4.1 INTRODUCTION

In this chapter, we discuss the 'Common-cause failure models' in the processes of evaluating the reliability characteristics viz., reliability and MTTF, which reveal the important measures of 'effectiveness' of the system in the reliability analysis. In the present chapter the emphasis is to concern mainly with the Markov model approach for 'common-cause shock failures' in order to evaluate the important reliability characteristics mentioned above. The discussion begin with consideration of Markov modelling aspect of the common-cause shocks to study the influence of common-cause shock failures in the reliability analysis.

(i) When the common-cause shocks hitting the system with certain probability. This
particular type of model we refer in this thesis as 'chance common-cause shock failure model' (CCS Model).

(ii) When the system is encountered with non-lethal common-cause shock failures in addition to individual failures. This type of model we refer in this thesis as 'Non-lethal common-cause shock model' (NCS Model).

(iii) Thirdly we consider when the system is influenced by Lethal shocks, in addition to individual and non-lethal shock failures. This type of model is referred in this thesis as 'Lethal common-cause shock model' (LCS Model).

We develop under the influence of the common-cause failures, the expression for the measures like reliability, MTTF using the above three categories of mathematical models under consideration.
Also we compare the reliability and MTTF of the system in the presence of common-cause failures with that of the event when the system is encountered only with 'individual failures'. For the present analysis we consider the 2-component system and the results of the same for both series and parallel redundant systems are discussed and presented.

4.2 COMMON-CAUSE SHOCK MODELS - RELIABILITY ANALYSIS

Reliability evaluation in the presence of the common-cause shock failures assume prime importance since the event of common-cause shocks degrade the reliability of the system when they act on the system. The common-cause models that are discussed in Chapter-III, however concerned with the aspects of the estimation of the basic parameters of the common-cause shock failures. These models did not touch upon the effect of such common-cause shock failures on the 'effectiveness' of the system, which in reality is facing with such common-cause shock failures. Reliability analyst
is much concerned with performance of the systems in realistic conditions. In what follows here we develop the models using Markov approach considering the common-cause shocks hitting the system along with individual failures.

4.2.1 CCS Model - Reliability Analysis

Markov approach had a great deal of attention to analyse the reliability of the systems. Now we formulate a Markov model to discuss the chance common-cause shock model and to derive the expression for reliability function of the system and also we study the effect of common-cause shocks on the reliability of the system. We develop Markov model for both series and parallel redundant configurations since they are most applicable and suitable configurations in many real applications.

Assumptions of CCS Model

1. System has two components which are identically
2. The components in the system are subjected to fail singly (individually and independently), or at the same instant (common-cause failures).

3. The arrival stream of individual failures forms a Poisson process with arrival rate \( c_1 \lambda_a \).

4. The arrival stream of common-cause failures forms a Poisson process with arrival rate \( c_2 \lambda_a \); \( c_1 + c_2 = 1 \).

5. The service of the components takes place singly (i.e., single server) and service times of the components are exponentially distributed with rate of service as \( \mu_s \).

Under the above assumptions we formulated a Markov model and developed the reliability function of the system under the presence of common-cause shock failures.

Notation

\( \lambda_a \) rate of individual failures or common-cause
shock failures (rate of both individual) and common-cause failures are assumed to be same)\).

\(c_1\) chance of individual failure, occur on the system.

\(c_2\) chance of common-cause failure, occur on the system.

\(\mu_s\) service rate.

\(R_{cs}(t)\) reliability of the system in the case of CCS model.

\(E_{cs}(T)\) MTTF of the system in the case of CCS model.

4.2.2 Reliability Function - CCS Model

The general reliability Markov graph for the CCS Model is seen in Fig. (4.1).
Using the above Markov graph the following Markov equations can be formulated

\begin{align*}
    p_0'(t) &= -(\lambda_0 + \lambda_1) p_0(t) + \mu_1 p_1(t) \\
    p_1'(t) &= \lambda_0 p_0(t) - (\mu_1 + \lambda_2) p_1(t) \quad (4.2.1) \\
    p_2'(t) &= \lambda_1 p_0(t) + \lambda_2 p_1(t)
\end{align*}

Using the Laplace transformation and the initial conditions, the set of equations given in (4.2.1) can be rewritten in matrix notation as
The solution vector \( \vec{P}^*(s) \) can be obtained using the condition that

\[
\sum_{i=0}^{2} P_i^*(s) = 1/s
\]

thus

\[
\vec{P}^*(s) = \begin{cases}
\frac{(s + \mu_1 + \lambda_2) / (s^2 + s(\lambda_0 + \lambda_1 + \mu_1 + \lambda_2) + \mu_1 + \lambda_0 \lambda_2 + \lambda_1 \lambda_2)}{
\lambda_0 P_0^*(s) / (s + \mu_1 + \lambda_2)} \\
(1/s) - [(P_0^*(s) + P_1^*(s)]
\end{cases}
\]
The vector \( \vec{r}(t) = [P_0(t) \ P_1(t) \ P_2(t)] \)

can be solved using the inverse Laplace transformation on the terms given in the parentheses of (4.2.4) and the solution is

\[
P_0(t) = \left[ (r_1^2 + \mu_1 + \lambda_2) \exp(r_1 t) - (r_2^2 + \mu_1 + \lambda_2) \exp(r_2 t) \right] / (r_1 - r_2) \quad (4.2.5)
\]

\[
P_1(t) = \lambda_0 \left[ \exp(r_1 t) - \exp(r_2 t) \right] / (r_1 - r_2) \quad (4.2.6)
\]

\[
P_2(t) = 1 - (P_0(t) + P_1(t)) \quad (4.2.7)
\]

where

\[
r_1, r_2 = \left[ -(\lambda_0 + \lambda_1 + \mu_1 + \lambda_2) \pm \sqrt{\left(\lambda_0 + \lambda_1 + \mu_1 + \lambda_2\right)^2 - 4(\mu_1 \lambda_1 + \lambda_0 \lambda_2 + \lambda_1 \lambda_2)} \right] / 2 \quad (4.2.8)
\]

and \( r_1 \) and \( r_2 \) are distinct because the term under square root is \( > 0 \), i.e., \( r_1 \neq r_2 \) if \( \lambda_0, \lambda_1, \mu_1, \mu_2 > 0 \).
The quantities in \((4.2.8)\) say \(\lambda_0, \lambda_1, \mu_1\) in the case of CCS Model are

\[
\lambda_0 = \lambda_2 = \lambda_a c_1, \quad \lambda_1 = \lambda_a c_2 \quad \text{and} \quad \mu_1 = \mu_s (4.2.9)
\]

Therefore substituting these quantities in \((4.2.5)\) to \((4.2.7)\), the chance that the system can stay in the states '0', '1', or '2' (i.e., \(p_i(t) \forall i = 0,1,2\)) can be expressed in terms of parameters \(\lambda_a, \mu_s, c_1\) and \(c_2\).

THEOREM (4.1) : The quantities \(p_1\) and \(p_2\) that are associated with probability of the system be in states '0', '1', and '2' are

(i) real
(ii) negative

for all input parameters of the model say \(\lambda_a, \mu_s \geq 0\) and \(c_1\) and \(c_2 \in [0,1]\)
PROOF:

(1) The necessary condition for the \( P_1 \) or \( P_2 \) to be real, is that the term under the square root of (4.2.8) must be non-negative.

\[
\lambda_0^2 + \lambda_1 + \mu_1 + \lambda_2)^2 - 4(\mu_1 \lambda_1 + \lambda_0 \lambda_2 + \lambda_1 \lambda_2) \geq 0
\]  

(4.2.10)

Using (4.2.9) this can be expressed as

\[
\lambda_a^2 + \lambda_a c_1 + \mu_1^2 + 2 \lambda_a c_1 - 2 \mu_s \lambda_a + 6 \lambda_a c_1 \mu_s \geq 0
\]

(4.2.11)

If \( c_1 = 0 \) (4.2.11) reduces to \((\lambda_a - \mu_s)^2 \geq 0\), which is true always, \( \forall \lambda_a, \mu_s \geq 0 \).

Also if \( c_1 = 1 \) (4.2.11) becomes \( \mu_s^2 + 4 \lambda_a \mu_s \geq 2 \mu_s \lambda_a \), which is true always,

and (4.2.11) is true \( \forall c_1 \in [0,1] \), \( \lambda_a, \mu_s \geq 0 \).
Since $c_1 \in (0,1)$ all the quantities on the L.H.S. of (4.2.11) are non-negative.

Hence it proves that (4.2.10) is true for all input parameters.

(ii) $r_1$ and $r_2$ are negative iff

(a) $r_1 < 0$

(b) $r_1 r_2 > 0$

To prove $r_1 < 0$ then, the condition

$$(\lambda_0 + \lambda_1 + \mu_1 + \lambda_2) > \text{SQRT} \left[ \left( \lambda_0 + \lambda_1 + \mu_1 + \lambda_2 \right)^2 - 4(\mu_1 \lambda_1 + \lambda_0 \lambda_1 + \lambda_1 \lambda_2) \right]$$

is to be satisfied (4.2.12)

this implies that

$$(\lambda_0 + \lambda_1 + \mu_1 + \lambda_2)^2 > (\lambda_0 + \lambda_1 + \mu_1 + \lambda_2)^2$$

$$(\lambda_0 + \lambda_1 + \mu_1 + \lambda_2)^2 > (\lambda_0 + \lambda_1 + \mu_1 + \lambda_2)^2 - 4(\mu_1 \lambda_1 + \lambda_0 \lambda_1 + \lambda_1 \lambda_2)$$

(4.2.12)

since both LHS and RHS of (4.2.12) are positive quantities.
The condition (4.2.12) is true if \( \lambda_a, \mu_s \geq 0 \) and \( c_1, c_2 \in [0,1] \).

Since \( u_1, \lambda_1, \lambda_o, \lambda_2 > 0 \) the second term on RHS of (4.2.12) is always non-negative.

Hence it proves that

\[ \eta_1 < 0 \]  \hspace{1cm} (4.2.13)

Also

\[ \eta_1 \eta_2 = \mu_1 \lambda_1 + \lambda_o \lambda_2 + \lambda_1 \lambda_2 \]  \hspace{1cm} (4.2.14)

using (4.2.9), (4.2.14) can be rewritten as

\[ \lambda_a \mu_s (1 - c_1) + \lambda_a^2 c_1 \]  \hspace{1cm} (4.2.15)

First term of the expression (4.2.15) is always greater than or equal to zero for all \( c_1 \in [0,1] \) and, of course, the second term is always non-negative.

Thus it implies that

\[ \eta_1 \eta_2 > 0 \text{ for all } \lambda_a, \mu_s > 0 \]  \hspace{1cm} (4.2.16)
Hence it follows from (4.2.13) and (4.2.16) that

\[ \gamma_2 < 0 \]

Thus the proof is completed.

4.3 CCS MODEL - RELIABILITY FUNCTION

The 2-component system would have assumed to be a series system or parallel redundant configuration. If the 2-component system be a series or parallel configuration then the fault-free operation of the system during the time \((0,t)\) can be derived using the information of the probability components of the system states derived in sub-section (4.2.2).

4.3.1 Series Configuration - Reliability Function

Following the discussion in Section (4.2.2), we now derive the expression of the reliability function. In the case of series configuration '0' is the only
state which represents the successful operation of the system and hence the probability of the successful operation of the system with CCS model is obtained as

\[ R_{cs}(t) = P_0(t) \]

Where \( P_0(t) \) is given by the expression (4.2.5)

Thus

\[ R_{cs}(t) = \frac{\left[ (r_1 + \mu_1 + \lambda_2) \exp(r_1 t) - (r_2 + \mu_1 + \lambda_2) \exp(r_2 t) \right]}{(r_1 - r_2)} \]  

(4.3.1)

where \( \lambda_0 = \lambda_a c_1 \), \( \lambda_1 = \lambda_a c_2 \), \( \mu_1 = 0 \), \( \lambda_2 = 0 \) 

(4.3.2)

because in the case of series configuration, once the system reaches state '1', it is already in down-state and hence no further transition is allowed from state '1' to '2' (hence \( \lambda_2 = 0 \)). Also, when the system is already in failed condition no repair is considered in
the case of series system, since reliability is the fault-free operation of the system during \((0, t)\). On substitution of the quantities given in (4.3.2) in the expression (4.3.1) the formula for reliability in the case of CCS Model for the series system be obtained as

\[
R_{\text{scs}}(t) = \exp \left[ -\lambda_a (c_1 + c_2) t \right]
\]  

(4.3.3)

From (4.3.3) we can see that the system will be working if neither of the type of failures hit the system in that interval of time and the probability of such a condition is given by (4.3.3). \( R_{\text{scs}}(t) = 1 \), if \( t = 0 \), which is true because we expect the system to be in perfect working condition at the starting point. It is also seen that from (4.3.3) that the reliability function will always takes values

\[
0 \leq R_{\text{scs}}(t) \leq 1
\]

This is obviously be the requirement for any reliability function.
Also $R_{sCS}(t) \to 0$, as $t \to \infty$. This comes from the fact that $r_1^o, r_2^o$ are negative and real. Hence for large values of 't' the first and the second terms in (4.3.1) are negligibly very small. This reveals that the chance of uninterrupted working of the system will be naturally going down as the time 't' goes up and the fault-free operation probability of the system will automatically decreases with respect to increase in time.

4.3.2 Parallel Configuration - Reliability Function

For 2-component parallel redundant configuration and satisfactory functioning is ensured if either one or two components operating successfully during the time $(0, t)$. Therefore, the probability of uninterrupted operation of the system during $(0, t)$ is expressed in the case of CCS Model as

$$R_{pcs}(t) = P_0(t) + P_1(t)$$
where $P_0(t)$ and $P_1(t)$ are given by the expressions in (4.2.5) and (4.2.6). Substituting them we see the expression of reliability of the parallel configuration in the case of CCS Model as

$$R_{pcs}(t) = \left[ (r_1^2 + \mu_1 + \lambda_2 + \lambda_0) \exp(r_1^1 t) \right. - \left( r_2^2 + \mu_1 + \lambda_2 + \lambda_0 \right) \exp(\frac{r_2}{2}) \right] / (r_1 - r_2)$$

(4.3.4)

where $\lambda_0 = \lambda_2 = \lambda_a \ c_1$, $\lambda_1 = \lambda_a \ c_2$, $\mu_1 = \mu_s \ (4.3.5)$

Substituting the above mentioned quantities, reliability expression of parallel system in the case of CCS Model be obtained as

$$R_{pcs}(t) = \left[ (r_1^2 + \mu_s + 2\lambda_a \ c_1) \exp(r_1^1 t) \right. - \left( r_2^2 + \mu_s + 2\lambda_a \ c_1 \right) \exp(\frac{r_2}{2}) \right] / \lambda$$

(4.3.6)

where

$$r_1, r_2 = \left[ -(\mu_s + \lambda_a (2-c_2))/2 \pm \sqrt{((\mu_s - \lambda_a c_2)^2 + \lambda_a \mu_s (1-c_2))} \right]$$

and $\lambda = 2 \sqrt{((\mu_s - \lambda_a c_2)^2 + \lambda_a \mu_s (1-c_2))} \ (4.3.7)$
Mean time to system failure is an important measure of a system, which reveals the average time the system will be working before it is down. In maintained systems with facility to service MTTF of the system may be improved. Thus one is particularly interested in this measure of 'effectiveness' in the case of maintained systems. In what follows, we develop the expression for the MTTF for both series and parallel configurations in the case of CCS Model.

4.4.1 MTTF-Series Configuration

Suppose the 2-component system that considered for the study is a maintained system then the expected time the system works satisfactorily before it fails can be derived for the series system in the case of CCS Model as

\[ E_{scs}(T) = \int_0^\infty R_{scs}(t) \, dt \quad (4.4.1) \]
using the result in (1.3.3)

where $R_{scs}(t)$ is the reliability in the case of series system for CCS Model. Thus

$$E_{scs}(T) = \int_0^\infty \exp \left[-\lambda_a (c_1 + c_2)\right] dt = \frac{1}{\lambda_a}$$

(4.4.2)

because $c_1 + c_2 = 1$

It can be seen that the MTTF of the series configuration that is developed in the case of CCS Model also will not be influenced by the repair, since the MTTF is seldom a function of service rate ($\mu_s$).

In fact, it can be seen that in the case of CCS Model, there is no effect on reliability and MTTF of the system of series system with changes in chance of occurrence of common-cause failures until the failure rate of the both individual as well as common-cause shock failures are equal.
4.4.2 MTTF Parallel System

The expression of MTTF for 2-component parallel system can be derived in the similar lines that of series case, however, to develop the MTTF of parallel system, of the CCS Model we substitute the reliability expression of the parallel system developed and seen in (4.3.6).

Thus the MTTF of the parallel system for CCS Model is

\[
E_{\text{pcs}}(T) = \int_0^\infty R_{\text{pcs}}(t) dt = \int_0^\infty \left[ P_1 \exp(\lambda_1 t) - P_2 \exp(\lambda_2 t) \right] dt
\]

\[
= \left[ \left. \frac{\exp(\lambda_1 t)}{\lambda_1} \right|_0^\infty \right] P_1 - \left[ \left. \frac{\exp(\lambda_2 t)}{\lambda_2} \right|_0^\infty \right] P_2
\]

\[
= \left[ \frac{1}{\lambda_1} - \frac{1}{\lambda_2} \right] P_1 - \frac{1}{\lambda_2} P_2 \tag{4.4.3}
\]

where \( P_1 = \gamma_1 + \mu_s + 2\lambda a c_1 \)
\[ P_2 = r_2^a + \mu_s + 2\lambda_a c_1 \]

\[ A = r_1^a - r_2^a \]

and \( r_1^a \) and \( r_2^a \) are defined in (3.3.6)

Using the result given in Theorem (4.2), (4.4.3) can be simplified as

\[ \frac{(\mu_s + 2\lambda_a c_1)}{\lambda_a^2(1 - c_2) + \lambda_a \mu_s c_2} \quad (4.4.4) \]

The result developed and seen in (4.4.4) will however agree with the MTTF expression already developed (E. Balaguruswamy [12] p.125) in the individual failures case, assuming that common-cause failures does not hit the system at all (i.e., \( c_2 = 0 \)). It is interesting to see that MTTF is likely to be improved with consideration of maintenance for the parallel system unlike in the case of series system. The gain in MTTF due to maintenance in the parallel system thus be derived as

\[
\text{GAIN} = \left[ \frac{E_{pcs}(T)}{E_{pcs}(T)} - \frac{E_{pcs}(T)}{E_{pcs}(T)} \right] \quad (4.4.5)
\]
\[ \text{GAIN} = \left[ (\mu_s \lambda_a + 2 \lambda_a^2 c_1) / (2 \lambda_a^2 (1 - c_2)) \right. \\
\left. + \lambda_a \mu_s c_2 \right] \times 100 \quad (4.4.6) \]

It can be shown that the gain in MTTF is increasing with increase in service rate, following the example given in (4.1).

If rate of 'individual failures' \( (\lambda') \) and 'common-cause failures' \( (\lambda''') \) are different the reliability function of series system is

\[ R_{\text{scs}}(t) = \exp\left[-(2 \lambda' c_1 + \lambda''' c_2) t\right] \quad (4.4.7) \]

and that of parallel system is

\[ R_{\text{pcs}}(t) = \left[ (r_1 + 3 \lambda' c_1 + \mu_s) \exp(r_1 t) \right. \\
\left. - (r_2 + 3 \lambda' c_1 + \mu_s) \exp(r_2 t) \right] \lambda_1 - r_2 \]

\quad (4.4.8)

where \( r_1 \) and \( r_2 \) are defined as
Also Mean Time to Failure (MTTF) in the case of series system is

$$E_{scs}(T) = \int_0^{\infty} \exp \left[ - (2 \lambda' c_1 + \lambda'' c_2) t \right] \cdot dt$$

$$= \frac{1}{(2 \lambda' c_1 + \lambda'' c_2)} \quad (4.4.9)$$

and that of parallel system is

$$E_{pcs}(T) = \int_0^{\infty} \left[ p_1 \exp(\gamma_1 t) - p_2 \exp(\gamma_2 t) \right] / \lambda \cdot dt$$

$$= \frac{1}{\lambda} \quad (4.4.10)$$

and $p_1$ and $p_2$ in (4.4.6) are defined as

$$p_1 = \frac{\gamma_1}{1 + 3 \lambda' c_1 + \mu_s}$$
It is interesting to note that the formulae for reliability for and MTTF in the case of series system seen in (4.4.7) and (4.4.9) will agree with the result already developed in the case of individual failure by letting $c_2 = 0$ and $c_1 = 1$ (this implies that there are no common-cause failures). Similarly the result in the case of parallel system seen in (4.4.8) and (4.4.11) also agrees with the individual failures case already developed (E. Balaguruswamy [12] p.125).

EXAMPLE (4.1) : If individual failure rate is $\lambda = 0.1$ fail/hr and for various values of service rate $\mu_s = 0.1(0.1)^2$, the gain percentage in MTTF was
calculated for various values of probability of common-cause occurrence, using (4.4.6). The percentage of gain is seen in Table 4.1. From the table we can conclude few important points.

(i) The percentage gain in MTTF is maximum when the system is affected with only individual failures (i.e., when the individual failures alone can hit the system).

(ii) On the other hand, if the common-cause shock failures are certain to hit the system then despite that it is parallel system the gain in MTTF is zero.

(iii) It is to understand that for a 2-component parallel redundant system the MTTF will not be improved with single repair unit when chance of common-cause occurrence is unity in the case of CCS Model.
<table>
<thead>
<tr>
<th>$\mu_s$</th>
<th>$c_2=0$</th>
<th>$c_2=0.25$</th>
<th>$c_2=0.50$</th>
<th>$c_2=0.75$</th>
<th>$c_2=1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>100</td>
<td>75.00</td>
<td>50.00</td>
<td>25.00</td>
<td>0</td>
</tr>
<tr>
<td>0.2</td>
<td>200</td>
<td>120.00</td>
<td>66.67</td>
<td>28.57</td>
<td>0</td>
</tr>
<tr>
<td>0.3</td>
<td>300</td>
<td>150.00</td>
<td>75.00</td>
<td>30.00</td>
<td>0</td>
</tr>
<tr>
<td>0.4</td>
<td>400</td>
<td>171.43</td>
<td>80.00</td>
<td>30.77</td>
<td>0</td>
</tr>
<tr>
<td>0.5</td>
<td>500</td>
<td>187.50</td>
<td>83.33</td>
<td>31.25</td>
<td>0</td>
</tr>
<tr>
<td>0.6</td>
<td>600</td>
<td>200.00</td>
<td>85.71</td>
<td>31.58</td>
<td>0</td>
</tr>
<tr>
<td>0.7</td>
<td>700</td>
<td>210.00</td>
<td>87.50</td>
<td>31.82</td>
<td>0</td>
</tr>
<tr>
<td>0.8</td>
<td>800</td>
<td>218.18</td>
<td>88.89</td>
<td>32.00</td>
<td>0</td>
</tr>
<tr>
<td>0.9</td>
<td>900</td>
<td>225.00</td>
<td>90.00</td>
<td>32.14</td>
<td>0</td>
</tr>
<tr>
<td>1.0</td>
<td>1000</td>
<td>230.77</td>
<td>90.90</td>
<td>32.26</td>
<td>0</td>
</tr>
<tr>
<td>1.1</td>
<td>1100</td>
<td>235.71</td>
<td>91.67</td>
<td>32.35</td>
<td>0</td>
</tr>
<tr>
<td>1.2</td>
<td>1200</td>
<td>240.00</td>
<td>92.31</td>
<td>32.43</td>
<td>0</td>
</tr>
<tr>
<td>1.3</td>
<td>1300</td>
<td>243.75</td>
<td>92.86</td>
<td>32.50</td>
<td>0</td>
</tr>
<tr>
<td>1.4</td>
<td>1400</td>
<td>247.06</td>
<td>93.33</td>
<td>32.56</td>
<td>0</td>
</tr>
<tr>
<td>1.5</td>
<td>1500</td>
<td>250.00</td>
<td>93.75</td>
<td>32.61</td>
<td>0</td>
</tr>
<tr>
<td>1.6</td>
<td>1600</td>
<td>252.63</td>
<td>94.12</td>
<td>32.65</td>
<td>0</td>
</tr>
<tr>
<td>1.7</td>
<td>1700</td>
<td>255.00</td>
<td>94.44</td>
<td>32.69</td>
<td>0</td>
</tr>
<tr>
<td>1.8</td>
<td>1800</td>
<td>257.14</td>
<td>94.73</td>
<td>32.73</td>
<td>0</td>
</tr>
<tr>
<td>1.9</td>
<td>1900</td>
<td>259.09</td>
<td>95.00</td>
<td>32.76</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 4.1: Gain percentage due to renewal in parallel system- CCS MODEL.
4.5 NCS MODEL RELIABILITY ANALYSIS

The model gets nomenclature because the system is affected by a non-lethal common-cause failures in addition to individual failures. Some of the practical examples of this model are cited in Chapter-III. We now state the assumptions of the NCS Model for the reliability analysis.

4.5.1 Assumptions of NCS Model

1. The system has 2-components.

2. The components fail individually with failure rate $\lambda$.

3. A random number of components fail at the same time, (simultaneously) due to non-lethal common-cause shock occurring at the rate say $\beta$.

4. The random number of components that fail due to non-lethal common-cause shocks is
governed by Binomial law with parameters $(2, p)$ and thus a specific component that fails due to non-lethal common-cause shock is probability $p$.

5. Service takes place singly with rate $\mu_3$.

Under the assumptions stated above we develop a Markov model to evaluate the measures of our concern viz., reliability function, MTTF of the system. Following the assumptions given in (4.5.1 Section) Markov graph for the NCS Model is seen in Fig. (4.2).

Fig. (4.2) Markov Graph of 2-component system - NCS Model
Here 0,1,2 are the states of the system showing the status of the system i.e., zero component fail, one component fail and two components fail respectively with the parameters $\lambda_0$, $\lambda_1$, $\lambda_2$, $\mu_1$, $\mu_2$ as

$$
\lambda_0 = 2[\lambda + \Theta p]\quad \lambda_1 = \Theta p^2
$$

$$
\lambda_2 = \lambda + 2p
$$

$$
\mu_1 = \mu_2 = 0
$$

For the case of NCS Model the Markovian equations can be developed and the following differential equations result.

$$
P_0'(t) = -(\lambda_0 + \lambda_1) P_0(t) + P_1(t) \mu_1
$$

$$
P_1'(t) = \lambda_0 P_0(t) - (\mu_1 + \lambda_2) P_1(t)
$$

$$
P_2'(t) = \lambda_1 P_0(t) + \lambda_2 P_1(t)
$$

Using Laplace transformation and initial conditions i.e.
\[ P_0(0) = 1 \quad \text{and} \quad P_1(0) = P_2(0) = 0 \]

the system of equations in (4.5.1) can be arranged in the matrix notation as

\[
\begin{pmatrix}
(s + \lambda_0 + \lambda_1) & -\mu_1 & 0 \\
-\lambda_0 & (s + \mu_1 + \lambda_2) & 0 \\
-\lambda_1 & -\lambda_2 & s
\end{pmatrix}
\begin{pmatrix}
P_0^*(s) \\
P_1^*(s) \\
P_2^*(s)
\end{pmatrix}
= \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}
\]

The vector \( \tilde{P}^*(s) \) of the system of equations in (4.5.2) can be solved and is seen as in (4.2.4) and the solution is as follows:

\[
P_0(t) = \left[ (\sigma_1 + \mu_1 + \lambda_2) \exp(\alpha_1 t) \\
- (\sigma_2 + \mu_1 + \lambda_2) \exp(\alpha_2 t) \right] / (\alpha_1 - \alpha_2)
\]

\[
(4.5.3)
\]
\[ P_1(t) = \frac{\lambda_0 \exp(\sigma_1 t) - \exp(\sigma_2 t)}{(\sigma_1 - \sigma_2)} (4.5.4) \]

\[ P_2(t) = 1 - \frac{1}{i=0} \sum P_i(t) \quad (4.5.5) \]

where

\[ \sigma_1, \sigma_2 = \left[ -(\lambda_0 + \lambda_1 + \mu_1 + \lambda_2) \pm \sqrt{\left((\lambda_0 + \lambda_1 + \mu_1 + \lambda_2)^2 - 4(\mu_1 \lambda_1 + \lambda_0 \lambda_2 + \lambda_1 \lambda_2)\right)} \right] / 2 \quad (4.5.6) \]

4.5.2 NCS Model-Reliability Function

(i) Series System

2-components system under the consideration is a series one, then the probability that the system function successfully is given by

\[ R_{s2}(t) = \left[ Q_1 \exp(\sigma_1 t) - Q_2 \exp(\sigma_2 t) \right] / B \quad (4.5.7) \]

where

\[ Q_1 = \sigma_1 + \mu_1 + \lambda_2 \]
\( \Omega_2 = \sigma_2^2 + \mu_1 + \lambda_2 \)

\( B = \sigma_1 - \sigma_2 \), and \( \sigma_1 \) and \( \sigma_2 \) are given by the expression in (4.5.6).

The quantities in the expression (4.5.7) i.e., \( \lambda_0, \mu_1, \mu_2, \lambda_1, \lambda_2 \) of the present model are to be substituted as

\[ \lambda_0 = 2(\lambda + 3pq) \]
\[ \lambda_1 = 2p^2 \]
\[ \lambda_2 = \mu_1 = \mu_2 = 0 \]

Thus the reliability expression for the 2-component series system in the case of NCS Model would become

\[ R_{sns}(t) = \exp \left[-(2\lambda + 2(3pq + 3p^2)t)\right] \quad (4.5.8) \]

The result given in (4.5.8) for the NCS Model will agree with the result for the 2-component series system with individual failures case, if we assume
that the non-lethal common-cause failures occur with rate zero (i.e., $\beta = 0$).

4.5.3 Reliability Comparison

In this section, we compare the reliability of the series system influenced by individual failures with that of the one when it is influenced by common-cause shock failures.

PROPOSITION (4.1): The reliability of the series system with respect to NCS Model is less than or equal to reliability estimate with respect to individual failures.

PROOF: Consider $D = R_s(t) - R_{sns}(t)$ (4.5.9)

$R_s(t)$ Reliability of the series system with consideration of only individual failures.

$R_{sns}(t)$ Reliability of the series system with the
consideration of common-cause failures along with individual failures.

Substituting the expressions of $R_s(t)$ and $R_{sns}(t)$ in (4.5.9) and simplifying the same we obtain

$$D = \left[\frac{1 - \exp(-(2(3pq + (3p^2))t)}{\exp(2 \lambda t)}\right]$$

RHS of (4.5.10) is the ratio of two non-negative quantities, hence $D$ is always non-negative $\forall \lambda, (\beta \geq 0$ and $p \in [0,1]$). Hence the proof is completed.

Thus it proves that the reliability of the system which is affected by a non-lethal shock will however be less than when it is not hit by such failures.

EXAMPLE (4.2): For a 2-component system, suppose if individual failure rate is $\lambda = 0.1$ failure/hr. and for various values of common-cause shock failures $\beta = 0 (0.1) 0.5$, the reliability of the series system is calculated using (4.5.8) and tabulated in Table (4.2).
The values of reliability are plotted against time 't' for different values of $\beta$ (seen in Fig. (4.3)). The plot of the reliability curve with $\beta = 0$ will refer to the reliability curve with individual failures. And the rest of the curves belong to reliability curves with various values of $\beta$.

It is clear from the Fig. (4.3) that the increase in $\beta$ (rate of non-lethal shock failures) will result in steep decline in the reliability of the system.

<table>
<thead>
<tr>
<th>TIME (hr)</th>
<th>$\beta = 0$</th>
<th>$\beta = 0.1$</th>
<th>$\beta = 0.2$</th>
<th>$\beta = 0.3$</th>
<th>$\beta = 0.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0.6703</td>
<td>0.6060</td>
<td>0.5488</td>
<td>0.4966</td>
<td>0.4066</td>
</tr>
<tr>
<td>4</td>
<td>0.4493</td>
<td>0.3678</td>
<td>0.3066</td>
<td>0.2466</td>
<td>0.1653</td>
</tr>
<tr>
<td>6</td>
<td>0.3013</td>
<td>0.2231</td>
<td>0.1653</td>
<td>0.1225</td>
<td>0.0672</td>
</tr>
<tr>
<td>8</td>
<td>0.2019</td>
<td>0.1353</td>
<td>0.0907</td>
<td>0.0608</td>
<td>0.0273</td>
</tr>
<tr>
<td>10</td>
<td>0.1353</td>
<td>0.0820</td>
<td>0.0498</td>
<td>0.0302</td>
<td>0.0111</td>
</tr>
<tr>
<td>12</td>
<td>0.0907</td>
<td>0.0498</td>
<td>0.0273</td>
<td>0.0150</td>
<td>0.0045</td>
</tr>
<tr>
<td>14</td>
<td>0.0608</td>
<td>0.0388</td>
<td>0.0150</td>
<td>0.0074</td>
<td>0.0018</td>
</tr>
<tr>
<td>16</td>
<td>0.0407</td>
<td>0.0183</td>
<td>0.0082</td>
<td>0.0037</td>
<td>0.0007</td>
</tr>
<tr>
<td>18</td>
<td>0.0273</td>
<td>0.0111</td>
<td>0.0040</td>
<td>0.0018</td>
<td>0.0003</td>
</tr>
<tr>
<td>20</td>
<td>0.0183</td>
<td>0.0067</td>
<td>0.0020</td>
<td>0.0009</td>
<td>0.0001</td>
</tr>
</tbody>
</table>

Table (4.2) Reliability values for various values of $\beta$ as a function of 't' series system – NCS Model.
FIG(4.3): Reliability curves for various $\beta$-Series System- NCS Model.
4.5.4 NCS Model—Reliability of Parallel System

Following the discussion in the Sub-Section (4.3.2) the reliability of parallel system of the NCS Model is derived using (4.5.4) and (4.5.5).

Thus expression of reliability in the case of parallel configuration can be seen as

\[ R_{\text{par}}(t) = \frac{\eta_1 \exp(\eta_1 t) - \eta_2 \exp(\eta_2 t)}{B} \quad (4.5...) \]

where

\[ \eta_1 = \sigma_1 + \lambda_0 + \mu_1 + \lambda_2 \]
\[ \eta_2 = \sigma_2 + \lambda_0 + \mu_1 + \lambda_2 \]

\[ B = \sigma_1 - \sigma_2 \] and \( \sigma_1 \) and \( \sigma_2 \) are given by the expression in (4.5.6). The quantities that appear in (4.5.11) are to be substituted using (4.5.1). The expression of reliability in the case of parallel system is lengthy and the numerical evaluation of reliability can be obtained using the COMPUTER PROGRAM-I. (This Program is adopted on CG-2000 COLOUR GENIE, PC compatible system).
COMPUTER PROGRAM-I.

10 REM PROGRAM TO CALCULATE RELIABILITY
20 REM OF PARALLEL CONFIGURATION-NCS MODEL.
30 INPUT ''INDIVIDUAL FAILURE RATE'' ; L
32 INPUT ''SERVICE RATE'' ; M
34 INPUT ''PROBABILITY OF SPECIFIC COMPONENT FAIL'' ; P
36 REM FOR VARIOUS VALUES OF NCS RATES
40 FOR B= 0 TO 0.5 STEP 0.1
50 LO = 2*L+2*B*P*(1-P)
60 L1 = B*P↑2 ; L2 = L+B*P
70 M1 = M
80 A = -(LO+L1+M1+L2)
90 D = 4*(L1*M1+LO*L2+L1*L2)
100 A1 =(-A+ABS(SQR( A↑2 - D)))/2
110 A2 =(-A-ABS(SQR( A↑2 - D )))/2
120 Q1 = (LO+M1+L2+A1)
130 Q2 = (LO+M1+L2+A2)
140 FOR T= 0 TO 10
150 Z = Q1*EXP(A1*T) - Q2*EXP(A2*T)
160 R = Z/(A1 - A2)
170 PRINT T,B,R
180 NEXT T,B
200 END
Considering the example given in (4.2), the reliability values of parallel configuration are evaluated using the COMPUTER PROGRAM-I and given in Table (4.3). However, this program can be used to assess the reliability of the parallel system for any given \( \lambda, \mu_s, p \) and for various values of \( \beta \) ranging from 0 to the specific value of interest.

\[
\lambda = 0.1 \quad \mu_s = 1 \quad p = 0.5
\]

<table>
<thead>
<tr>
<th>TIME (hr)</th>
<th>( \beta = 0 )</th>
<th>( \beta = 0.1 )</th>
<th>( \beta = 0.2 )</th>
<th>( \beta = 0.3 )</th>
<th>( \beta = 0.5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.9932</td>
<td>0.9652</td>
<td>0.9373</td>
<td>0.9097</td>
<td>0.8552</td>
</tr>
<tr>
<td>1</td>
<td>0.9803</td>
<td>0.9229</td>
<td>0.8674</td>
<td>0.8139</td>
<td>0.7135</td>
</tr>
<tr>
<td>2</td>
<td>0.9658</td>
<td>0.8805</td>
<td>0.8002</td>
<td>0.7254</td>
<td>0.5921</td>
</tr>
<tr>
<td>3</td>
<td>0.9511</td>
<td>0.8392</td>
<td>0.7376</td>
<td>0.6460</td>
<td>0.4909</td>
</tr>
<tr>
<td>4</td>
<td>0.9364</td>
<td>0.7998</td>
<td>0.6797</td>
<td>0.5751</td>
<td>0.4069</td>
</tr>
<tr>
<td>5</td>
<td>0.9364</td>
<td>0.7622</td>
<td>0.6264</td>
<td>0.5120</td>
<td>0.3372</td>
</tr>
<tr>
<td>6</td>
<td>0.9220</td>
<td>0.7264</td>
<td>0.5772</td>
<td>0.4558</td>
<td>0.2795</td>
</tr>
<tr>
<td>7</td>
<td>0.9077</td>
<td>0.6923</td>
<td>0.5319</td>
<td>0.4057</td>
<td>0.2316</td>
</tr>
<tr>
<td>8</td>
<td>0.8937</td>
<td>0.6598</td>
<td>0.4902</td>
<td>0.3612</td>
<td>0.1920</td>
</tr>
<tr>
<td>9</td>
<td>0.8799</td>
<td>0.6288</td>
<td>0.4517</td>
<td>0.3216</td>
<td>0.1591</td>
</tr>
</tbody>
</table>

Table (4.3) Reliability values for various values of \( \beta \) as a function of time 't' parallel system - NCS Model.
Reliability curves are plotted in the case of parallel configuration against the time 't' for various values of $\beta$, and given in Fig. (4.4). The reliability curve for $\beta = 0$, refer to the reliability without the consideration of common-cause failures, which corresponds to individual failures only. Thus the graph (4.4) indicates that as in the case of series, in parallel case also the reliability of the system significantly decreases as the non-lethal common-cause shock failure rate $(\beta)$ increases.
FIG(4.4): Reliability curves for various $\beta$ as a function of time $t$: PARALLEL SYSTEM - NCS Model.
4.6 NCS MODEL - MTTF OF THE SYSTEM

As in the case of CCS Model, we now develop the mean time to failure of the system in the case of NCS Model for both configurations.

4.6.1 Series Configuration

Mean time to failure of the system in case of series configuration can be derived as

\[ E_{\text{sns}}(T) = \int_0^\infty R_{\text{sns}}(t)dt, \]

using the result in (4.5.8)

\[ E_{\text{sns}}(T) = \int_0^\infty \left[ e^{-\frac{t}{\lambda}} \right] dt = \frac{1}{\lambda} \]

The MTTF in the case of series configuration is not a function of \( \mu_s \). This tells that series system will not be influenced by the maintenance.
This coincides with the results already established in the series case. It can be shown that the occurrence of non-lethal common-cause failures will certainly decrease the MTTF of the series system when compared to the MTTF with the influence of individual failures only.

ILLUSTRATION (4.1)

Suppose the individual failure rate is 0.1 failures/hr, and let non-lethal common-cause failure rate is (\(\beta\)) 0.01 failures/hr., then

MTTF with individual failures is

\[ E_s(T) = 5 \text{ hrs. using (1.3.3)} \]

Similarly, MTTF calculated with NCS Model is (i.e., under the influence of non-lethal common-cause failures along with individual failures).

\[ E_{sn}(T) = 4.82 \text{ hrs. using (4.6.1)} \]
Thus the percentage of decrease with the influence of common-cause failures is 3.6.

On the other hand, if we assume $\beta = 0.1$ failures/hr., then

$$E_{sn}(T) = 3.64 \text{ hrs.}$$

Hence the percentage of decrease in this case rises to 27. Thus it can be established that the percentage of decrease in MTTF due to non-lethal common-cause failures will increase as its rate increases.

4.6.2 NCS Model - MTTF of Parallel System

For the parallel system, MTTF in the case of NCS Model can be derived as

$$E_{pns}(T) = \int_0^\infty R_{pns}(t)dt$$

using the result in (4.5.12)

$$E_{pns}(T) = \int_0^\infty \left[Q_1 \exp(\sigma_1 t) - Q_2 \exp(\sigma_2 t)\right]/(\sigma_1 - \sigma_2)dt$$

(4.6.2)
On simplification, the above expression reduces to

$$E_{pns}(T) = \frac{\mu_s + 3\lambda + 3p - 2p^2}{2\lambda^2 + \beta p^2 \mu_s + 4\lambda \beta p + 2(\beta p^2 - \beta \lambda p^2 - \beta^2 p^3)}$$  \hspace{1cm} (4.6.3)

The result developed in this context is agreeing with the result in the case of individual failures by assuming $\beta = 0$. It is seen that the MTTF of the parallel system certainly is a function of service rate $\mu_s$. Thus it can be established that MTTF of parallel system in the case of a NCS Model be less when service is not considered than it is considered.

Thus the percentage of gain in MTTF with renewal can be obtained in the case of NCS Model using the formula

$$GAIN = \left[\frac{E_{pns}(T)}{E_{pns}(T) \mid \mu_s = 0}\right] - 1 \times 100$$

On simplification Gain percentage in MTTF can be obtained as
GAIN = \left\{ (1 + G_1)(1 + G_2)^{-1} - 1 \right\} \times 100 \quad (4.6.5)

where

\begin{align*}
G_1 &= \frac{\mu_s}{(3\lambda + 3\beta p - 2\beta p^2)} \\
G_2 &= \frac{\beta p^2 \mu_s}{(2\lambda^2 + 4\lambda \beta p + 2\beta^2 p^2 - 3\beta p^2 - 3p^3)}
\end{align*}

In the case of individual failures, the percentage gain in MTTF \([12]\) is

\[ \text{GAIN} = 33.3(\mu_s/\lambda) \]

We consider the comparison of the gain percentage in MTTF in the case of NCS Model with that of individual failures case.

EXAMPLE (4.3): Let us consider a system for which the individual failure rate is \(\lambda = 0.1\) failures/hr., and the non-lethal common-cause failure rate is \(\beta = 0.01\) failures/hr. Also the probability that a component fail when the system is hit by a non-lethal
common-cause shock is $p = 0.5$, then MTTF is calculated for both the cases (seen in Table (4.3)), with different levels of service rates ($\mu_s$). From the example considered, it is interesting to note that the gain percentage in MTTF both for individual failures case and common-cause failures is almost same for lower service rates (seen in the Table 4.3), at the top portion). However, if the service rate is larger the gain percentage in the case of common-cause failures is half when compared to individual failures case. This amounts to say that the effect of non-lethal common-cause failures is significant on the MTTF of the system.

Table (4.3a to 4.3d) indicate the MTTF values of the above values and also gain percentages in both cases as a function of service rate and for various levels of non-lethal common-cause failure rates.
<table>
<thead>
<tr>
<th>Service rate ($\mu_g$)</th>
<th>MTTF NCS-model</th>
<th>GAIN PERCENTAGES NCS-model</th>
<th>Individual case</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>18.6047</td>
<td>30.75</td>
<td>33.30</td>
</tr>
<tr>
<td>0.2</td>
<td>22.8828</td>
<td>60.82</td>
<td>66.60</td>
</tr>
<tr>
<td>0.3</td>
<td>27.0660</td>
<td>90.23</td>
<td>99.90</td>
</tr>
<tr>
<td>0.4</td>
<td>31.1574</td>
<td>118.98</td>
<td>133.20</td>
</tr>
<tr>
<td>0.6</td>
<td>39.0768</td>
<td>174.64</td>
<td>199.80</td>
</tr>
<tr>
<td>0.8</td>
<td>46.6632</td>
<td>227.90</td>
<td>266.40</td>
</tr>
<tr>
<td>1.0</td>
<td>53.9372</td>
<td>279.08</td>
<td>333.00</td>
</tr>
<tr>
<td>5.0</td>
<td>154.8670</td>
<td>988.44</td>
<td>1665.00</td>
</tr>
<tr>
<td>10.0</td>
<td>220.3580</td>
<td>1448.73</td>
<td>3330.00</td>
</tr>
</tbody>
</table>

Table (4.3a): MTTF values for non-lethal common-cause failures and gain percentages for (i) individual failures (ii) Common-cause failures when $\lambda=0.1$, $\beta=0.01$, $p = 0.5$.

<table>
<thead>
<tr>
<th>Service rate ($\mu_g$)</th>
<th>MTTF NCS-Model</th>
<th>GAIN PERCENTAGES NCS-model</th>
<th>INDIVIDUAL CASE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>16.3265</td>
<td>24.659</td>
<td>33.30</td>
</tr>
<tr>
<td>0.2</td>
<td>19.5662</td>
<td>51.71</td>
<td>66.60</td>
</tr>
<tr>
<td>0.4</td>
<td>25.5356</td>
<td>98.00</td>
<td>133.20</td>
</tr>
<tr>
<td>0.6</td>
<td>30.9099</td>
<td>139.67</td>
<td>199.80</td>
</tr>
</tbody>
</table>

Cont.....
<table>
<thead>
<tr>
<th>Service rate ($\mu_s$)</th>
<th>MTTF</th>
<th>GAIN PERCENTAGES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NCS-Model</td>
<td>NCS-Model</td>
</tr>
<tr>
<td>0.1</td>
<td>14.5455</td>
<td>23.38</td>
</tr>
<tr>
<td>0.2</td>
<td>16.4316</td>
<td>39.37</td>
</tr>
<tr>
<td>0.3</td>
<td>19.4393</td>
<td>64.89</td>
</tr>
<tr>
<td>0.4</td>
<td>21.6216</td>
<td>83.40</td>
</tr>
<tr>
<td>0.6</td>
<td>25.5462</td>
<td>116.69</td>
</tr>
<tr>
<td>0.8</td>
<td>28.9764</td>
<td>145.78</td>
</tr>
<tr>
<td>1.0</td>
<td>32.0000</td>
<td>171.43</td>
</tr>
<tr>
<td>5.0</td>
<td>58.0339</td>
<td>392.25</td>
</tr>
<tr>
<td>10.0</td>
<td>66.9091</td>
<td>467.53</td>
</tr>
</tbody>
</table>

Table (4.3b): when $\lambda=0.1$, $\beta=0.03$, $p=0.5$.

Table (4.3c): WHEN $\lambda=0.1$, $\beta=0.05$, $p=0.5$. 
### Table (4.3d) When $\lambda = 0.1$, $\sigma = 0.05$, $p = 0.5$

<table>
<thead>
<tr>
<th>Service rate ($\mu_s$)</th>
<th>MTTF NCS-Model</th>
<th>GAIN PERCENTAGES NCS-Model</th>
<th>Individual</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>11.4286</td>
<td>17.86</td>
<td>30.30</td>
</tr>
<tr>
<td>0.2</td>
<td>12.9730</td>
<td>33.78</td>
<td>66.60</td>
</tr>
<tr>
<td>0.3</td>
<td>14.3590</td>
<td>48.07</td>
<td>99.90</td>
</tr>
<tr>
<td>0.4</td>
<td>15.6098</td>
<td>60.97</td>
<td>133.20</td>
</tr>
<tr>
<td>0.6</td>
<td>17.7778</td>
<td>83.33</td>
<td>199.80</td>
</tr>
<tr>
<td>0.8</td>
<td>19.5918</td>
<td>102.04</td>
<td>266.40</td>
</tr>
<tr>
<td>1.0</td>
<td>21.1321</td>
<td>117.93</td>
<td>333.00</td>
</tr>
<tr>
<td>5.0</td>
<td>32.4812</td>
<td>234.96</td>
<td>1665.00</td>
</tr>
<tr>
<td>10.0</td>
<td>35.7082</td>
<td>268.24</td>
<td>3333.00</td>
</tr>
</tbody>
</table>

4.7 LCS MODEL-RELIABILITY ANALYSIS

The data obtained in real applications would reveal that a second kind of common-cause shocks known as Lethal common-cause shocks be affecting the operation of the systems. Lethal common-cause is found applicable in many systems. Therefore, in this chapter of the thesis, our objective is to evaluate
reliability characteristics in which we are interested. Thus the main emphasis is to see the effect of lethal common-cause shock failures acting on the system. Hence the LCS model is an extension of the NCS Model, where we study the effect of lethal shocks in addition to individual failures and non-lethal shock failures. Thus the assumptions stated in sub-section (4.5.1) will be retained for the model and in addition to that the following assumptions be stated for the present model.

1. Individual failures, common-cause failures (non-lethal) and lethal common-cause shock failures occur independently of each other.

2. The rate of lethal shock failure occurring is $\dot{\omega}$. Thus the rate at which a specific component fail due to either individual failures or due to a shock is

$$r_1 = \lambda + \beta p + \dot{\omega}$$
Further, the rate at which a specific set of k-components fail simultaneously due to a shock (either due to lethal or non-lethal) is given by

\[ r_k \]

Under the assumptions already stated the Markov graph for the LCS Model be seen in Fig.(4.5).

---

**Fig.(4.5)** Markov graph of 2-component system
LCS Model
The quantities that appear in this model are to be seen as

\[ \lambda_0 = 2(\lambda + \beta pq) \]
\[ \lambda_1 = \beta p^2 + \omega \]
\[ \lambda_2 = \lambda + \beta p \]
\[ \mu_1 = \mu_s \]
\[ \mu_2 = 0 \]

The Markov equations in this model after taking LT and using the initial conditions can be expressed as shown in (4.5.2) but however the various parameters in it will be defined as seen in (4.7.1).

Thus the solution of Markov equations is

\[ P_0(t) = \left[ R_1 \exp(w_1t) - R_2 \exp(w_2t) \right] / (w_1 - w_2) \] (4.7.2)
\[ P_1(t) = \frac{\lambda_0 [\exp(w_1 t) - \exp(w_2 t)]}{(w_1 - w_2)} \quad (4.7.3) \]

\[ P_2(t) = 1 - \left( P_0(t) + P_1(t) \right) \quad (4.7.4) \]

where

\[ R_1 = w_1 + \mu_1 + \lambda_2 \quad R_2 = w_2 + \mu_1 + \lambda_2 \quad (4.7.5) \]

and

\[ w_1, w_2 = \left[ -(\lambda_0 + \lambda_1 + \mu_1 + \lambda_2) \pm SQR \left( (\lambda_0 + \lambda_1 + \mu_1 + \lambda_2)^2 \right. \right. \\
- 4(\mu_1 \lambda_1 + \lambda_0 \lambda_2 + \lambda_1 \lambda_2) \left. \right] / 2 \quad (4.7.6) \]

and the quantities that appear in the expressions (4.7.3) to (4.7.7) has to be substituted as shown in (4.7.1).

4.7.1 LCS Model - Reliability of Series System

If the 2-component system under consideration is a series one then it needs the system be in state
'0' for the successful operation of the system. Therefore, the reliability of the series system for the LCS-Model is

\[ R_{sls}(t) = \frac{[R_1 \exp(w_1 t) - R_2 \exp(w_2 t)]}{(w_1 - w_2)} \]  

(4.7.7)

Thus the reliability in the case of LCS Model is a function of \( \lambda_0, \lambda_1, \mu_1 \) (as defined in 4.7.1) and \( \lambda_2 = \mu_2 = 0 \), and substituting them in the reliability expression given in (4.7.7), the formula for reliability of series system in the case of LCS-Model be obtained as

\[ R_{sls}(t) = \exp\left[-(2\lambda + 2\beta p - \beta p^2 + \omega)\right] \]  

(4.7.8)

This result also, as in the case of other models agree with the result already developed in the case of individual failures assuming \( \beta = \omega = 0 \).
4.7.2 LCS Model - Reliability of Parallel System

If the 2-component system has parallel configuration, then the reliability of the system for the LCS-Model be obtained as

\[ R_{\text{pl}}(t) = \frac{[R_1 \exp(w_1 t) - R_2 \exp(w_2 t)]}{w_1 - w_2} \]  \hspace{1cm} (4.7.9)

where

\[ R_1 = w_1 + \mu_1 + \lambda_2 + \lambda_0 \]
\[ R_2 = w_2 + \mu_1 + \lambda_2 + \lambda_0 \]

and \( w_1 \) and \( w_2 \) are given by the expression (4.7.6).

4.7.3 LCS Model - MTTF of Series System

Following the result in (4.7.8) we derive the MTTF of the series system in the case of LCS Model as
\[ E_{sls}(T) = \int_0^\infty R_{sls}(t) dt \]
\[ = \int_0^\infty \exp\left[-(2\lambda - \beta p^2 + 2\beta p + \omega)t\right] dt \]
\[ = \frac{1}{(2\lambda - \beta p^2 + 2\beta p + \omega)} \quad (4.7.10) \]

This result, however, will agree with the one studied under the influence of individual failures case, assuming \( \beta = \omega = 0 \).

In LCS-Model also, just like in the case of NCS Model, MTTF of the series system is seldom a function of service rate \( \mu_s \). This confirms that the series system will never be influenced by service or renewal.

Further it can be shown that MTTF of the series system be get reduced by the influence of lethal common-cause failures, in addition to individual and non-lethal shock failures in the system. Thus the
percentage of decrease in MTTF can be obtained as

\[
PD = \left[1 - \frac{E_{s1s}(T)}{E_s(T)}\right] \times 100
\]

\(E_{s1s}(T)\) is the average time the system is working before failure, under the influence of common-cause failures.

\(E_s(t)\) is the average time the system is working before failure under the influence of individual failures.

On simplification of the above expression the formula for percentage decrease in MTTF can be obtained as

\[
PD = \left[1 - \frac{2\lambda}{(2\lambda - (3p^2 + 2(3p + \omega))}\right] \times 100
\]

\[(4.7.11)\]
ILLUSTRATION (4.2)

Suppose individual failure rate is $\lambda = 0.1$ failures/hr. and non-lethal shock failure rate is $\beta = 0.01$ failures/hr., and let lethal shock failure rate is $\omega = 0.01$ failures/hr., and if $p = 0.5$ then,

Percentage decrease in MTTF due to influence of lethal shock is 8. However, if $\omega = 0.03$ this increases to 16 per cent and further if we assume that $\omega = 0.05$ this decrease will go up to 25 per cent. Thus the result shows that increase in lethal shock failure rate will significantly decrease the MTTF of the series system.

4.7.4 MTTF of Parallel System - LCS Model

For the parallel system in the case of LCS Model MTTF can be derived following the result (4.7.10).

Thus the MTTF for the parallel configuration is
\[ E_{pls}(T) = (3\lambda + \mu_s + 3(\beta p - 2(\beta p^2))/\alpha \text{ (4.7.12)} \]

where

\[ \alpha = (\beta \mu_g p^2 + \mu_g \omega + \omega \lambda + 2\lambda^2 + 4\beta p \lambda \]

\[ - (3\beta p^2 + 2\beta p^2 - \beta^2 p^3 - \omega \beta p \]

The result obtained here will agree with the one already developed for the case of individual failures when we assume that \( \beta = \omega = 0 \). Also from (4.7.12) it can be seen that MTTF of parallel system is certainly a function of renewal and is influenced by the renewal. Thus one can anticipate the gain in MTTF due to maintenance in this case.

**ILLUSTRATION (4.3)**

Suppose the individual failure rate is \( \lambda = 0.1 \) failures/hr., non-lethal shock failure rate is \( \beta = 0.01 \) failures/hr. and lethal shock failure rate
is $\omega = 0.01$ failure/hr. and if $p = 0.5$, then MTTF for the LCS Model is obtained using (4.7.12) for various levels of service rates $\mu_s = 0.1, 0.2, \ldots 10.0$ (see in Table (4.4a and 4.4b). The gain percentages in MTTF both for LCS Model and individual failures are obtained. The example clearly shows that the gain percentage decreases significantly with increase in $\beta$ and $\omega$.

Also from the example it is observed that gain percentage in MTTF in the case of LCS Model is 75 per cent when compared to individual failures case for the lower service rates. But the same for the larger service rates is shown to be 12 per cent. Thus it tells us the fact that gain percentage in MTTF for parallel system under the influence of common-cause failures is negligibly small.

Thus all these facts further emphasises that the system when it is being influenced by individual as well as common-cause shock failures, naturally
it is significant to consider the common-cause failures also in order to assess the measure of effectiveness of the system with high degree of precision. This will lead to predict and design the systems suitably, and correctly, for most real applications.

A computer program is developed in order to calculate the MTTF of parallel system in the case of LCS Model for any set of parameters $\lambda$, $\beta$, $\omega$, and $\mu_s$. The program also enable us to compare and calculate the percentage of gain in MTTF as a function of service rate, for common-cause failures and individual failures.

COMPUTER PROGRAM-II

10 REM A BASIC PROGRAM TO CALCULATE MTTF, GAIN PERCENTAGE
20 REM FOR LCS-MODEL AND INDIVIDUAL FAILURES CASE
30 INPUT "RATE OF INDIVIDUAL FAILURES"; L
40 INPUT "RATE OF NLS FAILURES"; B
50  INPUT "'RATE OF LS FAILURES'"; W : INPUT "'SERVICE RATE'"; M
60  INPUT "'PROBABILITY OF SPECIFIC COMPONENT FAIL'"; P
70  GOSUB 1000
80  PRINT "'MTTF WITH SERVICE-LCS MODEL='; Q
90  Z1 = Q
100 INPUT L,B,W,P
110 M=0
120 GOSUB 1000
130 PRINT "'MTTF WITH OUT SERVICE-LCS MODEL='; Q
140 Z2 = Q
150 G1 = (Z1/Z2 - 1)*100
160 REM PERCENTAGE GAIN IN INDIVIDUAL FAILURES CASE
170 INPUT "'INDIVIDUAL FAILURE RATE'"; L
180 INPUT "'SERVICE RATE'"; M
190 G2 = (33.3)*(M/L)
200 PRINT M,G2,G1
210 STOP
1000 REM SUBROUTINE STARTS
1010 G1 = 3*L*M+3*B*P-2*B*P↑2
Using the above COMPUTER PROGRAM--II, MTTF in the case of LCS Model is calculated and the percentage of gain in MTTF with consideration of service is also calculated for the example discussed in ILLUSTRATION (4.2) for varied values of $\beta$ and $\omega$, and presented in the Tables (4.4a - 4.4c).
<table>
<thead>
<tr>
<th>Service rate ($\mu_s$)</th>
<th>MTTF LCS-Model</th>
<th>GAIN PERCENTAGES LCS Model</th>
<th>INDIVIDUAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>17.0213</td>
<td>25.39</td>
<td>33.30</td>
</tr>
<tr>
<td>0.2</td>
<td>20.1283</td>
<td>48.28</td>
<td>66.60</td>
</tr>
<tr>
<td>0.3</td>
<td>22.9431</td>
<td>69.02</td>
<td>99.90</td>
</tr>
<tr>
<td>0.4</td>
<td>25.5052</td>
<td>87.89</td>
<td>133.20</td>
</tr>
<tr>
<td>0.6</td>
<td>29.9959</td>
<td>120.98</td>
<td>199.80</td>
</tr>
<tr>
<td>0.8</td>
<td>33.8028</td>
<td>149.02</td>
<td>266.40</td>
</tr>
<tr>
<td>1.0</td>
<td>37.0711</td>
<td>173.10</td>
<td>333.00</td>
</tr>
<tr>
<td>5.0</td>
<td>62.2235</td>
<td>358.40</td>
<td>1665.00</td>
</tr>
<tr>
<td>10.0</td>
<td>69.7387</td>
<td>413.761</td>
<td>3333.00</td>
</tr>
</tbody>
</table>

Table(4.4a): MTTF values for LCS-model and gain percentages for (i) Individual failures (11) common-cause failures when $\lambda=0.1$, $\beta=0.01$, $\omega=0.01$, $p=0.5$.

<table>
<thead>
<tr>
<th>Service rate ($\mu_s$)</th>
<th>MTTF LCS-Model</th>
<th>GAIN PERCENTAGES LCS Model</th>
<th>INDIVIDUAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>14.5455</td>
<td>17.00</td>
<td>33.30</td>
</tr>
<tr>
<td>0.2</td>
<td>16.2227</td>
<td>30.50</td>
<td>66.60</td>
</tr>
<tr>
<td>0.3</td>
<td>17.5856</td>
<td>41.46</td>
<td>99.90</td>
</tr>
<tr>
<td>0.4</td>
<td>18.7150</td>
<td>50.55</td>
<td>133.20</td>
</tr>
<tr>
<td>0.6</td>
<td>20.4782</td>
<td>64.73</td>
<td>199.80</td>
</tr>
<tr>
<td>0.8</td>
<td>21.7914</td>
<td>75.30</td>
<td>266.40</td>
</tr>
<tr>
<td>1.0</td>
<td>22.8074</td>
<td>83.47</td>
<td>333.00</td>
</tr>
<tr>
<td>5.0</td>
<td>28.3294</td>
<td>127.89</td>
<td>1665.00</td>
</tr>
<tr>
<td>10.0</td>
<td>29.4624</td>
<td>137.01</td>
<td>3333.00</td>
</tr>
</tbody>
</table>

Table(4.4b): when $\lambda=0.1$, $\beta=0.01$, $\omega=0.03$ and $p=0.5$.  
<table>
<thead>
<tr>
<th>Service rate ($\mu_s$)</th>
<th>MTTF LCS-Model</th>
<th>Gain Percentages LCS-Model</th>
<th>INDIVIDUAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>13.1148</td>
<td>15.40</td>
<td>33.30</td>
</tr>
<tr>
<td>0.2</td>
<td>14.5056</td>
<td>27.64</td>
<td>66.60</td>
</tr>
<tr>
<td>0.3</td>
<td>15.6375</td>
<td>37.59</td>
<td>99.90</td>
</tr>
<tr>
<td>0.4</td>
<td>16.5768</td>
<td>45.86</td>
<td>133.20</td>
</tr>
<tr>
<td>0.6</td>
<td>18.0451</td>
<td>58.78</td>
<td>199.80</td>
</tr>
<tr>
<td>0.8</td>
<td>19.1404</td>
<td>68.42</td>
<td>266.40</td>
</tr>
<tr>
<td>1.0</td>
<td>19.9887</td>
<td>75.88</td>
<td>333.00</td>
</tr>
<tr>
<td>5.0</td>
<td>24.6147</td>
<td>116.59</td>
<td>1665.00</td>
</tr>
<tr>
<td>10.0</td>
<td>25.5669</td>
<td>124.97</td>
<td>3333.00</td>
</tr>
</tbody>
</table>

Table(4.4c): When $\alpha = 0.1$ and $\gamma = 1 - 0.5 = 0.5$. 