ABSTRACT

Mathematical Models in Finance have been most active areas of research since their inception in 1973 as Black-Scholes model. Various mathematical models in the form of linear partial differential equations, nonlinear partial differential equations, partial-integro differential equations and fractional order partial differential equations were developed in the financial market (commodity market as well as securities market) arena. In commodity market most of the mathematical models are in the form of linear partial differential equations. However, models of all varieties as mentioned above can be found in securities market.

The models in financial market as categorized above were solved effectively using the discretization techniques such as finite difference method (FDM), operator splitting method, alternating directions implicit (ADI) method, higher order compact (HOC) method, front-fixing method, exponential time integration (ETI) method, hybrid finite difference method, penalty method etc. Apart from the above mentioned discretization techniques, it is found in the literature that the non-discretization techniques such as Adomian decomposition method (ADM), variational iteration method (VIM), homotopy perturbation method (HPM) and homotopy analysis method (HAM) are also very effective in solving linear partial differential equation (Black-Scholes equation) and obtained the solution in the form of an approximate polynomial. At the same time analytical solution techniques such as first integral method (FIM), tanh-coth and sine-cosine methods in various fields of engineering and science also offer effective and exact solution for nonlinear partial differential equations.
The present study focusses on the solution of nonlinear Black-Scholes partial differential equation of securities market and the solution of linear partial differential equations of commodity market. This work is devoted to study the efficacy of the mentioned analytical methods on the nonlinear Black-Scholes equation and the non-discretization techniques on the linear partial differential equations with variable coefficients proposed by Schwartz and termed as one factor, two factor and three factor commodity price models.

This study is divided into nine Chapters and an Appendix. The first chapter is devoted for introduction to understand the financial market and need of mathematical modeling in financial mathematics. The second chapter is devoted to the literature survey of financial mathematical models and its solution techniques. Chapter 3 presents the description of the non-discretization and analytical solution techniques which are to be used in the study. Chapter 4 is devoted to the effectiveness of the analytical solution techniques on nonlinear Black-Scholes equation. Chapters 5, 6, and 7 devote to testing the efficiency of non-discretization solution techniques ADM, VIM, HPM and HAM on one factor, two factor and three factor Commodity Price models respectively. Chapter 8 is devoted to the conclusions. Finally, Chapter 9 is devoted to the future scope based on this study. The Appendix contains the solutions of the commodity price models and its convergence results obtained using ADM, VIM and HAM.