CHAPTER 6

TWO NOVEL LOW COMPLEXITY SCHEMES FOR PAPR REDUCTION IN OFDM SYSTEMS USING CYCLICALLY SHIFTED SEQUENCES

6.1 TIME DOMAIN BASED PAPR REDUCTION IN OFDM SYSTEM

In the literature PAPR reduction in time domain has received significant attention due to low computational complexity. Certain DFT properties were employed to produce alternate candidate signal equivalent to the OFDM signal and finally the candidate with minimum PAPR is selected for transmission. However use of basic properties to reduce PAPR value in OFDM system does not ensure optimal BER performance. Hence this chapter proposes two methods which employs cyclically shifted sequences to generate multiple candidates with different PAPR value.

6.2 PAPR REDUCTION AND CYCLICALLY SHIFTED SEQUENCES

Two time domain based PAPR reduction techniques are proposed to generate multiple candidates each with different PAPR values by employing cyclically shifted sequences and finally the candidate with minimum PAPR is selected for transmission. Two low complexity time domain methods employs a basic signal processing operation in time domain, which involves linear combination of one OFDM sequence with cyclically shifted and scaled version of the other OFDM sequence. The first method employs CSS with varied delays and fixed phase rotation vector defined by
the scaling factor while the second method employs fixed delay and calculates the optimum phase factor such that candidate with minimum PAPR is produced.

6.2.1 Linear Combination of CSS with Different Delays (LCCSS-D)

Let $x^k$ denote the time domain OFDM signal produced by taking IFFT on the signal represented in frequency domain. In the proposed LC method one time domain sequence $x^1$ is fixed and the other sequence $x^2$ is cyclically to generate new sequence $x^2_{m,CS}$ is given by

$$x^2_{m,CS} = \text{shift}[x^2 (m \times CS)]$$ (6.1)

Where CS denotes the cyclically shifting number defined as $CS \in (1, 2, \ldots LN)$ and $m \in \left(1, 2, \ldots \left(\frac{LN-1}{CS}\right)\right)$.

![Figure 6.1 Block diagram of LCCSS-D scheme](image)

Equivalent candidates of an OFDM signal generated by linear superpositioning $x^1$ with $x^2_{m,CS}$ is given as

$$\tilde{x}_m = x^1 + \alpha^m x^2_{m,CS}$$ (6.2)

Where $\alpha$ is a linear scaling factor defined between 0 and 1.
The block diagram of LCCSS-D is given in Figure 6.1. The number of candidates generated can be adjusted by varying the parameters $k$ and CS. The candidate with minimum PAPR is transmitted along with the scaling factor and cyclic shifting number to the receiver.

6.2.2 Linear Combination of CSS with Optimal Phase Factors (LCCSS-OFF)

Assume a random phase sequence of length $L$ given as $c_k = [c_0, c_1, \ldots, c_{L-1}]$ where $c_k = e^{j\phi_k}$ and $\phi_k$ is uniformly distributed in the interval $[0, 2\pi]$. The periodic extension of $c_k$ is given by

$$c((k))_L = \sum_{i=-\infty}^{i=\infty} c_{k+iL}$$  \hspace{1cm} (6.3)

Where $c((k))_L$ is $k \mod L$. The first $N$ elements of $c((k))_L$ are chosen using a rectangular window to form a $N$ length phase sequence in the following manner

$$b_k = c((k))_L \cdot r_k$$  \hspace{1cm} (6.4)

Where $r_k = \begin{cases} 1, & 0 \leq k \leq N - 1 \\ 0, & \text{elsewhere} \end{cases}$

Since $N = ML$, the phase sequence $b_k$ is of length $N$ and contains $M$ periods of $c((k))_L$.

$L$ different Cyclically shifted phase sequence, $b_k^{(l)}$ with $L$ samples are generated in the following manner

$$b_k^{(l)} = \sum_{i=-\infty}^{i=\infty} c_{k+l+iL} \cdot r_k, \quad l = 0, 1, \ldots (L-1) \text{ and } k = 0, 1, \ldots (N-1)$$  \hspace{1cm} (6.5)
and are employed in the proposed method for PAPR reduction in OFDM systems

Figure 6.2 Block diagram of LCCSS-OPF scheme

The candidates generated based on Cyclically Shifted Sequences (CSS) with optimal phase factors $b_k^{(l)}$ is given by

$$\tilde{x}_n = x + \sum_{k=1}^{D} x_{m,CS}^k \cdot b_k^{(l)}$$  \hspace{1cm} (6.6)

The block diagram of LCCSS-OPF is given in Figure 6.2. The steps involved in LCCSS-OPF is summarized as follows

(a) Compute the time domain sequence $x_n$ using one N point IFFT, i.e., $x_n = \text{IFFT}\{X_k\}$.
(b) Generate the cyclically shifted sequence.
(c) Calculate the optimal phase sequence $b_k^{(l)}$ using Eqn. (6.5)
(d) Compute the candidates using Eqn. (6.6).
(e) Select the candidate $\tilde{x}_n$ with minimum PAPR
6.3 COMPUTATIONAL COMPLEXITY ANALYSIS

The first method requires the delay value and the second method demands the optimal phase factors to be transmitted as side information in order to recover the actual OFDM signal at the receiver. In the process of identifying the side information to be transmitted for the proposed LCCSS methods, complex multiplications and additions has to be performed. The amount of complex additions and multiplications that has to be performed to compute the delay value for the LCCSS-D and optimal phase factors for LCCSS-OPF are presented in Table 6.1.

Table 6.1 Computational complexity comparison of PTS, SLM and the proposed schemes

<table>
<thead>
<tr>
<th>Schemes</th>
<th>No. of Complex Additions</th>
<th>No. of Complex Multiplications</th>
</tr>
</thead>
<tbody>
<tr>
<td>PTS</td>
<td>( \frac{N(M-1)}{N \log^2 N} )</td>
<td>( \frac{1}{2} ) ( M \log^2 N )</td>
</tr>
<tr>
<td>SLM</td>
<td>( \frac{N(M-1)}{N \log^2 N} )</td>
<td>( \frac{1}{2} ) ( N \log^2 N )</td>
</tr>
<tr>
<td>LCCSS – D</td>
<td>( \frac{N \log^2 N + (CS-1)N}{N \log^2 N + CSN} )</td>
<td>( \frac{1}{2} ) ( N \log^2 N )</td>
</tr>
<tr>
<td>LCCSS – OPF</td>
<td>( \frac{N \log^2 N + (CS-1)N}{N \log^2 N + CSN} )</td>
<td>( \frac{N \log^2 N}{N \log^2 N} )</td>
</tr>
</tbody>
</table>

where M refers to number of sub blocks and CS refers to the number of cyclic shift.

6.4 NUMERICAL RESULTS AND PERFORMANCE ANALYSIS

The proposed LCCSS assisted PAPR reduction schemes are evaluated by performing several simulations on 10^4 OFDM symbols that are modulated using randomly generated orthogonal subcarriers and compared
with the conventional SLM and PTS scheme with number of sub blocks, \( M = 2 \) and \( M = 4 \). The proposed LCCSS methods PAPR reduction and BER performance is compared with that of SLM and PTS (\( M = 2 \) and \( M = 4 \)) by performing simulation over AWGN, fading and SUI-5 channel. Throughout the simulation the linear scaling factor \( \alpha \) is assumed as 0.72 in order to achieve better PAPR reduction performance. However for different values of \( \alpha \), the PAPR reduction and BER performance of the OFDM signal is greatly varied.

### 6.4.1 CCDF Performance Analysis

Based on the simulation performed, from Figure 6.3 it is obvious that the performance of the proposed LCCSS - OPF for a QPSK modulated OFDM signal is better than the SLM scheme, PTS (\( M = 2 \)) and PTS (\( M = 4 \)) by 2.29 dB, 1.92 dB and 0.14 dB respectively. While LCCSS - OPF for a 16-QAM modulated OFDM signal over AWGN channel is presented in Fig. 6.4. It outperforms the SLM scheme, PTS (\( M = 2 \)) and PTS (\( M = 4 \)) by 2.08 dB, 1.71 dB and 0.30 dB respectively.

![CCDF curves of the referred PAPR reduction schemes for a QPSK modulated OFDM signal](image)

**Figure 6.3** CCDF curves of the referred PAPR reduction schemes for a QPSK modulated OFDM signal
Figure 6.4  CCDF curves of the referred PAPR reduction schemes for a 16-QAM modulated OFDM signal

The second PAPR reduction method, LCCSS-D for a QPSK modulated OFDM signal performs better than SLM scheme and PTS (M = 2) by 1.02 dB and 0.65 dB respectively, where as it performs worse than PTS (M = 4) and LCCSS-OPF by 1.24 dB and 1.06 dB. While the CCDF curve of a LCCSS-D for a 16-QAM modulated OFDM signal surpasses the SLM scheme and PTS (M = 2) by 0.99 dB and 0.65 dB respectively, where as its performance is degraded when compared to PTS (M = 4) and LCCSS-OPF by 0.76 dB and 1.06 dB respectively.

Table 6.2  CCDF Comparative Analysis of LCCSS methods and the referred schemes.

<table>
<thead>
<tr>
<th>OFDM Symbol</th>
<th>PAPR (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Original</td>
</tr>
<tr>
<td>QPSK</td>
<td>12.33</td>
</tr>
<tr>
<td>16-QAM</td>
<td>13.42</td>
</tr>
</tbody>
</table>
The CCDF plot comparison for QPSK and QAM modulated OFDM signal processed using LCCSS methods along with the referred schemes are given in Table 6.2. It is perceptible from the tabulation that when compared to the original OFDM signal the LCCSS-OPF method achieves PAPR reduction by 5.28 dB and 5.24 dB respectively while the LCCSS-D methods achieves PAPR reduction by 4.01 dB and 4.18 dB respectively.

6.4.2 BER Performance Analysis

Based on the simulation performed it is evident from Figure 6.5 that over an AWGN channel, the BER performance of the proposed LCCSS-OPF for a QPSK modulated OFDM signal is better than that of PTS (M = 4), LCCSS-D and SLM scheme by 0.15 dB, 0.72 dB and 1.02 dB respectively where as its BER performance is poorer than that of PTS (M = 2) by 0.15 dB. While the proposed LCCSS – D for a QPSK modulated OFDM signal performs worse than PTS (M = 4), PTS (M = 2) and LCCSS-OPF by 0.57 dB, 0.87 dB and 0.72 dB, respectively and better than SLM scheme by 0.30 dB. Likewise from Figure 6.6 it can be manifested that for a 16-QAM modulated OFDM signal, the BER performance of the proposed LCCSS-OPF over an AWGN channel outperforms the PTS (M = 4), LCCDD-D and the SLM scheme by 0.76 dB, 0.45 dB, and 0.87 dB but provides slightly worse BER performance than that of PTS (M = 2) by 0.10 dB.

![Figure 6.5 BER vs. SNR plot of the referred PAPR reduction methods over AWGN channel for a QPSK modulated OFDM signal](image)

Figure 6.5 BER vs. SNR plot of the referred PAPR reduction methods over AWGN channel for a QPSK modulated OFDM signal
Figure 6.6 BER vs. SNR plot of the referred PAPR reduction methods over AWGN channel for a 16-QAM modulated OFDM signal

Figure 6.7 BER vs. SNR plot of the referred PAPR reduction methods over fading channel for a QPSK modulated OFDM signal
Figure 6.8  BER vs. SNR plot of the referred PAPR reduction methods over fading channel for a 16-QAM modulated OFDM signal

While the proposed LCCSS-D for a QAM modulated OFDM signal performs worse than PTS (M = 2) and LCCSS-OPF by 0.55 dB and 0.45 dB, and its performance is better than SLM scheme by 0.42 dB.

Over a fading channel, the BER performance of the proposed LCCSS-OPF for a QPSK modulated OFDM signal is presented in Figure 6.7. It is apparent from the simulation that the LCCSS-OPF is better than that of PTS (M = 4), LCCSS-D and SLM scheme by 1.32 dB, 0.92 dB and 1.71 dB where as its BER performance is poorer than that of the PTS scheme (M = 2) by 0.10 dB. While the proposed LCCSS-D for a QPSK modulated OFDM signal performs worse than PTS (M = 2) and LCCSS-OPF by 1.02 dB and 0.92 dB and better than PTS (M = 4) and SLM scheme by 0.40 dB and 0.79 dB respectively.

From Figure 6.8 the BER performance of the proposed LCCSS-OPF for a 16-QAM modulated OFDM signal over a fading channel outperforms the PTS (M = 4), LCCSS-D and SLM scheme by 0.54 dB, 1.23 dB and 1.98 dB respectively but provides slightly degraded BER performance when compared to that of PTS (M = 2) and SLM scheme by 0.12 dB. While
the proposed LCCSS – D for a QPSK modulated OFDM signal performs better than SLM scheme by 0.75 dB and worse than LCCSS-OPF, PTS (M = 2) and PTS (M = 4) by 1.23 dB, 1.35 dB and 0.69 dB respectively.

Figure 6.9 BER vs. SNR plot of the referred PAPR reduction methods over SU1-5 channel for a QPSK modulated OFDM signal

Figure 6.10 BER vs. SNR plot of the referred PAPR reduction methods over SU1-5 channel for a 16-QAM modulated OFDM signal
It is obvious from Figure 6.9 that the BER performance of the proposed LCCSS-OPF for a QPSK modulated OFDM signal over a SUI-5 channel is better than that of PTS (M = 4), LCCSS-D and SLM scheme by 0.54 dB, 3.82 dB and 6.39 dB respectively where as its BER performance is poorer than that of PTS (M = 4) by 0.54 dB. While the proposed LCCSS-D for a QPSK OFDM signal performs worse than PTS (M = 2), PTS (M = 4) and LCCSS-OPF by 5 dB, 3.28 dB and 3.82 dB respectively, and better than SLM scheme by 2.57 dB.

For a 16-QAM modulated OFDM signal, the BER performance of the proposed LCCSS-OPF over a SUI-5 channel is given in Figure 6.10. The LCCSS-OPF outperforms the PTS (M = 4), LCCSS-D and SLM scheme by 1.12 dB, 3.39 dB and 7.28 dB respectively where as its BER performance is poorer than that of PTS (M = 2) by 0.51 dB. While the proposed LCCSS-D for a QPSK modulated OFDM signal performs worse than PTS (M = 2), PTS (M = 4) and LCCSS-OPF by 3.90 dB, 2.27 dB and 3.39 dB and better than SLM scheme by 3.89 dB.

Table 6.3  Comparative Analysis of BER vs. SNR plot for the proposed LCCSS methods and the referred schemes.

<table>
<thead>
<tr>
<th>Channel</th>
<th>OFDM Symbol</th>
<th>SLM Scheme</th>
<th>PTS (M = 2)</th>
<th>LCCSS-D</th>
<th>PTS (M = 4)</th>
<th>LCCSS-OPF</th>
</tr>
</thead>
<tbody>
<tr>
<td>AWGN</td>
<td>QPSK</td>
<td>6.63</td>
<td>6.91</td>
<td>7.78</td>
<td>7.21</td>
<td>7.06</td>
</tr>
<tr>
<td></td>
<td>16-QAM</td>
<td>7.24</td>
<td>7.55</td>
<td>8.10</td>
<td>8.41</td>
<td>7.65</td>
</tr>
<tr>
<td>Fading</td>
<td>QPSK</td>
<td>9.13</td>
<td>9.62</td>
<td>10.64</td>
<td>11.04</td>
<td>9.72</td>
</tr>
<tr>
<td></td>
<td>16-QAM</td>
<td>10.31</td>
<td>10.90</td>
<td>12.25</td>
<td>11.56</td>
<td>11.02</td>
</tr>
<tr>
<td>SUI-5</td>
<td>QPSK</td>
<td>21.99</td>
<td>24.51</td>
<td>29.51</td>
<td>26.23</td>
<td>25.69</td>
</tr>
<tr>
<td></td>
<td>16-QAM</td>
<td>23.10</td>
<td>25.81</td>
<td>29.71</td>
<td>27.44</td>
<td>26.32</td>
</tr>
</tbody>
</table>
The BER performance of the LCCSS methods over AWGN, fading and SUI-5 channel along with other referred schemes are presented in Table 6.3. It is apparent that the LCCSS-OPF method records a slightly degraded BER performance than the PTS (M = 2) method. While the LCCSS-D method has a poor BER performance than the PTS and LCCSS-OPF method

6.4.3 Constellation Plot Analysis

Constellation plot of both QPSK and 16-QAM modulated OFDM signal processed using the proposed LCCSS-OPF and transmitted over three different channels namely

(a) AWGN channel are presented in Figure 6.11 and Figure 6.12 respectively
(b) Fading channel are presented in Figure 6.13 and Figure 6.14 respectively
(c) SUI – 5 channel are presented in Figure 6.15 and Figure 6.16 respectively

![Figure 6.11](image1.png)  
**Figure 6.11** Constellation diagram of a QPSK OFDM signal processed using the proposed LCCSS-OPF over AWGN channel.
Figure 6.12 Constellation diagram of a 16-QAM OFDM signal processed using the proposed LCCSS-OPF over AWGN channel.

Figure 6.13 Constellation diagram of a QPSK OFDM signal processed using the proposed LCCSS-OPF over fading channel.
Figure 6.14 Constellation diagram of a 16-QAM OFDM signal processed using the proposed LCCSS-OPF over fading channel.

Figure 6.15 Constellation diagram of a QPSK OFDM signal processed using the proposed LCCSS-OPF over SU1-5 channel.
Figure 6.16  Constellation diagram of a 16-QAM OFDM signal processed using the proposed LCCSS-OPF over SUI-5 channel.

The constellation plot of the received OFDM signal shows that the digital modulation technique namely QPSK and 16-QAM used as signal mapper in order to maintain orthogonality among the subcarriers at the OFDM transmitter which when processed using the proposed LCCSS – OPF preserves the constellation signature after transmission through the AWGN, fading and SUI - 5 at the receiver. Since the modulation features performed at the transmitter on the OFDM signal are preserved by the proposed LCCSS-OPF at the receiver, the proposed method can be used in OFDM systems for effective PAPR reduction.

Constellation plot of both QPSK and QAM modulated OFDM signal processed using the proposed LCCSS-D and transmitted over three different channels namely

(a) AWGN channel are presented in Figure 6.17 and Figure 6.18 respectively
(b) Fading channel are presented in Figure 6.19 and Figure 6.20 respectively
(c) SUI-5 channel are presented in Figure 6.21 and Figure 6.22 respectively.

Figure 6.17 Constellation diagram of a QPSK OFDM signal processed using LCCSS–D over AWGN channel.

Figure 6.18 Constellation diagram of a 16-QAM OFDM signal processed using LCCSS–D over AWGN channel.
Figure 6.19  Constellation diagram of a QPSK OFDM signal processed using LCCSS-D over fading channel.

Figure 6.20  Constellation diagram of a 16-QAM OFDM signal processed using LCCSS-D over fading channel.
Figure 6.21 Constellation diagram of a QPSK OFDM signal processed using LCCSS-D over SU1-5 channel.

Figure 6.22 Constellation diagram of a 16-QAM OFDM signal processed using LCCSS-D over SU1-5 channel.
The constellation plot of the received OFDM signal shows that the digital modulation technique namely QPSK and 16-QAM used as signal mapper at the OFDM transmitter when processed using the proposed LCCSS-D for PAPR reduction preserves the constellation signature but the constellation energy is slightly degraded compared to LCCSS-OPF after transmission through the different communication channels at the receiver. Since the modulation features performed at the transmitter along with the proposed LCCSS-D to reduce PAPR of the OFDM signal preserves the constellation points at the receiver, it can be used in OFDM systems for effective PAPR reduction.

6.4.4 Power Spectral Density Analysis

The spectral characteristics of both QPSK and QAM modulated OFDM signal processed using the proposed LCCSS - OPF and LCCSS – D are compared with that of the original OFDM signal in order to infer the robustness of the proposed LCCSS methods. The PSD plot of QPSK and QAM OFDM signal processed using the LCCSS methods are presented in Figure 6.23 and Figure 6.24 respectively.

![Figure 6.23 PSD plot of the QPSK modulated OFDM signal and LCCSS methods](image)

**Figure 6.23** PSD plot of the QPSK modulated OFDM signal and LCCSS methods
Figure 6.24 PSD plot of the 16-QAM modulated OFDM signal and LCCSS methods

It is evident from the above PSD plots that the proposed LCCSS methods for both QPSK and 16-QAM modulated OFDM signal, the proposed LCCSS methods leads to little spectral regrowth compared to the original OFDM signal and hence the PAPR reduction using the proposed LCCSS methods is more immune to out of band radiation of OFDM signal

6.5 SUMMARY

The proposed LCCSS methods with less computational complexity achieve significant PAPR reduction in OFDM systems. The OFDM signal processed using LCCSS-OPF when compared with the original QPSK OFDM and QAM OFDM signal achieves PAPR reduction by 5.28 dB and 4.01 dB respectively, while LCCSS-D based method achieves PAPR reduction by 5.24 dB and 4.18 dB respectively. The performance of the proposed time domain methods over different channels also shows that LCCSS-OPF outperforms the LCCSS-D method both in terms of PAPR reduction and BER performance.