CHAPTER-1

INTRODUCTION

1.1 FINANCIAL MARKETS

Financial market is mainly categorized into two types such as Securities Market and Commodity Market. The main dissimilarity among these two markets is the goods operated. On commodities markets, futures contracts for real commodities are bought and sold, while on the securities market, financier’s trade shares of stock in firms. However, the trading of stocks and commodities on these markets are similar. Mostly trading happens on the physical exchanges, such as the New York Stock Exchange (NYSE) and the Chicago Mercantile Exchange (CME). Though, ample trading also happens off the exchange. Stocks in the securities market, sold-off an exchange are said to be sold "over the counter." Commodities sold-off a controlled exchange are said to be sold on the "spot market."

The securities and commodity markets are linked in many ways. In concept, the securities market increases and decreases based on the reported earnings and projected earnings of the companies with stock trading on an index. When business decelerates, or the cost of producing products increases, earnings will fall and so will stock prices. At times, when the commodity markets are rallying, the rise in prices of those commodities means corporate earnings of users of those commodities will decline. Sometimes interference by central banks makes a situation where both commodity markets and securities markets rally or drop.
1.1.1 SPOT & FUTURE PRICES

Trading in financial products has been an important component of world economy. Financial contracts in ‘Spot’ and ‘Future’ trading for both financial securities and commodities have dominated in the international markets. In simple terms, ‘Spot price’ refers to prices at which immediate delivery/receipt of a product is made. ‘Future price’ refers to prices at which delivery/receipt of a product is made on a future date. Financial securities include commercial bills, treasury bills, shares, bonds and so on. Commodities include metals such as gold, silver, copper; crude oil, agricultural products, and so on.

1.1.2 FINANCIAL SECURITIES & COMMODITIES

The trading of these financial securities and commodities happen both at national and international levels involving huge amount of money between the sellers and buyers with the assistance of registered brokers.

Contracts that started on mere gut feel of the participants in the early stages has slowly transformed into trade agreements based on logical analysis for the market conditions and demand and supply factors. Factors such increasing number of multi-national participants, multi-variant products, and high risk of financial manipulations, coupled with policies of different governments across the world mandated for a robust analysis of the price movements of the securities and commodities traded.

1.1.3 STOCHASTIC STUDY – MATHEMATICAL MODELING

The term Stochastic, as given in Oxford Dictionary “characterized by a sequence of random variables” and refers to process. Stochastic study aims to understand the influence in variations of different variables that affect the prices of a product. The financial industry has both stimulated and benefited from advances in various disciplines of mathematical sciences like probability, differential equations,
Optimization, statistics and numerical analysis. The study of financial systems is centered on the study of the behavior of economic agents in allocating and deploying their resources, both spatially and across time, in an uncertain environment. Time and uncertainty are central elements which influence the financial behavior.

During 1970-1980, a significant research was commenced in development of new models and refinement of existing mathematical models (Black & Scholes [24], Brennan & Schwartz [26]-[28]). By this time the financial databases to support these models were extremely larger and the feasibility of implementing these models was also much greater. The developments in computing and telecommunication technologies made possible the formation of new financial markets and the same technologies made feasible the numerical solution of complex models of multivariate partial differential equations. During this period many mathematicians got attracted to the financial services industry by high salaries and challenging problems.

The advent of mathematical modeling, during the 20\textsuperscript{th} Century, for the financial markets has brought a shift in the paradigm and made the contracts of spot, future, and forwards more reliable. Numerous models were proposed and developed to support contracts in the financial market arena (Andersen & Andersen [12], Company & et al [30], Cont & Voltchkova [31], Esekon [41], Jensen & et al. [75], Kumar & et al. [79], Oosterlee [100], Rodrigoa [108], Schwartz [111], Tangman & et al. [119], Toivanen [122], Vencer [125], Yun [136], [137]). However, all of these evolved in the developed western economies and were dominated the features of those economies.

Economic reforms taken up by the Indian government during 1990’s have contributed for development of their financial systems. With the development of economy, financial markets have also advanced and demands for robust analysis
for market trends gained prominence. Models proposed by western researchers are being applied with necessary modifications.

Financial mathematics, also termed as financial engineering, mathematical finance, and computational finance, is the application of mathematical methods to the solution of problems in finance. It imbibes tools from applied mathematics, computer science, statistics, and economic theory for financial models. Investment banks, commercial banks, hedge funds, insurance companies, corporate treasuries, regulatory agencies and commodity traders apply these methods of financial mathematics to problems such as derivative securities valuation, portfolio structuring, risk management, scenario simulation and commodity prices. Quantitative analysis has brought efficiency and rigor to financial markets and to the investment process and is becoming increasingly significant in regulatory concerns. With the pace of financial innovation, the need for highly qualified people with specific training in financial mathematics intensifies.

Finance, offspring of economics, concerns itself with the valuation of assets and financial instruments as well as the apportionment of resources. Centuries of history and experience have resulted in fundamental theories on the way economies function and the way we value assets. Mathematics acts as a suitable tool because it allows theoreticians to model the relationships between variables and represent randomness in a manner that can lead to useful analysis. Mathematics, then, becomes a useful reserve from which researchers can draw to solve problems, provide insights and make the intractable model tractable.

Mathematical finance gathers from the disciplines such as probability theory, statistics, scientific computing and partial differential equations to provide models and derive relationships between fundamental variables like asset prices, market movements, interest rates and convenience yield. These mathematical tools allow
us to infer conclusions that can be otherwise difficult to find or not immediately obvious from human intuition.

Support of modern computational techniques help in storage of vast quantities of data and model numerous variables simultaneously, leading to the ability to model quite large and complicated systems. So, it may be inferred that techniques of scientific computing, such as numerical computations, Monte Carlo simulation, and optimization are an important part of financial mathematics.

1.2 BLACK-SCHOLES MODEL

A large part of any science is the ability to develop testable hypotheses based on a fundamental understanding of the objects of study and prove or contradict the hypotheses through repeatable studies. In this light, mathematics is a language to represent theories and provides tools for testing their validity. For instance, in the theory of option pricing due to Black, Scholes and Merton, a model [24], for the movement of stock prices is posited, and in conjunction with basic theory which states that a riskless investment will result in risk-free rate of return, the researchers reasoned that a value can be assigned to an option that is independent of the expected future value of the stock.

This theory, for which Scholes and Merton were awarded the Nobel Prize, is a classic illustration of the interaction between math and financial theory, which ultimately led to a surprising insight into the nature of option prices. The mathematical contribution was the basic stochastic model (Geometric Brownian motion) for movement of stock prices and the partial differential equation (PDE) and its solution providing the relationship between the option's value and other market variables. Their analysis also helped in providing a completely specified strategy for managing option investment which permits practical testing of the model's consequences. This theory, which would not have been possible without the fundamental participation of mathematics, today plays a significant role in a trillion dollar industry.
Over the past three decades the deficiencies of the Black-Scholes model have become progressively clear, with some academic observers continually ringing the “death knell” of the formula as its weaknesses become more obvious and it can lead to substantial discrepancies between actual market prices and prices calculated using the model. These discrepancies between market and theoretical prices are obvious in the observation of different implied volatilities as per the exercise prices (smile or skew) and maturities (term structure). Therefore, despite their popularity and wide spread use, the model is built on some non-real life assumptions about the market. One problem with the Black-Scholes analysis, however, is that the mathematical skills that are required in the derivation and solutions of the model are fairly advanced and probably unfamiliar to many economists.

Fig. 1 represents the classification of Financial Markets and Models

Critics of Black-Scholes Model, while contributing on the deficiencies, have proposed further studies in extending the models and also different solution techniques of these models to address numerous financial products. In real world, both the enhanced models and their techniques of tracing suitable solution are
significant due to the dynamics of market. They help in prediction of price movement of the financial products with highest precision. These models, often times, are in the form of statistical models and PDE models. However, tracing the accurate solution for these equations is a herculean task which went arrived would help in the development of new mathematical techniques. Statistical models include STARX (McMillan [91]), TAR and STAR (Valterri [124]), ARIMA (Khan & et al. [78]), and GARCH (Ramirez & Fadiga [106]), etc. PDE models include linear, nonlinear partial differential equations (NPDE) and partial-integro differential equations (PIDE).

Esekon, J. E. [41], developed a nonlinear Black-Scholes model for hedging of derivatives in illiquid markets, and has obtained analytical solution using transformation of variables.

1.3 MATHEMATICAL TECHNIQUES

Arriving at analytical solution is phenomenal assignment since most equations do not have exact methodology for application. However, there are some special methods such as First integral method (FIM) proposed by Feng [49], Tanh-Coth method proposed by Wazwaz [127] and Sine-Cosine method proposed by Alquran & Al-Khaled [11] to get the exact solution of nonlinear PDEs. But, these methods have their own limitations for their applicability such as involving variable coefficients, finding the parameters in Tanh-Coth method and Sine-Cosine methods.

Researchers attempted to develop techniques that could arrive at approximate solution with desired accuracy. These approximate solution techniques have been classified into two categories like discretization and non-discretization techniques.
1.3.1 DISCRETIZATION TECHNIQUES

These include, Finite difference method (FDM) (Brennan & Schwartz [26]-[28]), Alternating Directions Implicit method (ADI) (Andersen & Andersen [12]), Front-fixing method & Penalty method (Nielsen & et al. [96]), Backward difference formula BDF2 (Oosterlee [100]), Operator splitting method (Ikonen & Toivanen [71]), Explicit-implicit finite difference method (Cont & Voltchkova [31]), hybrid finite difference method (Cen & et al. [29]), high-order front-tracking finite difference method (Toivanen [122]), exponential time integration (ETI) method (Tangman & et al. [119]), Higher Order Compact (HOC) (Kumar & et al. [79]), are discretization methods in solving the PDE models aroused in financial market.

1.3.2 NON-DISCRETIZATION TECHNIQUES

These techniques include, Adomian Decomposition Method (ADM) proposed by Adomian [5], Variational Iteration Method (VIM) proposed by He [59], Homotopy Perturbation Method (HPM) proposed by Liao [83], and Homotopy Analysis Method (HAM) proposed by Liao [87], are non-discretization methods used in solving the Black-Scholes equation. The methods were developed for non-linear and ordinary differential equations (ODEs) and PDEs, it is proved by later researches that it can be applied to partial integro-differential equations subject to the satisfaction of initial boundary conditions.

The above discretization and non-discretization methods were developed for solving the nonlinear PDEs. However, further researchers have tested them for linear PDEs with successful results in different fields of engineering and science.

1.4 PRESENT STUDY

The present study “Stochastic Behavior of Spot and Future Commodity Prices: Numerical Methods Approach” focusses primarily on the PDE models of linear
(commodities market) and non-linear (securities market) and is based on the scope for further study, as suggested Esekon [41] “Future work will also involve solving the nonlinear Black-Scholes equation using the hyperbolic tangent (Tanh) method.” The ‘Tanh’ method suggested provides an opportunity to arrive at analytical solutions.

Tan-Coth method, the extension of Tanh method, First Integral Method (FIM), and Sine-Cosine Method are three powerful methods identified to solve non-linear PDEs in the literature. Taking these researches as cue the present study aims to test applicability of FIM, Tan-Coth method, and Sine-Cosine methods to solve non-linear Black-Scholes equation.

At the same time, the present study proposes to consider testing of non-discretization techniques such as ADM, VIM, HPM and HAM. These non-discretization techniques are successfully applied by Allahviranloo [8], on linear Black-Scholes equation which becomes the base of the present study to test its applicability to solve one-factor, two-factor and three-factor commodity price models given by Schwartz [111].