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Notations

a.s. : Almost surely

d.f. : Distribution function

d.f.s : Distribution functions

i.i.d. : Independent and Identically Distributed

r.v. : Random variable

r.v.s : Random variables

LIL : Law of the Iterated Logarithm

i.o. : Infinitely often

S.V. : Slowly varying

m.g.f. : Moment generating function

iff : if and only if

$r(x)$ : The right extremity of $x$

$\epsilon, c, q$ : Positive constants (with or without suffix)

$n, k, m$ : Positive integer constants (with or without suffix)

$\{X_n\}$ or $(X_n)$ : Sequence of random variables

$[x]$ : Largest integer less than or equal to the positive number $x$

$L(x)$ : Slowly varying function

$\ell(x)$ : Inverse of $L(x)$

lim sup : Limit Superior

lim inf : Limit Inferior

$\overset{d}{\rightarrow}$ : Convergence in distribution

$\overset{p}{\rightarrow}$ : Convergence in probability

$\overset{a.s.}{\rightarrow}$ : Almost sure convergence

$X \overset{d}{=} Y$ : X and Y have the same distribution

$\approx$ : Asymptotically equal to

$\sim$ : $f(x) \sim g(x)$ as $x \to x_0$ means $\lim_{x \to x_0} \frac{f(x)}{g(x)} = 1$

$\equiv$ : Equivalent to

$\in$ : Belongs to