APPENDIX 1

DERIVATION FOR TRANSMIT-SIDE SELECTION OF
EXTENDED GHRAVI & GERSHMAN ALGORITHM
DISSA

Let us assume that \( L_R \) antennas have been selected out of \( M_R \) antennas at the receive side by using the algorithm given in (Gharavi & Gershman 2004). The original \( \mathbf{H} \) matrix was of \( M_R \times M_T \). The \( \mathbf{H} \) matrix after the receive side selection will be of \( L_R \times M_T \). The capacity expression after selection only at the receiver is

\[
C = \log_2 \det \left[ \mathbf{I}_{L_R} + \frac{\rho}{L_T} \mathbf{H} \mathbf{H}^H \right] \quad (A1.1)
\]

where \( \mathbf{H} \) is of \( L_R \times M_T \).

Now, assume that \( n \) antennas have been already selected at the transmit side and the present problem is to select \( (n+1) \)th antenna. The capacity of a system with \( (n+1) \) transmit antennas and \( L_R \) receive antennas, notated as \( C(\mathbf{H}_{n+1}) \) is given by

\[
C(\mathbf{H}_{n+1}) = \log_2 \det \left[ \mathbf{I}_{L_R} + \frac{\rho}{L_T} \mathbf{H}_{\mathcal{L}_T^{n+1}} \mathbf{H}_{\mathcal{L}_R^{n+1}}^H \right] \quad (A1.2)
\]

where \( \mathbf{H}_{n+1} \) is of \( L_R \times (n + 1) \) and \( \mathcal{L}_T^{n+1} \) refers to a set of transmit antennas, whose size is \( n+1 \).

\( \mathbf{H}_{n+1}^H \mathbf{H}_{n+1} \) can be written in terms of \( \mathbf{H}_n \) which is of \( L_R \times n \).
\[ H_{n+1}^H H_{n+1}^H = H_n H_n^H + h_j^H h_j \]  
(A1.3)

where \( h_j \) is the Hermitian transpose of the submatrix formed of newly-selected \( j \)th column of \( H \) and \( \mathcal{E}_R \) set of rows. That is, \( h_j = H_{\mathcal{E}_R}^H \). The order of \( h_j \) is \( 1 \times L_R \); that is \( h_j \) is a row vector. \( H_n \) is a submatrix formed of \( \mathcal{E}_R \) set of rows and \( n \) selected columns of \( H \). The order of \( H_n \) is \( L_R \times n \). \( C(H_{n+1}) \) can be written in terms of \( H_n \) by using (A1.3)

\[
C(H_{n+1}) = \log_2 \det \left[ I_{L_R} + \frac{\rho}{L_T} [H_n H_n^H + h_j^H h_j] \right] \quad (A1.4)
\]

\[
= \log_2 \det \left[ I_{L_R} + \frac{\rho}{L_T} H_n H_n^H + \frac{\rho}{L_T} h_j^H h_j \right]
\]

We have matrix determinant lemma as

\[
\det(A+uv^T) = (1+v^TA^{-1}u)\det(A),
\]

(A1.5)

where \( u \) and \( v \) are column vectors. In the present case, \( A = I_{L_R} + \frac{\rho}{L_T} H_n H_n^H \), \( u = h_j^H \) and \( v = \frac{\rho}{L_T} h_j \). The determinant calculation is started with \( A = I_{L_R} \), that is, with no antenna or column selected

\[
C(H_{n+1}) = \log_2 \{ \det( [I_{L_R} + \frac{\rho}{L_T} H_n H_n^H] ) \}
\]

\[
[1 + \frac{\rho}{L_T} h_j \left( I_{L_R} + \frac{\rho}{L_T} H_n H_n^H \right)^{-1} h_j^H ]; \quad (A1.6)
\]

Now, \( C(H_{n+1}) = \log_2 \det \left[ \left( I_{L_R} + \frac{\rho}{L_T} H_n H_n^H \right) \right] + \log_2 \left[ 1 + \frac{\rho}{L_T} h_j \left( I_{L_R} + \frac{\rho}{L_T} H_n H_n^H \right)^{-1} h_j^H \right] \quad (A1.7) \)
\[ C(H_n) + \log_2 \left[ 1 + \frac{\rho}{L_T} h_j \left( I_{L_R} + \frac{\rho}{L_T} H_0 H_0^H \right)^{-1} h_j^H \right] \]

\[ = C(H_n) + \Delta C(H_n) \]  
(A1.8)

The inverse of \( \left( I_{L_R} + \frac{\rho}{L_T} H_0 H_0^H \right)^{-1} \) can be found by using Sherman-Morrison formula, which is given as (A1.9)

\[ (A + uv^T)^{-1} = A^{-1} - \frac{A^{-1} uv^T A^{-1}}{1 + v^T A^{-1} u}, \]

where \( A \) is an invertible matrix, \( u \) and \( v \) are column vectors.

\[
C(H_1) = \log_2 \det \left[ \left( I_{L_R} + \frac{\rho}{L_T} H_0 H_0^H \right) \right] \\
+ \log_2 \left[ 1 + \frac{\rho}{L_T} h_j \left( I_{L_R} + \frac{\rho}{L_T} H_0 H_0^H \right)^{-1} h_j^H \right]
\]

In this case, the first log term is 0, \( A \) is \( I_{L_R} \) and \( u \) and \( v \) are null vectors.

\[
C(H_2) = \log_2 \det \left[ \left( I_{L_R} + \frac{\rho}{L_T} H_1 H_1^H \right) \right] + \log_2 \left[ 1 + \frac{\rho}{L_T} h_j \left( I_{L_R} + \frac{\rho}{L_T} H_1 H_1^H \right)^{-1} h_j^H \right] \]  
(A1.10)

\[
C(H_3) = \log_2 \det \left[ \left( I_{L_R} + \frac{\rho}{L_T} H_2 H_2^H \right) \right] \\
+ \log_2 \left[ 1 + \frac{\rho}{L_T} h_j \left( I_{L_R} + \frac{\rho}{L_T} H_2 H_2^H \right)^{-1} h_j^H \right]
\]

The first term of \( C(H_3) \) is \( C(H_2) \). In the second term,
\((I_{LR} + \frac{\rho}{\lambda_T} H_2 H_2^H)^{-1}\) is in the form of \((A + uv^T)^{-1}\) where \(A = I_{LR} + \frac{\rho}{\lambda_T} H_j H_j^H\) and \(u\) and \(v^T\) are respectively \(h_j^H\) and \(\frac{\rho}{\lambda_T} h_j\) in accordance with (A1.3). Based on these observations, the following definitions can be made.

Define

\[
B_n \equiv \left( I_{LR} + \frac{\rho}{\lambda_T} H_n H_n^H \right)^{-1}.
\]  \hspace{1cm} (A1.11)

Initially, there are no antennas selected and hence \(B\) will be initialised to \(I_{LR}\).

\[
B_0 = I_{LR}
\]

Define,

\[
\alpha_{j,n} \equiv h_j B_n h_j^H.
\]  \hspace{1cm} (A1.12)

Then, \(C(H_{n+1})\) can be written as

\[
C(H_{n+1}) = C(H_n) + \log_2 \left[ 1 + \frac{\rho}{\lambda_T} \alpha_{j,n} \right].
\]  \hspace{1cm} (A1.13)

where \(\alpha_{j,n}\) corresponds to \(\max \Delta C(H_n)\)

\(J = \text{arg} \max \alpha_{j,n}\). The \(B\) matrix will now be updated as follows in accordance with (A1.9)

\[
B_{n+1} = B_n - \frac{B_n \frac{\rho}{\lambda_T} h_j^H h_j B_n}{1 + h_j B_n \frac{\rho}{\lambda_T} h_j^H}
\]  \hspace{1cm} (A1.14)

\[
= B_n - \frac{B_n h_j^H h_j B_n}{\frac{\rho}{\lambda_T} \alpha_{j,n}}
\]
Define,

\[ a = \frac{1}{\sqrt{(l_T/\rho) + \alpha_j n}} \, B_n h_j^H \]  \hspace{1cm} (A1.15)

Then,

\[ a^H = \frac{1}{\sqrt{(l_T/\rho) + \alpha_j n}} \, h_j B_n^H \]

\[ = \frac{1}{\sqrt{(l_T/\rho) + \alpha_j n}} \, h_j B_n \]

We get,

\[ B_{n+1} = B_n - aa^H \]  \hspace{1cm} (A1.17)

It must be noted that in the beginning when we are looking for the first antenna, the first term of (A1.7) will be zero and the second term will be

\[ \log_2 \left[ 1 + \frac{\rho}{l_T} \, h_j (I_{l_R})^{-1} h_j^H \right] \]  \hspace{1cm} (A1.18)

The required expressions are

\[ B_0 = I_{l_R} \]

\[ a = \frac{1}{\sqrt{(l_T/\rho) + \alpha_j n}} \, B_n h_j^H \]

\[ a^H = \frac{1}{\sqrt{(l_T/\rho) + \alpha_j n}} \, h_j B_n^H \]

\[ B_{n+1} = B_n - aa^H \]

\[ \alpha_{j,n} = h_j B_n h_j^H. \]

The full algorithm is stated in Table 3.2 of chapter 3.