CHAPTER 5

A NOVEL COMPANDING TRANSFORM FOR PAPR REDUCTION IN OFDM SYSTEMS

In companding scheme, the OFDM signal is processed directly in order to reduce PAPR value by increasing the average signal power while maintaining the peak value constant. However this method achieves PAPR reduction but leads to significant degradation in BER performance. In this chapter a novel companding transform which attains significant tradeoff between PAPR reduction, transmission power, implementation complexity and BER performance is proposed.

5.1 NEED FOR COMPANDING TRANSFORM

The adverse effect of OFDM signal with large PAPR demands HPA with large dynamic range which can be eliminated by using a scheme called companding. The idea of companding enables the OFDM signal to be transmitted by means of a HPA with smaller dynamic range (Wang et al 1999). The dynamic range of the OFDM signal is compressed at the transmitter before transmission and is restored to the original value at the receiver.

5.2 PROPOSED COMPANDING TRANSFORM

A Companding Transform (CT) is proposed to treat large, small and average amplitude samples of OFDM signals on different scales and its BER performance is evaluated over AWGN, fading and SUI-5 channel respectively. The proposed CT with two inflexion points achieves significant Bit Error Rate (BER) and PAPR reduction performance with minimum Out of
Band Interference (OBI) when compared with μ-Law Scheme and EC scheme.

In OFDM systems, spectral leakage from one sub channel to other is unavoidable and results in OBI. One way to restrict the OBI is to employ soft compression methods rather than performing hard clipping operation. The OBI due to companding can be quantified by computing the Power Spectral Density (PSD) of the companded signal.

This section put forward the analytical expressions of the proposed method and examines its suitability to perform companding operation on OFDM signal so that PAPR reduction is achieved with significant BER performance. Let the random variables X and Y denote the amplitude of the input and output companding function with their respective Cumulative Distributive Function’s (CDFs) given by \( F_X(x) \) and \( F_Y(y) \). Since Y is a uniform distribution in the interval \( \frac{\sigma}{\sqrt{8}} < |x(n)| \leq \frac{\sigma}{\sqrt{6}} \) the CDF of Y is

\[
F_Y(y) = \frac{y}{2\beta} + \frac{1}{2}
\]

The companding function with two inflection points is given by

\[
C(x) = \begin{cases} 
  u_1 x(n), & |x(n)| \leq \frac{\sigma}{\sqrt{8}} \\
  \text{sgn}(x(n)) \beta \sqrt{\frac{1}{2} - \frac{1}{2} \exp \left(1 - \frac{a^2 x(n)^2}{\sigma^2}\right)}, & \frac{\sigma}{\sqrt{8}} < |x(n)| \leq \frac{\sigma}{\sqrt{6}} \\
  \frac{1}{u_2} x(n), & |x(n)| > \frac{\sigma}{\sqrt{6}}
\end{cases}
\]

where the parameter \( u_1 > 1, 0 \leq u_2 \leq 1 \),

\( x(n) \) is a complex valued OFDM signal,

\( a \) varies the degree of companding and

\( \beta \) maintains the companded output signal power to be same as the average input power which is defined as
\[
\beta = \sqrt{\frac{E[|x(n)|^2]}{\frac{1}{2} \cdot \frac{1}{3} \exp\left(1 - \frac{\alpha |x(n)|^2}{\sigma^2}\right)}}
\]

The companded signal is given by \(x_c(n) = C(x(n))\), where \(C(.)\) denotes the companding function.

The inverse companding function is given by

\[
C^{-1}(x) = \begin{cases}
\frac{\beta}{\alpha} x(n), & n \in \varphi_1\left(\frac{\sigma}{\sqrt{8}}\right) \\
\frac{1}{\alpha} \text{sgn}(x(n)) \sqrt{-c^2 \ln \left(6 - \frac{12|x(n)|}{\beta}\right)}, & n \in \varphi_1\left(\frac{\sigma}{\sqrt{8}}, \frac{\sigma}{\sqrt{6}}\right) \\
u_2 x(n), & n \in \varphi_1\left(\frac{\sigma}{\sqrt{6}}\right)
\end{cases}
\]  

(5.3)

\[
\varphi_1\left(\frac{\sigma}{\sqrt{8}}\right) = \left\{ n \forall |x(n)| \leq \frac{\sigma}{\sqrt{8}} \right\}
\]

\[
\varphi_1\left(\frac{\sigma}{\sqrt{8}}, \frac{\sigma}{\sqrt{6}}\right) = \left\{ n \forall \frac{\sigma}{\sqrt{8}} < |x(n)| \leq \frac{\sigma}{\sqrt{6}} \right\}
\]

\[
\varphi_1\left(\frac{\sigma}{\sqrt{6}}\right) = \left\{ n \forall |x(n)| > \frac{\sigma}{\sqrt{6}} \right\}
\]

where \(\varphi_1\left(\frac{\sigma}{\sqrt{8}}\right)\), \(\varphi_1\left(\frac{\sigma}{\sqrt{8}}, \frac{\sigma}{\sqrt{6}}\right)\) and \(\varphi_1\left(\frac{\sigma}{\sqrt{6}}\right)\) are the index set of OFDM samples.

\(F_X(x)\) is a monotonic increasing function in the interval \(|x(n)| \leq \frac{\sigma}{\sqrt{8}}\), a uniform distribution in the interval \(\frac{\sigma}{\sqrt{8}} < |x(n)| \leq \frac{\sigma}{\sqrt{6}}\) and a decreasing function in the interval \(|x(n)| > \frac{\sigma}{\sqrt{6}}\).

The proposed companding function with two inflexion points transforms the average Gaussian distributed OFDM signal into a uniform distribution such that the non-linear distortion during transmission to the
receiver is reduced and also treats the small and large OFDM signals on different scale. The use of distribution based transformation of average OFDM signals and linear function based transformation of small and large OFDM signals greatly improves the tradeoff between PAPR reduction and BER performance. Since the function C(.) is infinitely differentiable, its leads to a minimum amount of OBI which is verified based on the simulation results. The companded OFDM signal at the receiver can be written as

\[ r(n) = x_c(n) \otimes h(n) + w_{wn}(n) \]  \hspace{1cm} (5.4)

where \( x_c(n) \) denotes companded OFDM signal i.e., \( x_c(n) = f(x(n)) \), \( h(n) \) denotes impulse response of the channel and \( w_{wn}(n) \) refers to AWGN with variance \( \sigma_w^2 \).

Equalization is performed at the receiver to reduce channel distortion by convolving the received signal \( r(n) \) with \( h_{\text{inv}}(n) \) which the inverse function of \( h(n) \), i.e., \( h(n) \otimes h_{\text{inv}}(n) = \delta(n) \).

Hence the received signal after equalization is written as

\[ r(n) = x_c(n) + w(n) \]  \hspace{1cm} (5.5)

where \( w(n) = w_{wn}(n) \otimes h_{\text{inv}}(n) \)

Applying decompanding function to Equation (5.5), we get

\[ r(n) = C^{-1}(x_c(n) + w(n)) \]
\[ r(n) = x(n) + C^{-1}(w(n)) \]  \hspace{1cm} (5.6)

The proposed method employs two inflexion points so that the PAPR reduction performance is varied in accordance with the degree of companding. This method overcomes the drawback of \( \mu \) - Law companding scheme which enlarges only small signals and EC scheme whose performance remains unchanged for different levels of companding \( (d > 4) \).
5.3 NUMERICAL RESULTS AND PERFORMANCE ANALYSIS

The proposed CT's PAPR reduction and BER performance is compared with that of μ-Law CT, EC scheme and Trapezoidal Companding (TC) scheme by performing simulation over a non linear AWGN, fading and SU1-5 channel separately. The effect of Inter Symbol Interference (ISI) is reduced by employing a cyclic prefix with a length of one-fourth of OFDM symbol duration. The simulation results for the proposed CT are obtained by assigning the parameters $u_1$, $u_2$ and $\alpha$ with values 1.32, 0.73 and 1.67 respectively.

The EC transform employed for performance comparison assumes a degree of companding, $d = 2$ for simulation since the performance of EC scheme for $d > 4$ is found to be similar to that of $d = 4$ hence it is ignored.

5.3.1 CCDF Performance Analysis

PAPR reduction performance is approximated by employing Complimentary Cumulative Distributive Function (CCDF) which is a probability that PAPR exceeds a specific threshold. CCDF for PAPR reduction performance is plotted based on the 10000 random QPSK and 16-QAM modulated OFDM signals generated and also compared with that of μ-Law signals, EC signals with degree of companding $d = 2$, TC signals and the companded signals based on the proposed method.

CCDF plot of both the QPSK and 16-QAM modulated OFDM signal processed using the proposed CT are compared with that of μ-law CS, EC scheme and TC ($a = 0.2, b = 0.7$). The CCDF curves of the same are presented in Figure 5.1 and Figure 5.2. The PAPR reduction performance of the proposed companding scheme for a QPSK modulated OFDM signal is better than μ-Law ($\mu = 1$), EC Scheme, TC ($a = 0.2$ and $b = 0.7$) by 2.83 dB, 2.47 dB and 0.51 dB respectively. While the proposed scheme for a 16-QAM
modulated OFDM signal outperforms the $\mu$-Law ($\mu = 1$), EC Scheme, TC ($a = 0.2$ and $b = 0.7$) by 2.73 dB, 1.48 dB and 0.81 dB respectively.

Figure 5.1 CCDF curves of the referred companding schemes for a QPSK modulated OFDM signal

Figure 5.2 CCDF curves of the referred companding schemes for a 16-QAM modulated OFDM signal
Table 5.1  CCDF Comparative Analysis of the proposed CT and the referred schemes.

<table>
<thead>
<tr>
<th>OFDM Symbol</th>
<th>PAPR (dB)</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Original</td>
<td>μ-Law (μ = 1)</td>
<td>EC Scheme</td>
<td>TC a = 0.2, b = 0.7</td>
</tr>
<tr>
<td>QPSK</td>
<td>12.33</td>
<td>6.24</td>
<td>5.88</td>
<td>3.92</td>
</tr>
<tr>
<td>16-QAM</td>
<td>13.42</td>
<td>7.11</td>
<td>5.86</td>
<td>5.19</td>
</tr>
</tbody>
</table>

The performance analysis based on CCDF curves for PAPR reduction for QPSK and 16-QAM modulated OFDM signal are presented in the Table 5.1. Based on the simulation results it is observed that the OFDM signal companded based on the proposed method achieves better PAPR reduction performance that the other referred companded schemes.

5.3.2 BER performance Analysis

Simulation of BER versus Signal to Noise Ratio (SNR) of actual OFDM signal based on various companding techniques assuming the wireless channel to be (a) AWGN channel (b) fading channel (c) SU−5 channel are discussed in this section. Ideal or original OFDM signal refers to the case where the OFDM signal is directly transmitted into the channel without considering the effects of HPA and performing any transformation.

The BER curves of companded signals using μ-Law, EC scheme with d = 2, TC scheme and the proposed CT are located to the right of OFDM signal’s BER curve in both AWGN and fading channel due to the signal distortion introduced by companding operation but simultaneously achieves significant PAPR reduction.
BER vs. SNR plot of the OFDM signal processed using the proposed CT is compared with that of \( \mu \)-law CS, EC scheme and TC \((a = 0.2, b = 0.7)\). The simulation results are presented for both QPSK and 16-QAM modulated OFDM signal over

(a) AWGN channel in Figure 5.3 and Figure 5.4 respectively
(b) Fading channel in Figure 5.5 and Figure 5.6 respectively
(c) SUI—5 channel in Figure 5.7 and Figure 5.8 respectively.

Figure 5.3 BER vs. SNR plot of the referred companding schemes over AWGN channel for a QPSK modulated OFDM signal

Figure 5.4 BER vs. SNR plot of the referred companding schemes over AWGN channel for a 16-QAM modulated OFDM signal
Over AWGN channel, the BER performance of the proposed CT for a QPSK modulated OFDM signal is better than that of $\mu$-Law ($\mu = 1$) and EC scheme by 1.19 dB and 0.36 dB, where as its BER performance is poorer than TC ($a = 0.2$, $b = 0.7$) by 0.31 dB. For a 16-QAM modulated OFDM signal, the BER performance of the proposed CT over a AWGN channel outperforms the $\mu$-Law ($\mu = 1$) and EC scheme by 0.85 dB and 0.44 dB, but provides slightly worse BER performance than that of TC ($a = 0.2$, $b = 0.7$) by 0.41 dB.

Figure 5.5  BER vs. SNR plot of the referred companding schemes over fading channel for a QPSK modulated OFDM signal

Figure 5.6  BER vs. SNR plot of the referred companding schemes over fading channel for a 16-QAM modulated OFDM signal
Figure 5.7 BER vs. SNR plot of the referred companding schemes over SUI-5 channel for a QPSK modulated OFDM signal

Figure 5.8 BER vs. SNR plot of the referred companding schemes over SUI-5 channel for a 16-QAM modulated OFDM signal

Over fading channel, the BER performance of the proposed CT for a QPSK modulated OFDM signal is better than that of μ-Law and EC scheme
by 1.83 dB and 0.83 dB, where as its BER performance is poorer than TC \((a = 0.2, b = 0.7)\) by 0.59 dB. For a 16-QAM modulated OFDM signal, the BER performance of the proposed CT in a fading channel outperforms the µ-Law and EC scheme by 0.98 dB and 0.90 dB, but provides slightly worse BER performance than that of TC \((a = 0.2, b = 0.7)\) by 0.54 dB.

Over SUI-5 channel, the BER performance of the proposed CT for a QPSK modulated OFDM signal is better than that of µ-Law and EC scheme by 1.50 dB and 0.21 dB, where as its BER performance is poorer than TC \((a = 0.2, b = 0.7)\) by 0.97 dB. For a 16-QAM modulated OFDM signal, the BER performance of the proposed CT in a SUI-5 channel outperforms the µ-Law and EC scheme by 3.18 dB and 1.34 dB, but provides slightly worse BER performance than that of TC \((a = 0.2, b = 0.7)\) by 0.80 dB.

**Table 5.2 Comparative Analysis of BER vs. SNR plot for the proposed CT and the referred schemes**

<table>
<thead>
<tr>
<th>Channel</th>
<th>OFDM Symbol</th>
<th>SNR, ( \frac{c_{\text{SNR}}}{10^{-4}} ) dB at BER = 10^{-4}</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Original (\mu)-Law ((\mu = 1))</td>
</tr>
<tr>
<td>AWGN</td>
<td>QPSK</td>
<td>6.63</td>
</tr>
<tr>
<td></td>
<td>16-QAM</td>
<td>7.24</td>
</tr>
<tr>
<td>Fading</td>
<td>QPSK</td>
<td>9.13</td>
</tr>
<tr>
<td></td>
<td>16-QAM</td>
<td>10.31</td>
</tr>
<tr>
<td>SUI-5</td>
<td>QPSK</td>
<td>21.99</td>
</tr>
<tr>
<td></td>
<td>16-QAM</td>
<td>23.1</td>
</tr>
</tbody>
</table>

The BER performance of the proposed CT along with the referred schemes over three different channels are given in Table 5.2. Over AWGN, fading and SUI-5 channel, the BER performance of our proposed method is better than EC scheme and µ-Law scheme but performs worse than TC
scheme. Hence the use of linear and non-linear companding schemes in different intervals has achieved significant PAPR reduction but with a slightly degraded BER performance compared to TC scheme.

5.3.3 Constellation Plot Analysis

Constellation plot of both QPSK and QAM modulated OFDM signal processed using the proposed CT and transmitted over three different channels namely

(a) AWGN channel are presented in Figure 5.9 and Figure 5.10 respectively

(b) Fading channel are presented in Figure 5.11 and Figure 5.12 respectively

(c) SUb5 channel are presented in Figure 5.13 and Figure 5.14 respectively

Figure 5.9  Constellation diagram of a QPSK modulated OFDM signal processed using the proposed CT over AWGN channel.
Figure 5.10 Constellation diagram of a 16-QAM modulated OFDM signal processed using the proposed CT over AWGN channel.

Figure 5.11 Constellation diagram of a QPSK modulated OFDM signal processed using the proposed CT over fading channel.
Figure 5.12 Constellation diagram of a 16-QAM modulated OFDM signal processed using the proposed CT over fading channel.

Figure 5.13 Constellation diagram of a QPSK modulated OFDM signal processed using the proposed CT over SUI-5 channel.
Figure 5.14 Constellation diagram of a 16-QAM modulated OFDM signal processed using the proposed CT over SU-5 channel

The constellation plot of the received OFDM signal shows that the digital modulation technique namely QPSK and 16-QAM used as signal mapper at the OFDM transmitter is preserved by the proposed CT even after transmission through the different communication channels at the receiver. Since the modulation features performed at the transmitter on the OFDM signal are preserved by the proposed CT at the receiver, this method can be used in OFDM systems for effective PAPR reduction.

5.3.4 Power Spectral Density Analysis

The spectral analysis of the OFDM signal processed using the proposed CT for reducing PAPR is compared with that of the original OFDM signal. PSD plot of both QPSK and 16-QAM modulated OFDM signal processed using the proposed CT along with that of the original OFDM signal are presented in Figure 5.15 and Figure 5.16 respectively.
Figure 5.15 PSD of the QPSK modulated OFDM Signal processed by the proposed CT and the original signal

Figure 5.16 PSD of the 16-QAM modulated OFDM Signal processed by the proposed CT and the original signal

Based on the PSD plot of both QPSK and QAM modulated OFDM signal it is evident that the PAPR reduction using the proposed CT is more immune to the out of band radiation caused by the companding process. Hence the proposed CT is a viable model for PAPR reduction in OFDM systems.
5.4 SUMMARY

The proposed CT with two inflexion points combines linear and non-linear companding schemes to scale different signal levels independent of one another inorder to achieve a PAPR reduction of 8.92 dB and 9.04 dB compared to the original QPSK - OFDM and 16-QAM OFDM signal respectively. The significant PAPR reduction by 72.34% for QPSK OFDM signal and 67.36% for QAM OFDM signal is compromised with a slight degradation of BER performance over AWGN, fading and SUI-5 channel. With proper selection of companding parameters the proposed scheme meets the OFDM system requirement and HPA characteristics in order to achieve a significant tradeoff between PAPR reduction and BER performance.