition at any time and the system is restored to its operational use after a while of down-time. Thus we want in the case of availability the status of the system at any time instant 't', rather than 'failure-free' operation at an instant of time 't', of course, the latter
refers to reliability which we already discussed in Chapter-IV.

Therefore, in the Markov graph (seen in Fig.(4.1)) there is a slight change indicating that the transition be allowed from state '2' to '1' in the case of single server which is possible. Therefore in this case Markov equations for a general two component systems can be formulated as [4]

\[ P_0(t+dt) = [1 - (\lambda_0 + \lambda_1)dt] P_0(t) + \mu_1 P_1(t)dt \]

\[ P_1(t+dt) = \lambda_0 dt P_0(t) + \left[ 1 - (\mu_1 + \lambda_2)dt \right] P_1(t) + \mu_2 dt P_2(t) \]

\[ P_2(t+dt) = \lambda_1 dt P_0(t) + \lambda_2 dt P_1(t) + \left[ 1 - \mu_2 dt \right] P_2(t) \]

\[(5.2.1)\]

Using the LT and the initial conditions the set of equations (5.2.1) can be written in matrix notation
in the following way

\[
\begin{pmatrix}
(s + \lambda_0 + \lambda_1) & -\mu_1 & 0 \\
-\lambda_0 & (s + \mu_1 + \lambda_2) & -\mu_2 \\
-\lambda_1 & -\lambda_2 & (s + \mu_2)
\end{pmatrix}
\begin{pmatrix}
P_0^*(s) \\
P_1^*(s) \\
P_2^*(s)
\end{pmatrix}
= \begin{pmatrix}
1 \\
0 \\
0
\end{pmatrix}
\]

(5.2.2)

Under the conditions mentioned earlier we get the vector \( \mathbf{P}^*(s) \)

\[
\mathbf{P}^*(s) = \begin{pmatrix}
s^2 + sA + \mu_2 \mu_1 & s(s^2 + sB + C) \\
(s + \lambda_0 + \lambda_1) P_0^*(s) - 1 \\
(s) - P_0^*(s) + P_1^*(s)
\end{pmatrix}
\]

(5.2.3)

where

\[ A = \mu_1 + \lambda_2 + \mu_2 \]
\[ B = \mu_1 + \lambda_o + \lambda_1 + \mu_2 + \lambda_2 \]

\[ C = \mu_2 \mu_1 + \mu_2 \lambda_o + \mu_2 \lambda_1 + \lambda_1 \mu_1 + \lambda_2 \lambda_o + \lambda_2 \lambda_1 \]

\[ P_0(t) = \left[ \mu_2 \mu_1 / \alpha_1 \alpha_2 \right] + \left[ G_1 \exp(\alpha_1 t) - G_2(\alpha_2 t) \right] / (\alpha_1 - \alpha_2) \quad (5.2.4) \]

\[ P_1(t) = \left[ \lambda_o \mu_2 + \lambda_1 \mu_2 \right] / \alpha_1 \alpha_2 + \left[ H_1 \exp(\alpha_1 t) - H_2 \exp(\alpha_2 t) \right] / (\alpha_1 - \alpha_2) \quad (5.2.5) \]

\[ P_2(t) = 1 - (P_0(t) + P_1(t)) \quad (5.2.6) \]

where \( \alpha_1, \alpha_2 = (-p \pm \sqrt{q})/2 \) \quad (5.2.7)

\[ p = \lambda_o + \lambda_1 + \mu_1 + \mu_2 + \lambda_2 \]

\[ q = p^2 - 4(\lambda_o \mu_2 + \lambda_1 \mu_2 + \mu_2 \mu_1 + \mu_1 \lambda_1 + \lambda_2 \lambda_o + \lambda_1 \lambda_2) \]
and

\[ G_1 = \frac{\alpha_1^2 + \alpha_1(\mu_1 + \lambda_2 + \mu_2)}{\alpha_1} \]

\[ G_2 = \frac{\alpha_2^2 + \alpha_2(\mu_1 + \lambda_2 + \mu_2)}{\alpha_2} \]

(5.2.8)

\[ H_1 = \frac{\alpha_1 \lambda_0 + \lambda_0 \mu_2 + \lambda_1 \mu_2}{\alpha_1} \]

\[ H_2 = \frac{\alpha_2 \lambda_0 + \lambda_0 \mu_2 + \lambda_1 \mu_2}{\alpha_2} \]

and the quantities \( \lambda_0, \lambda_1, \lambda_2, \mu_1, \mu_2 \) are to be substituted as

\[ \lambda_0 = \lambda_2 = \lambda_a c_1 \]

\[ \lambda_1 = \lambda_a c_2, \quad \mu_1 = \mu_s \quad \text{and} \quad \mu_2 = 2\mu_s \quad (5.2.9) \]

since if the system is in state '2' and when a service facility is being used and if one of the components are restored then the system goes from state '2' to '1'.

5.2.1 CCS Model - Steady-State Availability
Series System

We are mostly interested to find the proportion of the time the system is in operational use in the long run usage of the system. This is usually be obtained by taking \( t \) very large i.e., \( t \to \infty \). Thus the steady-state availability is

\[
\pi_s(\infty) = \lim_{t \to \infty} (\text{PROB. system is working})
\]

Since we know that '0' is the only state of working of the system, the steady-state availability of the system be obtained as

\[
\pi_s(\infty) = \lim_{t \to \infty} \left[ P_0(t) \right]
\] (5.2.10)

Therefore the steady-state availability in the case of CCS Model is derived as

\[
\pi_{sCS}(\infty) = \lim_{t \to \infty} \left[ P_0(t) \right]
\] (5.2.11)
where $P_0(t)$ in (5.2.11) has to be substituted using (5.2.4).

Using final value theorem (Shooman [58] p.108), (5.2.11) can be written as

$$\pi_{scs}(\infty) = \lim_{s \to 0} \left[ sP_0^*(s) \right]$$

Thus the steady-state availability of the series system be derived as

$$\pi_{scs}(\infty) = \frac{\mu_2 \mu_1}{\left( \lambda_0 \mu_2 + \lambda_1 \mu_2 + \mu_2 \mu_1 + \mu_1 \lambda_1 + \lambda_2 \lambda_0 + \lambda_1 \lambda_2 \right)}$$

$$= \frac{2\mu_2}{2\lambda_a \mu_s c_1 + 2\mu_s \lambda_a c_2 + 2\mu_s^2 + \lambda_a \mu_s c_2} \quad (5.2.12)$$

defining 'operating Ratio' $\eta = \frac{\lambda_a}{\mu_s}$

We can express the steady-state availability given
in (5.2.12) as

$$\pi_{sCS}^{(\infty)} = \frac{1}{1 + \eta + (\eta c_2 / 2)} \quad (5.2.13)$$

The result developed in (5.2.13) agrees with the result already established in the case of single failures by assuming that the chance of occurrence of common-cause failures is zero i.e., by taking $c_2 \lambda_a = 0; \quad c_2 = 0$ and $\lambda_a = \text{constant.}$

5.2.2 CCS Model - Steady-State Availability - Parallel System

The formula for steady-state $\pi$ for the parallel system in the case of CCS Model be obtained using [12]

$$\pi_{pcs}^{(\infty)} = \lim_{t \to \infty} \left[ P_0(t) + P_1(t) \right]$$

Using the final value theorem

$$= \lim_{s \to 0} \left[ sP_0^*(s) + sP_1^*(s) \right]$$
Substituting the quantities $\lambda_0$, $\lambda_1$, $\lambda_2$, $\mu_1$, $\mu_2$ using (5.2.9) the steady-state availability for the parallel system be obtained as

$$\pi_{pcs}(\infty) = \frac{(1+\eta)}{(1+\eta+\eta c_2+\eta^2(1-c_2)/2)} \quad (5.2.15)$$

and the unavailability is expressed as

$$\eta_{pcs}(\infty) = \frac{(\eta c_2+\eta^2(1-c_2)/2)/(1 + \eta + \eta c_2 + \eta^2)(1-c_2)/}{(5.2.16)}$$

5.2.3 Nature of Steady-state Availability

Steady-state availability is, of course, a function of $\eta$, $c_1$ and $c_2$. Without loss of generality we assume that $\eta_i \leq 1$, since in many practical situations failure rate is less than service rate i.e., $\lambda_a < \mu_a$. 

THEOREM (5.1) Steady-state availability of the series system is a decreasing and convex function of $\eta < 1$, and $\forall c_2 \in [0, 1]$.

PROOF: The necessary conditions for the $\pi_s(\infty)$ to be decreasing as well as convex function are

(i) $\partial \pi_{scs}/\partial \eta < 0$ and $\partial^2 \pi_{scs}/\partial \eta^2 > 0$, $\forall \eta < 1$ and

(ii) $\partial \pi_{scs}/\partial c_2 < 0$ and $\partial^2 \pi_{scs}/\partial c_2^2 > 0$, $\forall c_2 \in [0, 1]$

consider the condition (i)

$$\frac{\partial \pi_{scs}}{\partial \eta} = -(1 + c_2/2)/(1 + n + nc_2/2)^2 \quad (5.2.17)$$

The RHS of (5.2.17) is always negative $\forall c_2 \in [0, 1]$ and $\eta < 1$

Also $\partial^2 \pi_{scs}/\partial \eta^2 = 2(1 + c_2/2) (1 + \eta + \eta c_2/2)^3 \quad (5.2.18)$
The RHS of (5.2.18) is always positive $\eta$, $\eta < 1$ and $c_2 \in [0, 1]$

Thus it follows from (5.2.17) and (5.2.18) that condition (i) is satisfied.

Further,

$$\frac{d\pi_{scs}}{dc_2} = -\frac{n}{2}/(1+\eta+\eta c_2/2)^2 \quad (5.2.19)$$

The RHS of the (5.2.19) is always negative $\forall c_2 \in [0, 1]$ and $\eta < 1$

and also

$$\frac{d^2\pi_{scs}}{dc_2^2} = \eta^4/2(1+\eta+\eta c_2/2)^3 \quad (5.2.20)$$

is always positive $\forall c_2 \in [0, 1]$ and $\eta < 1$

Thus it follows from (5.2.19) and (5.2.20) that condition (ii) is satisfied. Hence the proof is completed.
From the theorem it is established that steady-state availability in the case of series system is decreasing and convex function both for operating ratio ($\eta$) and chance of common-cause failures occurring. This result tells that more the chance of common-cause shock failures more the unavailability and also more the rate of the common-cause occurrence then also unavailability is more.

PARTICULAR CASES

(i) If the chance of occurrence of common-cause shock failures is zero (i.e., $c_2 = 0$) implies that $\lambda_a c_2 = 0$ then steady-state availability is $\pi_{scs}^{ss} = 1$.

(ii) If $c_2 = 1$ and $\lambda_a$ is very large $\Rightarrow \lambda_a c_2 = \infty$ then $\pi_{scs}^{ss} = 0$.

(iii) If $\lambda_a$ is neither too small nor too large and if $c_2 = 0$, i.e., chance of common-cause occurrence is negligibly
small, then availability of the series system reduces as the availability of the system influenced by individual failures only.

iv) If \( c_2 = 1 \), that means when the system is totally influenced by common-cause failures, the expression of availability is derived as

\[
\pi_{scs}(\infty) = \frac{1}{1+\eta+\eta/2}
\]

and unavailability

\[
Q_{scs}(\infty) = \frac{\eta+\eta/2}{(1+\eta+\eta/2)}
\]

(5.2.21)

PROPOSITION (5·1)

Series system which is influenced by only common-cause failures (no individual failures) will have more unavailability.
PROOF:

Consider

\[ Q_{scs}(\infty) - Q_{ss}(\infty) \]

\( Q_{scs}(\infty) \) Unavailability of the system when common-cause failures are only occurring.

\( Q_{ss}(\infty) \) Unavailability of the system when individual failures are only occurring.

\[ Q_{scs}(\infty) - Q_{ss}(\infty) = \eta(3/2-1)/(1+(3/2)\eta)(1+\eta) \quad (5.2.22) \]

The RHS of the above expression is always positive for all \( \eta < 1 \).

Hence the proof is completed.

In consequence to the result in the proposition (5.1) we obtain the percentage increase in unavailability of the system in the case of series configuration due to occurrence of common-cause failures as
The percentage of decrease in availability of
the series system depends on the failure rate \( \lambda_a \).
The percentage decrease in availability of the
series system due to common-cause shock failures as
compared to individual failures is calculated and
given in Table (5.1). It is observed that there is
atmost of 20 per cent decrease in availability due
to influence of common-cause shocks only. On the
similar lines as shown in Theorem (5.1) we can show
that steady-state availability in the case of
parallel system is a decreasing and concave function
for all \( \eta < 1 \).

PARTICULAR CASES :

(i) The availability of the parallel system
is \( \pi_{pca}(\infty) = 1 \), when \( c_2 = 0 \) which implies
that when the system is influenced by only individual failures, the system is always available certainly for use in long run.

(ii) The steady-state availability of the parallel system is \( \pi_{pcs}(\infty) = \frac{1}{2} \) when \( c_2 = 1 \), this implies that when the system is influenced by the common-cause failures certainly and if failure rate is large, the system is available fifty per cent.

(iii) If \( c_2 = 1 \) and failure rate is finite then the availability is given by

\[
\pi_{pcs}(\infty) = \frac{(1+\eta)}{(1+2\eta)} \tag{5.2.23a}
\]

PROPOSITION (5.2)

The steady-state availability of the parallel system, like in series case, is less under the influence of common-cause shock failures than
individual failures case.

PROOF

Consider \( \pi_{\text{pcs}}(\infty) - \pi_{\text{ps}}(\infty) \)

\( \pi_{\text{pcs}}(\infty) \) Steady-state availability of the parallel system under the influence of common-cause failures only.

\( \pi_{\text{ps}}(\infty) \) Steady-state availability of the parallel system under the influence of individual failures only.

\[
\pi_{\text{pcs}}(\infty) - \pi_{\text{ps}}(\infty) = \eta/(1+2\eta) \tag{5.2.24}
\]

RHS of (5.2.24) is always positive for all \( n < 1 \).
Hence the proof is completed.

The percentage decrease in \( \pi(\infty) \) due to the influence of the common-cause failures in the case of parallel system can be obtained for various levels
of values of $\eta$. It is seen that the maximum decrease is 30 per cent when $\eta < 1$.

The percentage decrease in availability both for series and parallel systems are plotted in Fig.\(5.3\) and Fig.\(5.4\). Also, unavailability for both cases are also plotted in Fig.\(5.1\) and Fig.\(5.2\). From the graphs we can observe that in series case, the unavailability increases steeply but in parallel case, however, the increase is not so. This unfolds the fact that the availability of the series system is very much influenced by the common-cause shock failures than the parallel system.

5.3 DISCUSSION ON TIME DEPENDENT SOLUTION

In this section, we investigate the condition under which time dependent availability is real in the case of CCS Model.
5.3.1 Time Dependent Solution - Series System

From (5.2.4) we derive the time dependent solution for the series system as

\[ \pi_{scs}(t) = \frac{2\mu_s^2}{(2\mu_s^2 + 2\mu_g \lambda_1 c_1 + 3\mu_s \lambda_1 c_2)} 
+ \left[ G_1 \exp(\alpha_1 t) - G_2 \exp(\alpha_2 t) \right] / (\alpha_1 - \alpha_2) \]

(5.3.1)

\[ \alpha_1, \alpha_2 = \left[ -(\lambda_1 + 3\mu_g) \pm \sqrt{(\lambda_1 + 3\mu_g)^2 - 4(2\mu_s^2 + 2\mu_g \lambda_1 c_1 + 3\mu_s \lambda_1 c_2)} \right] \]

(5.3.2)

Therefore we see that \( \pi_{scs}(t) \) is a function of \( \alpha_1, \alpha_2 \) and \( \lambda_0, \mu_s, c_1, c_2 \). \( \pi_{scs}(t) \) will not be guaranteed as a real valued function. This comes from the fact that the expression under root of the expression (5.3.2) may not always be positive for all \( \lambda_1, \mu_s, c_1 \) and \( c_2 \). Thus, we now enquire the condition for the existence of real value solution of \( \pi_{scs}(t) \).
FIG(5.1): Effect of common-cause failures on the Unavailability* of the Series System-CCS Model.

* Plotted on logarithmic scale.
FIG(5.2): Effect of Common-cause failures on the unavailability of the Parallel system-CCSModel plotted on logarithmic scale
FIG(5.3): Effect of common-cause failures on the percentage decrease in availability—Series system—CCS Model.
FIG(5.4): Effect of Common-cause failures on percentage decrease in $\pi(t)$—Parallel system-CCSModel.
PROPOSITION (5.3)

\( \pi_{scs}(t) \) is real iff the parametric condition is satisfied

\[
c_2 \leq \left[ \frac{\eta}{4 + 9/(4\eta)} - 2/\eta - 0.5 \right] \quad (5.3.3)
\]

PROOF

\( \pi_{scs}(t) \) is a function of \( \alpha_1, \alpha_2 \). But \( \alpha_1, \alpha_2 \) are in turn are functions of \( \lambda_s, \mu_s, c_1, c_2 \).

Therefore, \( \pi_{scs}(t) \) real iff that the term under the square root is non-negative

\[
\Rightarrow (\lambda_s + 3\mu_s)^2 \geq 4(2\mu_s^2 + 2\mu_s \lambda_a c_1 + 3\mu_s \lambda_a c_2)
\]

\[
\Rightarrow \left[ \frac{\lambda_s + 3\mu_s}{2} \right]^2 \geq 2 \mu_s^2 + 2\mu_s \lambda_a c_1 + 3\mu_s \lambda_a c_2 \quad (5.3.4)
\]

(5.3.4) further can be simplified to become

\[
c_2 \leq \left( \frac{\eta}{4 + 9/(4\eta)} - 2/\eta - 0.5 \right)
\]
so that the condition is necessary. Also to prove sufficiency let

\[ B = \left( \eta/4 + 9/(4n) - 2/n - 0.5 \right) \]

and let us substitute

\[ c_2 = B - \delta, \text{ where } \delta > 0, \text{ a small quantity.} \]

We get after substituting and simplification,
(5.3.4) as

\[ 4 \lambda_a \mu_s \delta \]

(5.3.5)

and (5.3.5) is always non-negative for all \( \lambda_a, \mu_s > 0 \)

This satisfies (5.3.2) and in turn (5.3.1). Hence sufficiency proved. Thus the condition stated in (5.3.3) is satisfied to be both necessary and sufficient one.

Thus the proof is completed.
5.3.2 Parallel System

The time dependent solution in the case of parallel system for the case of CCS model is

\[ \pi_{pcs}(t) = \left[ \frac{\mu_2(\mu_1 + \lambda_0 + \lambda_1)}{(\alpha_1\alpha_2)} \right] + \left[ (G_1 + H_1)\exp(\alpha_1 t) \right. \\
\left. - (G_2 + H_2)\exp(\alpha_1 t) \right]/(\alpha_1 - \alpha_2) \] (5.3.6)

**PROPOSITION (6.4)**

\[ \pi_{pcs}(t) \text{ is real iff the following parametric condition} \]

\[ c_2 \leq \left[ \frac{5}{\eta} - 2 \sqrt{\frac{6}{\eta^2} - \frac{1}{\eta}} \right] \] (5.3.7)

is satisfied.

**PROOF**

The necessary condition for the \( \pi_{pcs}(t) \) to be real implies that \( \alpha_1, \alpha_2 \) are to be real.
This implies that the term under the square root of the expression for $\alpha_1$ is non-negative.

Thus

$$(\lambda_0 + \lambda_1 + \mu_1 + \mu_2 + \lambda_2)^2 - 4(\mu_2\lambda_0 + \mu_2\lambda_1 + \mu_2\mu_1 + \lambda_0\lambda_2 + \lambda_1\lambda_2) \geq 0$$

$$\Rightarrow \left[(\eta(1+c_1) + 3)/2\right]^2 - 2 - 2\eta - \eta^2 \geq c_2(\eta - \eta^2)$$

$$\Rightarrow c_2 \leq \left[\frac{5}{2} \eta + 2 \text{ sech}(6/\eta^2 - 1/\eta)\right]$$

sufficiency of the theorem can be proved in the similar lines to that shown in series case.

Thus the proof is completed.

The result shows that given the parameters $\lambda_s, \mu_s$ (i.e., $\eta$) there exist an upper bound on the value of $c_2$ for which the $\alpha_1$ is real and hence $\pi_{\text{pcs}}(t)$ is real.
Also since $c_2$ is the probability of occurrence of common-cause failures, the maximum feasible value it can take is unity and hence in the condition (5.3.7) the maximum value be taken always is unity.

5.4 NCS MODEL - AVAILABILITY ANALYSIS

The assumptions in the sub-section (4.5.1) of the Chapter-IV, will be reconsidered here in order to discuss the availability measure of the system. In this analysis it is emphasised that the system is hit by both individual failures and common-cause (non-lethal) failures. Under the assumptions stated, Markov graph for the availability measure will have a slight change wherein the transition from '2' to '1' is allowed and possible.

Thus the Markov equations in this case are seen as

\[
P_\alpha(t+dt) = P_\alpha(t)[1 - (\lambda_0 + \lambda_1)dt] + P_1(t)[\mu_1 dt]
\]
Formulating differential equations, taking Laplace transformation and using initial condition, we get the vector $\mathbf{a}^*(s)$. Expressing the quantities in the vector $\mathbf{a}^*(s)$ by partial fractions and using inverse Laplace transformation we get

$$P_1(t+dt) = P_0(t) \left( \lambda_0 dt \right) + P_1(t) \left[ 1 - (\mu_1 + \lambda_2) dt \right] + P_2(t) \left( \mu_2 dt \right)$$

$$P_2(t+dt) = P_0(t) \left( \lambda_1 dt \right) + P_1(t) \left( \lambda_2 dt \right) + P_2(t) \left( 1 - \mu_2 dt \right)$$

$$P_0(t) = \left[ \frac{\mu_1 \mu_2}{\sigma_1 \sigma_2} \right] \left[ \sigma_1 \exp(\sigma_1 t) - \sigma_2 \exp(\sigma_2 t) \right] / (\sigma_1 - \sigma_2)$$

(5.4.2)

where

$$Q_1 = \left[ \sigma_1^2 + \sigma_1 (\mu_1 + \mu_2 + \lambda_2) + \mu_1 \mu_2 \right] / \sigma_1$$

$$Q_2 = \left[ \sigma_2^2 + \sigma_2 (\mu_1 + \mu_2 + \lambda_2) + \mu_1 \mu_2 \right] / \sigma_2$$

(5.4.3)
and

\[ \sigma_1, \sigma_2 = \left[ -p \pm \sqrt{q} \right] / 2 \quad (5.4.4) \]

where

\[ p = -(\lambda_0 + \lambda_1 + \lambda_2 + \mu_1 + \mu_2) \]
\[ q = 4(\lambda_0 \mu_2 + \mu_2 \lambda_1 + \mu_1 \mu_2 + \mu_1 \lambda_1 + \lambda_0 \lambda_2 + \lambda_1 \lambda_2) \]

\[ P_1(t) = \left[ \frac{\mu_2(\lambda_0 + \lambda_1)}{\sigma_1 \sigma_2} \right] + \frac{R_1 \exp(\sigma_1 t)}{\sigma_1 - \sigma_2} - \frac{R_2 \exp(\sigma_2 t)}{\sigma_1 - \sigma_2} \quad (5.4.5) \]

where

\[ R_1 = \left[ \frac{\sigma_1 \lambda_0 + (\lambda_0 + \lambda_2) \mu_2}{\sigma_1} \right] \]
\[ R_2 = \left[ \frac{\sigma_2 \lambda_0 + (\lambda_0 + \lambda_2) \mu_2}{\sigma_2} \right] \quad (5.4.6) \]

and \( \sigma_1, \sigma_2 \) are given in (5.4.4). The basic quantities in them say \( \lambda_0, \lambda_1, \lambda_2, \mu_1, \mu_2 \) are to be substituted as shown in (4.5.1).
\[ P_2(t) = 1 - \frac{1}{\sum_{i=0}^{n} P_i(t)} \]  \hspace{1cm} (5.4.7)

5.4.1 NCS Model - Steady-State Availability

The steady-state availability measure of the series system in the case of NCS Model will be

\[ \pi_{sna}(\infty) = \lim_{t \to \infty} P_0(t) \]

where \( P_0(t) \) is derived in (5.4.3).

Using the final value theorem

\[ = \lim_{s \to 0} \left[ sP_0^*(s) \right] \]

\[ = \frac{\mu_1 \mu_2}{(\lambda_0 \mu_2 + \lambda_1 \mu_2 + \mu_1 \mu_2 + \mu_1 \lambda_1)} \]  \hspace{1cm} (5.4.8)

In the case of the series configuration \( \lambda_2 = 0 \), and substituting the quantities given in (4.5.1) we
derive the formula for the steady-state availability of the series system in the case of NCS Model as

$$\pi_{s_{ns}}(\infty) = \frac{2\mu_s}{(2\mu_s+4\lambda+4(3\pi-(3\pi^2)) (5.4.9)}$$

The result obtained in this case will be seen agreeing with the one which was already developed in the context when the system is hit only with individual failures assuming $\beta = 0$ in (5.4.9).

5.4.2 NCS Model - Availability of Parallel System

For the parallel system the steady-state availability can be derived for the NCS Model as

$$\pi_{p_{ns}}(\infty) = \lim_{t \to \infty} \left[ p_0(t) + p_1(t) \right]$$

Using final value theorem

$$= \lim_{s \to 0} \left[ sF_0^*(s) + sF_1^*(s) \right]$$
by substituting the quantities the formula can be expressed as

\[ \pi_{\text{pns}}(\infty) = \frac{\theta}{\sigma} \]

where

\[ \theta = 2u_2^2 + 2u_2 (2\lambda + 2 (\beta p q + 3p^2)) \]

\[ \sigma = 2u_2^2 + 2\lambda^2 + 4\beta \mu_0 + 2\lambda^2 p + 2 (3^2 p^2 - 3p^2 \mu_0 - \lambda \beta p^2 - \beta^2 p^3 + 4\lambda \mu_0) \]

If \( \beta = 0 \), then the result derived in (5.4.10) will agree with the one discussed in the case of individual failures in the literature.

5.5 TIME DEPENDENT AVAILABILITY

Now we derive the time dependent availability
for both type of configurations in the case of NCS Model.

5.5.1 NCS Model - Availability of Series System

From the probability information, given in (5.4.2), we obtain the time dependent availability of the series system for the NCS model as

\[
\pi_{\text{sns}}(t) = \left[ \mu_1 \mu_2 / \sigma_1 \sigma_2 \right] + \left[ Q_1 \exp(\sigma_1 t) \right. \\
- \left. Q_2 \exp(\sigma_2 t) \right] / (\sigma_1 - \sigma_2) 
\]  
(5.5.1)

where \( Q_1, Q_2 \) and \( \sigma_1, \sigma_2 \) are given by (5.4.3) and (5.4.4) respectively.

The quantities \( \lambda_0, \lambda_1, \lambda_2, \mu_1, \mu_2 \) are to be substituted as

\[ \lambda_0 = 2 \left[ \lambda + \alpha \rho \sigma \right], \quad \lambda_1 = \alpha \rho^2, \quad \lambda_2 = 0 \]
\[ \mu_1 = \mu_s \quad \text{and} \quad \mu_2 = 2 \mu_s \]
A COMPUTER PROGRAM (COMPUTER PROGRAM-III) is developed to compute the values of time dependent availabilities for the given parameters for series case. However, one needs to supply the input parameters, individual failure rate ($\lambda$), non-lethal common-cause failure rate ($\beta$), probability a specific component fail due to common-cause shock ($p$) and the service rate ($\mu$).

5.5.2 Parallel System

The time dependent value of availability of the parallel system can be obtained by

$$\Pi_{pns}(t) = \left[\frac{\mu_2(\mu_1 + \lambda_0 + \lambda_1)}{\sigma_1 \sigma_2 + [\Omega_1 + R_1]\exp(\sigma_1 t)} - (\Omega_2 + R_2)\exp(\sigma_2 t)]}{(\sigma_1 - \sigma_2)} \quad (5.5.2)$$

and the quantities in (5.5.2) are given by the expressions (5.4.3) to (5.4.6).
As in the case of series system, even in parallel system time dependent availabilities can be numerically computed using the same COMPUTER PROGRAM i.e., COMPUTER PROGRAM-III.

COMPUTER PROGRAM-III

10 REM A BASIC PROGRAM TO FIND
20 REM TIME DEPENDENT AVAILABILITIES
30 REM FOR BOTH SERIES AND PARALLEL SYSTEMS
40 REM IN THE CASE OF NCS-MODEL
60 INPUT"INDIVIDUAL FAILURE RATE"; L
70 INPUT"COMMON-CAUSE FAILURE RATE"; B
80 INPUT"PROBABILITY OF SPECIFIC COM FAIL"; P
90 INPUT"SERVICE RATE"; M
100 INPUT"UPPER TIME BOUND OF INTEREST TO CALCULATE "; X
110 FOR T = 0 TO X
120 LO = 2*L+2*B*P*(1-P)
130 L1 = B*P^2
140 M1 = M
M2 = M*2
L2 = L+B*P
A = LO+L1+M1+M2+L2
B = LO+L1+M1+M2
D=4*(LO*M2+L1*M2+M1*M2)
A1=(-B+ABS(SQR(B↑2-D)))/2
A2=(-B-ABS(SQR(B↑2-D)))/2
C1=(-A+ABS(SQR(A↑2-C)))/2
C2=(-A-ABS(SQR(A↑2-C)))/2
Q1=(A1↑2+A1*(M2+M1)+M1*M2)/A1
Q2=(A2↑2+A2*(M2+M1)+M1*M2)/A2
P1=(C1↑2+C1*(M1+M2+L2)+M1*M2)/C1
P2=(C2↑2+C2*(M1+M2+L2)+M1*M2)/C2
R1=(LO*C1+(LO+L2)*M2)/C1
R2=(LO*C2+(LO+L2)*M2)/C2
F1=(M1*M2+LO*M2+L1*M2)/(C1*C2)
S = (P1+Q1)*EXP(C1*T)-(P2+Q2)*EXP(C2*T)
S1=S/(C1-C2)
Z1=F1+S1
Q =1 - Z1
360 S2 = Q1*EXP(A1*T)-Q2*EXP(A2*T)
370 E = S2/(A1-A2)
390 Z2 = E+G
400 PRINT L,B,Z1,Z2
410 INPUT "DO YOU WISH TO TRY WITH ANOTHER B"; N$
420 IF N$='YES' THEN 60
430 PRINT 'END OF JOB'
440 END

(This Program is developed for CG-2000 COLOUR GENIE)

5.6 NCS MODEL - COEFFICIENT OF EFFICIENCY OF RENEWAL

Coefficient of efficiency of renewal is the measure of efficiency of repair considered in the case of maintained systems. This tells us how best the service is and will improve the overall reliability of the system in the sense of improving its availability. Thus to arrive at this measure we consider the conditional probability of the system be uninte-
interruptedly working in the duration of time 't' starting at instant of time 't' if service is considered and we consider the same when service is not provided. Now the ratio of these two quantities will give us the measure of efficiency of renewal. The coefficient of efficiency of renewal in the NCS Model for the series case is

\[ K_{esns}(t) = \frac{R_{sns}(t, \tau)}{R_{sns}(t, \tau)} \bigg| \mu_s = 0 \quad (5.6.1) \]

\[ = \pi_{sns}(t) \frac{R_{sns}(\tau)}{R_{sns}(t) R_{sns}(\tau)} \bigg| \mu_s = 0 \]

\[ (5.6.2) \]

using the result in (2.2.12).

Now we arrive at the expression of \( K_{esns}(t) \) using the results in (4.5.8) and (5.5.1)

Thus the coefficient of efficiency of renewal is

\[ K_{esns}(t) = \pi_{sns}(t) \frac{\exp[-(2\lambda + 2\eta \beta - 3p^2)\tau]}{\exp[-(2\lambda + 2\eta \beta - 3p^2)(t+\tau)]} \]
\[ \pi_{sns}(t) / \exp[-(2\lambda + 2\beta p - 3\beta^2)t] \] (5.6.3)

The numerator of (5.6.3) can be replaced by the result in (5.5.1). The coefficient of efficiency of renewal arrived in the case of series configuration is lengthy and we develop the COMPUTER-IV to calculate the numerical values of 'coefficient'. Also the program is meant for calculating the time dependent availability and reliability of the series system.

COMPUTER PROGRAM-IV

10 REM A BASIC PROGRAM TO FIND RELIABILITY,
20 REM AVAILABILITY, \( R_{sns}(t,\tau) \), \( R_{sns}(t,\tau) \) WITHOUT
30 REM SERVICE FOR NCS-MODEL-SERIES SYSTEM.
40 INPUT "INDIVIDUAL FAILURES RATE"; L
50 INPUT "COMMON-CAUSE FAILURES RATE"; B
60 INPUT "TIME LENGTH IN WHICH FAILURE-FREE PROB.NEEDED"; TO
70 INPUT "PROB OF SPECIFIC COM.FAILURE"; P
80 INPUT "SERVICE RATE"; M
90 PRINT "TIME", "RELIABILITY", "AVAILABILITY",
110 L0 = 2*(L+B*P*(1-P))
120 L1 = B*P^2
130 M1 = M
140 M2 = 2*M
150 Z1 = LO*M2; Z2 = L1*M2; Z3 = M1*M2; Z4 = M1*L1
160 Z = Z1+Z2+Z3+Z4
170 F = (M2*M1)/Z
180 PRINT "STEADY-STATE AVAILABILITY = "; F
190 A = (LO+L1+M1+M2)
200 D = (LO*M2+L1*M2+M1*M2+M1*L1)
210 A1 = (-A+ABS(SQR(A^2-4*D)))^2
220 A2 = (-A-ABS(SQR(A^2-4*D)))^2
230 T1 = (A1 + A1*(M2+M1)+M2*M1)/A1
240 T2 = (A2 + A2+(M2+M1)+M2*M1)/A2
250 INPUT "UPPER BOUND ON TIME"; U
260 FOR T=0 TO U
270 S1 = T1*EXP(A1*T)-T2*EXP(A2*T)
280 S = S1/(A1-A2)
290 Q = S+F
ILLUSTRATION (5.1)

Suppose if individual failure rate is 0.1 f/hr., and for various non-lethal common-cause failure rates \( \alpha = 0, 0.05, 0.1, 0.5 \) and \( p = 0.5 \) coefficient of efficiency is calculated using the COMPUTER PROGRAM-IV, to study the effect of non-lethal common-cause shock failures on the system. The coefficient of efficiency is seen increasing as time increasing. This comes from the fact...
that service under consideration is keeping availability of the system at a high level say for example 0.972054 (see Table (5.1a)) while the reliability of the system comes down rigorously.

Also, when the non-lethal common-cause shock failure rate increases, the 'coefficient of efficiency of renewal' increases.

This further tells us the fact that the increase in non-lethal common-cause shock failures occurrence will result in very significant decrease in reliability of the system but however the availability is not decreasing to that extent. Therefore, it is resulted in increase in $K_{esns}(t)$ for larger (see Table (5.1d)).

The values of $K_{esns}(t)$ (coefficient factors) are plotted against time $'t'$ for various values of (seen in Fig. (5.5)). In the Fig. (5.5) the plot of $K_{esns}(t)$ for $\gamma = 0$ refer to the coefficient of renewal when the
system is affected with only individual failures.
We can easily observe that curves with regard to $\beta \geq 0$
of $K_{esns}(t)$ are all above the curve with $\beta = 0$. Also
we observe that $K_{esns}(t)$ increases with increase in $\beta$.

This gives an idea that the consideration of
service facility would however be highly justified
when the influence of non-lethal common-cause shock
is more on the system than it is not there ($\beta = 0$).
Finally the reliability and availability curves of
the series system are also plotted using the above
example and seen in Fig.(5.6). In the similar way,
for the parallel system also one can consider the
discussion on coefficient of efficiency of renewal.
| TIME | $H_{sns}(t,T)$ | $R_{sns}(t,T)|_{\mu_1=0}$ | AVAIL | RELIA | COEFFICIENT |
|------|----------------|-----------------------------|-------|-------|-------------|
| 0    | 0.81873        | 0.81873                     | 1     | 1     | 1           |
| 1    | 0.80268        | 0.67032                     | 0.98039 | 0.81873 | 1.1975     |
| 2    | 0.80268        | 0.54881                     | 0.98039 | 0.67032 | 1.4626     |
| 3    | 0.80268        | 0.44329                     | 0.98039 | 0.54881 | 1.7864     |
| 4    | 0.36788        | 0.36790                     | 0.30119 | 2.1819  |
| 5    | 0.24660        | 0.24660                     | 0.20190 | 4.8560  |
| 6    | 0.16530        | 0.16530                     | 0.11080 | 7.2442  |
| 7    | 0.09072        | 0.09072                     | 0.07427 | 10.8070 |
| 8    | 0.06081        | 0.06081                     | 0.04980 | 13.1997 |
| 9    | 0.04076        | 0.04076                     | 0.03337 | 16.1222 |
| 10   | 0.02732        | 0.02732                     | 0.02237 | 19.5917 |
| 11   | 0.01832        | 0.01832                     | 0.01499 | 24.0515 |
| 12   | 0.01499        | 0.01499                     | 0.00337 | 29.3766 |
| 13   | 0.00337        | 0.00337                     | 0.00273 | 35.8506 |
| 14   | 0.00273        | 0.00273                     | 0.00183 | 43.8247 |
| 15   | 0.001499       | 0.001499                    | 0.00183 | 53.5276 |

Table (5.1a): Coefficient Values of Series system-
NCS - Model when $\lambda=0.1$, $\beta=0$, $T0=1$, $p=0.5$. 
<table>
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<th>$R_{SN}(t, \tau)$</th>
<th>$r_s=0$</th>
<th>AVAIL</th>
<th>RELIA</th>
<th>COEFFICIENT</th>
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Table 5.1b: Coefficient values of Series system- NCS-Model when $\lambda=0.1$, $\beta=0.05$, $\Gamma=1$, $p=0.5$. 
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Table 5.1c: Coefficient values of Series system- NCS Model, When $\Lambda=0.1, \beta=0.1, p=0.5, \tau_0=1$. 
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Table (5.1d): Coefficient value of Series System-NCS Model, when $\lambda=0.1, \beta=0.5, IC=1, p=0.5$. 
FIG(5.5): Coefficient of efficiency of renewal ($K_{est}(t)$) (V/S) time for Various $\beta$ -Series system- NCS Model.
FIG(5.6): Reliability and Availability (V/S) Time as a function of \( \beta \)-Series system-NCS Model.
* Plotted on log Scale

**FIG(5.5):** Coefficient of efficiency of renewal \( K_{e_{rms}}(t) \) (Vs) time for Various \( \beta \) series system - NCS Model.
5.7 LCS MODEL - AVAILABILITY ANALYSIS

We derive already the reliability of the system when it is influenced by the lethal common-cause failures in addition to individual as well as non-lethal shock failures. We now consider the availability of the system under the influence of lethal shock failures. The assumptions regarding the model are already explained in Section (4.7) of the Chapter-IV in this thesis. Under the assumptions as far as availability analysis is concerned, we could see that $\mu_2$ is considered in the Markov graph (see Fig. (4.3)) and hence the Markov equations for the present model are

\[
P_0(t+dt) = P_0(t) \left[ 1 - (\lambda_0 + \lambda_1)dt + P_1(t)\mu_1dt \right]
\]

\[
P_1(t+dt) = P_0(t) \lambda_0 dt + P_1(t) \left[ 1 - (\mu_1 + \lambda_2)dt \right] + P_2(t)\mu_2dt
\]

\[
P_2(t+dt) = P_0(t) \lambda_1 dt + P_1(t)\lambda_2 dt + P_2(t)\left[ 1 - \mu_2 dt \right]
\]

(5.7.1)
The above set of equations can be solved using Laplace transformation technique and vector $\vec{p}(t)$ is

$$p_0(t) = \left[\mu_1\mu_2/\alpha_2^2\right] \left[ g_1 \exp(\alpha_1 t) - g_2 \exp(\alpha_2 t)\right] / (\alpha_1 - \alpha_2)$$

(5.7.2)

where

$$g_1 = \left[\alpha_1^2 + \alpha_1 (\mu_2 + \mu_1 + \lambda_2) + \mu_1 \mu_2\right] / \alpha_1$$

(5.7.3)

$$g_2 = \left[\alpha_2^2 + \alpha_2 (\mu_2 + \mu_1 + \lambda_2) + \mu_1 \mu_2\right] / \alpha_2$$

$$p_1(t) = \left[\left(\lambda_0 + \lambda_1\right) \mu_2 / \alpha_1 \alpha_2\right] + \left[H_1 \exp(\alpha_1 t) - H_2 \exp(\alpha_2 t)\right] / (\alpha_1 - \alpha_2)$$

(5.7.4)

where

$$H_1 = \left[\alpha_1 \lambda_0 + (\lambda_0 + \lambda_2) \mu_2\right] / \alpha_1$$

$$H_2 = \left[\alpha_2 \lambda_0 + (\lambda_0 + \lambda_2) \mu_2\right] / \alpha_2$$

(5.7.5)

$$\alpha_1, \alpha_2 = (-\lambda \pm \sqrt{D}) / 2$$
and \( A = - (\lambda_0 + \lambda_1 + \lambda_2 + \mu_1 + \mu_2) \)

\[ D = A^2 - 4(\lambda_0 \mu_2 + \mu_2 \lambda_1 + \mu_1 \mu_2 + \mu_1 \lambda_1 + \lambda_0 \lambda_2 + \lambda_1 \lambda_2) \]  

(5.7.6)

\[ P_2(t) = 1 - \sum_{i=0}^{1} P_i(t) \]  

(5.7.7)

5.7.2 LCS Model – Steady-State Availability

Series System

For the series system, the steady-state availability in the case of LCS Model is

\[ \pi_{sls}^{(\infty)} = \lim_{t \to \infty} [P_0(t)] \]

\[ = \lim_{s \to 0} [sP_0^*(s)], \text{ using the final value theorem.} \]

\[ = \frac{\mu_1 \mu_2}{\lambda_1 \lambda_2} \]  

(5.7.8)

\[ = \frac{2\mu_s}{(4\lambda + 2\mu_s + 3\omega + 4\beta p + 3\beta p^2)} \]  

(5.7.9)
5.7.3 Steady State - Availability Parallel System

For the parallel system, the steady-state availability in the case of LCS Model is

\[ \pi_{pls}(\infty) = \lim_{t \to \infty} \left[ P_0(t) + P_1(t) \right] \]

\[ = \lim_{s \to 0} \left[ sP_0(s) + sP_1(s) \right] \]

using the final value theorem.

Thus the expression of availability for the parallel system in the case of LCS model is

\[ \pi_{pls}(\infty) = \frac{N}{D} \]

where

\[ N = 4\lambda u_s + 4\beta pq u_s + 2\mu_s \beta p^2 + 2\mu_s \omega + 2\mu_s^2 \]

\[ D = 2\lambda^2 + 2\mu_s^2 + 4\lambda\beta p + 2\beta^2 p^2 + 3\omega u_s \]

\[ + 3 \mu_s \beta p^2 + 4\lambda u_s + 4 pq u_s + \omega + \omega \beta \omega - \lambda \beta p^2 - \beta^2 p^3 \]

\[ (5.7.10) \]
The result developed for the LCS Model is agreeing with the one already developed in the case of individual failures case by assuming $\beta = \omega = 0$. This has been verified for both series and parallel systems.

5.7.4 Time Dependent Availability

(1) Series System

The time dependent solution of availability is given by

$$
\pi_{s1s}(t) = \left[ \mu_1 \mu_2 / (\alpha_1 \alpha_2) \right] + \left[ G_1 \exp(\alpha_1 t) - G_2 \exp(\alpha_2 t) \right] / (\alpha_1 - \alpha_2) \tag{5.7.11}
$$

where

$G_1, G_2, \alpha_1$ and $\alpha_2$ are given in (5.7.3) and (5.7.6).
(ii) Parallel System

Following the results given in (5.7.2) to (5.7.4) the availability of parallel system as a function of 't' can be derived as

\[
\pi_{pls}(t) = \left[ \mu_2 \left( \lambda_0 + \lambda_1 + \mu_1 \right) / (\alpha_1 \alpha_2) \right] + \left[ (G_1 + H_1) \exp(\alpha_1 t) \right.
\]

\[
- (G_2 + H_2) \exp(\alpha_2 t) \right] / (\alpha_1 - \alpha_2)
\]

where

\[ G_1, H_1, \alpha_1, \alpha_2, G_2, H_2 \] are shown in (5.7.4) to (5.7.6)

5.8 EFFICIENCY OF RENEWAL - LCS MODEL

The coefficient of efficiency of renewal in the case of LCS Model can be obtained as

\[
K_{esls}(t) = R_{sls}(t, \tau) / R_{sls}(t, \tau) \mid \mu_s = 0 \quad (5.8.1)
\]

In the case of series system

\[
R_{sls}(t, \tau) = \pi_{sls}(t) \exp \left[ -(2 \lambda - \beta p^2 + 2 \beta p + \omega) t \right]
\]
and

\[ R_{sls}(t, \tau) \mid \mu_s = 0 = \exp \left[ -(2\lambda + 2\varphi_p - (3\varphi^2 + \omega)(t+\tau)) \right] \]

It could be seen that for LCS model the availability, steady-state availability, reliability and the coefficient of efficiency of service are all function of failure rates \( \lambda, \beta, \omega \) and \( p \). Similar to the case of NCS Model the expressions above mentioned are all lengthy and thus we developed a COMPUTER PROGRAM-V to enable one to obtain the numerical values of the above formulae.

We also discussed an example (Illustration (5.2)) in order to illustrate the influence of the lethal common-cause failures on the performance of the system.

**ILLUSTRATION (5.2)**

Suppose the individual failure rate \( \lambda = 0.1 \) f hr. and let for different values of \( \beta \) and \( \omega \) (see Table 5.2a). (5.2d)}.
We obtained the numerical values of $R_{sls}(t, \tau)$ with service and also without service, $n_{sls}(t)$ and $R_{sls}(\tau)$ by taking time $T = 0(1)$ 20 hrs. The curves are plotted for various values of $\beta$ and $\omega$ (seen in Fig.5.7).

It is seen from Fig.(5.7), the curve of coefficient of efficiency in respect of $\beta = \omega = 0$ belongs to the case of individual failures which is minimum most since there is no influence of lethal and non-lethal failures.

It is clear that as $\beta$ and $\omega$ increases the coefficient of efficiency is larger indicating that with service the system has high reliability than the one which did not support by the service and hence service in this case is highly justified in order to increase the reliability of the system.

It is also observed that when lethal shock is considered the "coefficient factor ($K_{esls}(t)$) curve
steeply increases. This implies that the lethal common-cause shock failures decrease the reliability steeply when compared to non-lethal common-cause shocks alone occur along with individual failures.

COMPUTER PROGRAM-V

10 REM A BASIC PROGRAM
20 REM TO FIND RELIABILITY, AVAILABILITY, CONDITIONAL
30 REM RELIABILITY OF THE SYSTEM
40 INPUT ''INDIVIDUAL FAILURE RATE''; L
50 INPUT ''N$ FAILURE RATE''; B
60 INPUT ''LS FAILURE RATE''; W : INPUT ''SERVICE RATE''; M
70 INPUT ''DURATION OF TIME''; TO
80 INPUT ''PROB OF SPECIFIC COM FAIL''; P
110 LO = 2*(L+B*P*(1-P))
120 L1 = B*P^2+W
130 L2 = L+B*P
140 M1 = M
150 M2 = 2*M
160  Z1 = M1*M2
170  Z2 = L0*M2
180  Z3 = L1*M2;  Z4 = M1*M2
190  Z = Z1+Z2+Z3+Z4
200  F = (M2*M1)/Z
210  PRINT 'STEADY-STATE AVAILABILITY='; F
220  D = (L0*M2+L1*M2+M1*M2+M1*L1)
230  A = (L0+L1+M1+M2)
240  A1 = (-A+ABS(SQR(A+2-4*D)))/2
250  A2 = (-A-ABS(SQR(A+2-4*D)))/2
270  G2 = (A2+2+A2*(M2+M1)+M1*M2)/A2
280  INPUT 'UPPER BOUND ON TIME'; U
290  FOR T = 0 TO U STEP 1
300  S1 = G1*EXP(A1*T)-G2*EXP(A2*T)
310  S = S1/(A1-A2)
320  Q = S+F
330  PRINT 'TIME DEPENDENT AVAILABILITY='; Q
340  Q1 = Q *EXP(-(2*L+2*B*P-B*P+2+W)*T)
350  R = EXP(-(2*L+2*B*P-B*P+2+W)*T)
360  Q2 = EXP(-(2*L+2*B*P-B*P+2+W)*(T+T0))
In the parallel case also we can proceed in the similar way and we can obtain 'coefficient factor' $K_{pls}(t)$ using the reliability, availability expressions derived already in Chapters IV and V.

Thus the expression of coefficient of efficiency in the case of LCS Model will be

$$K_{pls}(t) = \frac{R_{pls}(t,\tau)}{R_{pls}(t,\tau)} \Big|_{\mu_s = 0} (5.8.2)$$

where

$$R_{pls}(t,\tau) = \pi_{pls}(t) R_{pls}(\tau)/\pi_{pls}(t) R_{pls}(\tau) \Big|_{\mu_s = 0}$$
and the expressions are given in (5.8.1) and (4.7.10).

A COMPUTER PROGRAM-V is developed to obtain the values of reliability, availability, conditional reliability of the system and finally the values of coefficient factors in the case of series system for the LCS Model. In the similar way a Computer Program may be developed to obtain the above mentioned values for the parallel configuration also.
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Table (5.2a): Coefficient values against time 't'-LCS model

When \( \lambda=0.1, \beta=0, \omega=0, p=0.5, \mu_e=10 \).
<table>
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Table (5.2b): Coefficient Values against time 't' - LCS Model, when $\gamma = 0.1, \beta = 0.01, \omega = 0, \mu_s = 10, p = 0.5.$
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Table(5.2c): Coefficient values against time 't', When $\beta=0.01, \omega=0.01, \mu_s=10, T_0=1, p=0.5$. 
<table>
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<th>Time</th>
<th>$R_{sls}(t, \tau')$</th>
<th>$R_{sls}(t, T)$</th>
<th>AVAIL</th>
<th>RELIA</th>
<th>COEFFICIENT</th>
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Table (5.2d): Coefficient values against time $t'$ - LCS Model.

When $\lambda=0.1$, $\beta=0.01$, $\omega=0.03$, $T_D=1$, $p=0.5$, $\mu_s=10$. 
FIG(5.7): Coefficient of Efficiency of renewal( $K_{esis}(t)$) (V/S) time for various $\beta$ and $\omega$ - SeriesSystem-LCS Model.