A Biomimetic Hand: Prototype 1.0

This chapter details the development of a five fingered extreme upper limb prosthetic hand prototype: Prototype 1.0. The focus is on emulating the six grasping operations involved during 70% of daily using EMG based grasps classification.

The approach for development of an anthropomorphic hand should be to take reference of human hand (146). Biomimetic approach for development of a prosthetic hand has appeared as a significant opportune towards mimicking the natural counterparts (147) as well in tackling the challenges of present artificial hands (148). Biomimesis is the understanding of nature, its models, systems, processes and elements to emulate or take inspiration from these designs and processes; and imitate them into artificial system (149, 150). Although some of the laboratory prototypes (2, 14, 15) have been developed following biomimetic approaches, they are far from the human hand in terms of the static and dynamic constraints (12).

The biomimetic approach is followed to harmonize both physical and functional aspects of the human hand. The emulation of grasp types is through a two layered architecture: SHC and LHC. SHC is for recognition of user’s intended grasp based on EMG signals and LHC is for actuating the fingers in the prototype to emulate the identified grasp. Prototype 1.0 can perform the six grasp types under static and dynamic constraints, which are responsible for natural movement (151).
5.1 Biomimetic Approach

An ideal upper limb prototype should be perceived as a part of the natural body by the amputees. Towards this goal, Prototype 1.0 is developed following a biomimetic approach as shown in Figure 5.1. The approach comprises of five steps. It involves:

[i] Study of the human hand physiology
[ii] Material selection based on the expected properties in the prototype
[iii] development of bio-mechanical structure
[iv] development of control architecture
[v] development of biomimetic hand prototype.

Study of the human hand physiology in terms of the anatomy, grasping operations, static and dynamic constraints, EMG based actuation; the first stage of the biomimetic approach is detailed in Chapter 2.

Figure 5.1: Biomimetic Approach followed for development of Prototype 1.0

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1With reference to (152), bio-mechanical structure means the physical representation of human hand anatomy quantitatively for this research.
5.1 Biomimetic Approach

5.1.1 Material selection

The approach for building the prototype is to duplicate any of the features and properties which affect the characteristics of human hand. So the work seek to use materials that can mimic the human hand properties. The goal has always been for size and weight similar to human hands vis-a-vis grasp functionality. Following (153), two properties: specific gravity and coefficient of friction have been considered for material selection. Heavy weight is one of the main reasons for non-acceptance of artificial hands by the amputees (154). Moreover, an object grasped by the hand should not slip. These leads to specific gravity and coefficient of friction as the important properties for selection of the material to be used for development of the prototype. Following (153) and (155), four materials: nylon, teflon, steel and aluminum are under study. From Table 5.1, nylon and teflon are found to bear properties close to that of human hand. Nylon is stable, undeformable and has low friction and low specific gravity. Further, cost of nylon is more than five times lesser than that of teflon. Therefore, nylon is selected for the skeletal structure of the hand. The building blocks of the human hand and that of Prototype 1.0 are tabulated in Table 5.2.

Table 5.1: Comparison of primitive characteristics (required to replicate human hand properties) for four materials under study

<table>
<thead>
<tr>
<th>Material</th>
<th>Nylon</th>
<th>Teflon</th>
<th>Steel</th>
<th>Aluminum</th>
<th>Human Hand</th>
</tr>
</thead>
<tbody>
<tr>
<td>Specific Gravity</td>
<td>1.13</td>
<td>2.15</td>
<td>7.85</td>
<td>2.64</td>
<td>0.96 - 1.2</td>
</tr>
<tr>
<td>Co-efficient of friction</td>
<td>0.15 - 0.25</td>
<td>0.04</td>
<td>0.8</td>
<td>1.35</td>
<td>0.72 - 0.7</td>
</tr>
</tbody>
</table>

Table 5.2: Building blocks of Human hand versus the Prototype 1.0

<table>
<thead>
<tr>
<th>Human Hand</th>
<th>Prototype 1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>The skeletal of the hand</td>
<td>Nylon as links, joints and palm</td>
</tr>
<tr>
<td>Set of muscles, which are embedded in</td>
<td>A set of DC geared motors embedded in the palm</td>
</tr>
<tr>
<td>between the skin surface and skeleton</td>
<td></td>
</tr>
<tr>
<td>Mass-spring system, inter-linking the skin,</td>
<td>Tendon system interlinking the skeletal and actuators</td>
</tr>
<tr>
<td>skeleton and muscle</td>
<td></td>
</tr>
<tr>
<td>Joint hierarchy which matches the structure of</td>
<td>A joint hierarchy which matches the structure of natural hand</td>
</tr>
<tr>
<td>the skeleton</td>
<td></td>
</tr>
</tbody>
</table>
5.1.2 Bio-mechanical Structure

The development of the mechanical prototype is based on the knowledge gathered from the study of the human hand physiology. The skeletal structure of the prototype is developed using the material selected with reference to the study carried out in section 5.1.1.

Human anatomical terminologies has been used to describe Prototype 1.0. The prototype comprises of five fingers. Each finger consists of three links replicating the distal, middle and proximal phalanges of human finger; thumb consist of two links. The links are connected through revolute joints corresponding to DIP, PIP and MCP joints. The palm is two piece and can move inward and outward to form grasp modes. The palm accommodates the actuators. The wrist is of two concentric cylindrical structures made of nylon.

In the prototype, the joints have been obtained with an extended knuckle structure at each link to prevent the backward movement of the succeeding link. The curvature of the phalanges are with reference to the human anatomical structure and therefore can flex and extend in the joint range corresponding to the human finger as recorded in Table 5.3.

Table 5.3: Finger joint range of motion of the prototype (in degrees) measured using Jamar Plastic Goniometer

<table>
<thead>
<tr>
<th></th>
<th>Thumb</th>
<th>Index</th>
<th>Middle</th>
<th>Ring</th>
<th>Little</th>
</tr>
</thead>
<tbody>
<tr>
<td>MCP Flexion</td>
<td>0 to 100</td>
<td>0 to 90</td>
<td>0 to 90</td>
<td>0 to 90</td>
<td>0 to 90</td>
</tr>
<tr>
<td>PIP Flexion</td>
<td>0 to 110</td>
<td>0 to 110</td>
<td>0 to 110</td>
<td>0 to 110</td>
<td>0 to 110</td>
</tr>
<tr>
<td>DIP Flexion</td>
<td>0 to 90</td>
<td>0 to 70</td>
<td>0 to 70</td>
<td>0 to 70</td>
<td>0 to 70</td>
</tr>
</tbody>
</table>

Figure 5.2(a) shows the dimensional representation of the index finger. Figure 5.2(b) is a planner schematic structure of the index finger and 5.2(c) represents the routing of the tendons through the joints of the finger. Figure 5.2(b) and (c) are for kinematic and dynamic analysis of the prototype as detailed in section 5.1.3.
5.1 Biomimetic Approach

![Diagram](image)

Figure 5.2: (a) Dimensional representation of the index finger. The lengths of the proximal, middle and distal phalanges are 40 mm, 30 mm and 20 mm respectively. The radius of rotation ($R_1, R_2, R_3$) of the three joints (MCP, PIP and DIP) are 14 mm, 8 mm and 6 mm respectively. (b) A planner schematic structure of a finger (other than the thumb). Each link $L_i (i = 1, 2, 3)$ corresponds to the proximal, middle and distal phalanges. MCP, PIP and DIP joint angles are $\theta_1, \theta_2$ and $\theta_3$ respectively. (c) Tendon routing the finger joints. $d_1, d_2$ and $d_3$ are the distance of the center of mass of the phalanges from the respective joints MCP, PIP and DIP ($E_1, E_2$ and $E_3$) respectively. $I_1, I_2$ and $I_3$ are the moment of inertias of the three phalanges about an axis passing through their center of masses. $m_1, m_2$ and $m_3$ are the masses of the proximal, middle and distal phalanges respectively. $a$ and $b$ are half the finger width and distance of the tendon guides form the finger joints.

Each finger is actuated with two motors; one for flexion and another for extension. The little and ring finger are actuated through the common motors. Abduction and adduction is not implemented in Prototype 1.0. The prehension of the palm is obtained through one single motor. The wrist of the prototype is actuated through three DC motors placed in mutually perpendicular axes. The developed prototype possess a total of ($3 \times 3$ of fingers + 2 of thumb + 1 of the palm + 3 of wrist) = 15 DoF. Arrangement of the actuators and tendons as well as the digits of the prototype in its ventral view is shown in Figure 5.3. Each finger tip is equipped with film like force sensors to measure the fingertip force. The specification of the used actuator units (geared DC motors) are in Table 5.4.
5.1 Biomimetic Approach

Table 5.4: Specification of the Actuating Motors

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Finger Motor</th>
<th>Wrist Motor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gear Ratio</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>No load Speed</td>
<td>250 Revolutions</td>
<td>300 RPM</td>
</tr>
<tr>
<td></td>
<td>Per Minute (RPM)</td>
<td></td>
</tr>
<tr>
<td>No Load Torque</td>
<td>0.076 Nm</td>
<td>0.090 Nm</td>
</tr>
<tr>
<td>Diameter</td>
<td>160 mm</td>
<td>160 mm</td>
</tr>
<tr>
<td>Length</td>
<td>300 mm</td>
<td>350 mm</td>
</tr>
<tr>
<td>Diameter of motor pulley</td>
<td>10 mm</td>
<td>10 mm</td>
</tr>
</tbody>
</table>

Figure 5.3: Ventral View of Prototype 1.0

5.1.2.1 Tendon System

N + 1 tendon configuration is used as media to transmit forces from actuators to the finger’s joints. N+1 tendon configuration is one in which a single tendon pulls on all the joints in one direction and one or more additional tendons which generate torques in opposite direction (156). Such a system is important from a biomimetic point of view
5.1 Biomimetic Approach

as most of the muscles and tendons involve in flexion-extension of the human body parts form agonist-antagonist pair. In the prototype, the agonist and antagonist tendons mimic the flexor digitorum superficialis and flexor digitorum profundus; extensor digitorum communis and extensor indicis tendons of human finger (82). Thin tendons made of polymeric fibers have been used. Extensor and flexor tendons are placed on the dorsal and ventral side of each finger and connected to individual actuation unit in the palm. The tendons connected to the pulley of the motor, passing through a series of hollow guides are fixed at the finger tips. This replicates the agonist-antagonistic tendon system of human hand (157). Table 5.5 shows the characteristics of a human hand vis-a-vis Prototype 1.0.

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Human Hand</th>
<th>Prototype 1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of DoF</td>
<td>22</td>
<td>15</td>
</tr>
<tr>
<td>Wrist Mobility</td>
<td>03</td>
<td>03</td>
</tr>
<tr>
<td>Total Volume</td>
<td>50 cc</td>
<td>47 cc</td>
</tr>
<tr>
<td>Each finger length</td>
<td>92 mm</td>
<td>90 mm</td>
</tr>
<tr>
<td>Each finger diameter</td>
<td>14 mm</td>
<td>14 mm</td>
</tr>
<tr>
<td>Wrist width</td>
<td>65 mm</td>
<td>65 mm</td>
</tr>
<tr>
<td>Palm width and thickness</td>
<td>90 mm  and 45 mm</td>
<td>90 mm and 45 mm</td>
</tr>
<tr>
<td>Total weight</td>
<td>400 gram</td>
<td>520 gram</td>
</tr>
</tbody>
</table>

In this comparison, one of the distinguishing feature of the biomimetic hand from the human hand is the number of DoF. The thumb of Prototype 1.0 possess significantly lower DoF - two DoF as compared to the five DoF of the human hand thumb. Prototype 1.0 thumb possess two flexion-extension axis of rotation at DIP and MCP joint. However in human thumb, the DIP joint have one flexion-extension DoF, MCP and CMC joints have one flexion-extension and one abduction-adduction DoF in each. Further, the abduction-adduction of the remaining four fingers are not implemented in Prototype 1.0. This results in Prototype 1.0 with seven DoF lesser than that of the human hand.
5.1 Biomimetic Approach

5.1.3 Development of Control Architecture

The proposed control architecture is two layered: superior hand control (SHC) and local hand control (LHC). Figure 5.4 shows the schematic representation of the developed control architecture. SHC is to perceive the type of grasps attempted by the user and the LHC is to compute the grasp primitives i.e. finger joint torques and angles to emulate the identified grasp type.

![Diagram of Control Architecture](image)

**Figure 5.4**: Schematic of Control Architecture indicating two main states: Superior Hand Control and Local Hand Control
5.1 Biomimetic Approach

5.1.3.1 Superior Hand Control

The SHC provides the information about the grasp types to be performed by Prototype 1.0 based on EMG signals acquired from forearm muscles. In accordance to the visual information about the object to be grasped, the human brain plans the type of grasp to be formed for holding the object. Accordingly, the motor commands generated in the brain are sent down through the spinal cord to the forearm muscles. The motor commands initiates the forearm surface EMG. The grasp recognition architecture recognize the grasp type attempted by the user based on the forearm EMG.

The grasp recognition module comprises of four fundamental units: EMG Unit, MVC normalization Unit, Feature Extraction Unit followed by the Classifier Unit. The EMG unit comprises of the amplifier, band pass and notch filter. The EMG signals were acquired following the experimental protocol as detailed in section 3.1.1.1 in Chapter 3. The raw EMG signals extracted from the subjects required processing for accurate recording, display and analysis. The EMG signal obtained after filtration and amplification is called IEMG signal. The MVC normalization unit applies normalization algorithm to avoid the test case subjectivity and generates nIEMG signals. The characteristic patterns of a signal in a reduced dimension called features are extracted from nIEMG signals in the feature extraction unit and a feature vector is generated. The feature vector: PCA of TFD EMG feature is fed to the classifier. The classifier is a RBF kernel SVM. The details of the grasp recognition architecture is elaborate in Chapter 4 and here mentioned in brief for completeness. The information about the identified grasp type is passed to the LHC.

5.1.3.2 Local Hand Control

The LHC is the interface between the SHC and Prototype 1.0. Human hands are capable of grasping objects with dexterous motion. As such, fingers typically grasps by curling around the objects following stereotypical trajectories (158). Manual grasping is more stable and secure than the current prosthesis grasping. Therefore, it is of interest to reproduce the natural movement of fingers in order to perform stable grasping operations by the prosthetic hands.

LHC identifies the fingers to be actuated for performing the grasp type identified via SHC. The derivation of the grasp primitives are through kinematic, static and
dynamic analysis. Based on the kinematic and static analysis of the finger model as shown in Figure 5.2, the LHC determines the relation between the finger joint torques and the actuating motor torque. This is achieved satisfying the human finger dynamic constraints; responsible for stable grasping (151). Based on the joint angles for natural curling of the human finger (159), the finger joint torques are determined through dynamic analysis such that the joint angle trajectories are similar to that of the human finger. The corresponding finger-tip force is determined through kinematic and static analysis. A proportional integral derivative (PID) controller is implemented to maintain the fingertip force.

5.2 Kinematics, Statics and Dynamics

For kinematic, static and dynamic analysis, the index finger of Prototype 1.0 is schematically represented as shown in Figure 5.2(b). $L_{c1}, L_{c2}$ and $L_{c3}$ are the distance of the center of mass of the phalanges from the respective joints PIP, MCP and DIP shown as $d_1, d_2$ and $d_3$ respectively in Figure 5.2(b). $R_1, R_2$ and $R_3$ are the radius of the joints $E_1, E_2$ and $E_3$. $I_1, I_2$ and $I_3$ be the moment of inertias of the three phalanges about an axis passing through their center of masses. $m_1, m_2$ and $m_3$ are the masses of the proximal, middle and distal phalanges respectively. The distance from the end of tendon guide to the finger joint is $b$ and radius of the finger is $a$.

5.2.1 Kinematic Analysis of a Finger

The Denavit-Hartenberg parameters describing finger kinematics are illustrated in Table 5.6; where $\theta_i$ is the joint angle from $X_{i-1}$ axis to $X_i$ axis about $Z_{i-1}$ axis, $d_i$ is the distance from the origin of $(i - 1)^{th}$ coordinate frame to the intersection of $Z_{i-1}$ axis with $X_{i-1}$ axis along $Z_{i-1}$ axis, $a_i$ is the offset distance from intersection of $Z_{i-1}$ axis with $X_i$ axis and $\alpha_i$ is the offset angle from $Z_{i-1}$ axis to $Z_i$ axis about the $X_i$ axis with $i = 1, 2, 3$.

Direct kinematic equations are used to obtain the fingertip position and orientation according to the joint angles. With three revolute joints, the finger has three rotational DoF ($\bar{\theta} = \{\theta_1, \theta_2, \theta_3\}^T$) leading to the finger end effector having pose ($\bar{x} = \{x, y, \alpha\}^T$). To analyze the three joint link shown in Figure 5.2(b), the first step is to establish the mapping from joint angles (the vector of three generalized rotational coordinates
Table 5.6: Denavit-Hartenberg Parameters of the Finger

<table>
<thead>
<tr>
<th>Link</th>
<th>$\alpha_{i-1}$</th>
<th>$a_{i-1}$</th>
<th>$d_i$</th>
<th>$\theta_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$\theta_1$</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>$L_1 = 40$ mm</td>
<td>0</td>
<td>$\theta_2$</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>$L_2 = 30$ mm</td>
<td>0</td>
<td>$\theta_3$</td>
</tr>
</tbody>
</table>

$\bar{\theta} = \{\theta_1, \theta_2, \theta_3\}^T$ to link end point position and orientation of the finger for a given set of link lengths $L = \{L_1, L_2, L_3\}$. From the Denavit-Hartenberg parameters of the finger as stated in Table 5.6, the fingertip pose $\bar{x}$ with respect to the base frame can be computed as:

$$\bar{x} = G(\bar{\theta}) = \begin{bmatrix} G_x(\bar{\theta}) \\ G_y(\bar{\theta}) \\ G_\alpha(\bar{\theta}) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ \alpha \end{bmatrix} = \begin{bmatrix} L_1 C_1 + L_2 C_{12} + L_3 C_{123} \\ L_1 S_1 + L_2 S_{12} + L_3 S_{123} \\ \theta_1 + \theta_2 + \theta_3 \end{bmatrix}$$ \hspace{1cm} (5.1)

where $G(\bar{\theta})$ is the geometric model defined by the trigonometric equations for the end point position $\{x, y\}^T$ and orientation $\{\alpha\}$ of the last link as a function of $\bar{\theta}$ and link lengths of the finger $L$. $C_1, C_{12}$ and $C_{123}$ denotes $\cos(\theta_1), \cos(\theta_1 + \theta_2)$ and $\cos(\theta_1 + \theta_2 + \theta_3)$ and $S_1, S_{12}$ and $S_{123}$ denotes $\sin(\theta_1), \sin(\theta_1 + \theta_2)$ and $\sin(\theta_1 + \theta_2 + \theta_3)$ respectively.

### 5.2.1.1 Tendon Actuation

Flexion and extension of the fingers is performed by pulling and releasing the flexor and extensor tendons. The finger joint angles depends on the tendon length pulled $l_m$ and released $l_m'$ by the flexor motors. Tendon length while the finger is maximally extended is $l_o = L_1 + L_2 + L_3$. When the finger is flexed, the flexor tendon is pulled by the motor. Let $l_x$ be the resulting flexor tendon length and $\theta_1$, $\theta_2$, $\theta_3$ be the joint angles respectively. Change in flexor tendon length $l_m$ is the difference of $l_o$ and $l_x$.  

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\[ l_m = l_o - l_x = (L_1 + L_2 + L_3) - (L_1 C_1 + L_2 C_{12} + L_3 C_{123}) \] (5.2)

Both anatomical and empirical studies show a linear inter-joint angular relationships in human finger expressed as dynamic constraints (12) and results in a natural curling motion of the fingers. The dynamic constraints of human hand finger is represented using the following:

\[ DC1 = \frac{\theta_1}{\theta_2} = 0.5 \] (5.3)
\[ DC2 = \frac{\theta_2}{\theta_3} = 1.5 \] (5.4)

In order to replicate the natural motion of human finger into the prototype, the dynamic constraints of human fingers have been considered for computation of the tendon length. Substituting the constraints in equations (5.3) and (5.4) into equation (5.2), following relation between \( \theta_1 \) and \( l_m \) is obtained.

\[ l_m = (L_1 + L_2 + L_3) - (L_1 \cos(\theta_1) + L_2 \cos(2.\theta_1) + L_3 \cos(4.\theta_1/3)) \] (5.5)

In a similar way, the length of the extensor tendon released by the extensor motor is given as:

\[ l_m' = (L_1 + L_2 + L_3) + (L_1 \cos(\theta_1) + L_2 \cos(2.\theta_1) + L_3 \cos(4.\theta_1/3)) \] (5.6)

Since, \( l_m \) is the length of the tendon pulled by the motor; \( l_m' \) can be computed using equation 5.6 given diameter of the pulley connected to the motor, \( D \); time of rotation of the motor, \( \delta t \) and revolution per minute of the motor, \( N \).
5.2 Kinematics, Statics and Dynamics

\[ l_m = \pi D N \delta t \]  \hspace{1cm} (5.7)

The values of \( D \) and \( N \) are known a priori as in Table 5.4. \( \delta t \) is computed from force sensory feedback. The start time is achieved from initiation of the actuating signal to the motor and the time of contact is on receiving a feedback signal from fingertip sensor.

5.2.2 Static Analysis of a Finger

5.2.2.1 Joint Torques and Fingertip Forces

The joint torques exactly balances finger tip forces in static equilibrium situations. The rotational kinetic input to the end effector is net of three torques \((\tau = \{\tau_1, \tau_2, \tau_3\}^T)\) at MCP, PIP and DIP joints respectively to produce the output wrench vector

\[ \vec{W} = \begin{pmatrix} 0_{f_{\text{fingertip}}} \\ 0_{n_{\text{fingertip}}} \end{pmatrix} \]  \hspace{1cm} (5.8)

The Jacobian transpose maps finger tip forces into equivalent joint torques (156). The joint torques \( \tau \) which balances the wrench vector \( \vec{W} \) is given as:

\[ \tau = J(\theta)^T \vec{W} \]  \hspace{1cm} (5.9)

Where \( J(\theta) \) is the Jacobian matrix relating the joint space to the fingertip space.

The three link finger as shown in Figure 5.2(b) is applying force and moment through the fingertip on the environment. These are given by the following equations:

\[ 0_{f_{\text{fingertip}}} = (f_x, f_y, 0)^T \]  \hspace{1cm} (5.10)

\[ 0_{n_{\text{fingertip}}} = (0, 0, n_z)^T \]  \hspace{1cm} (5.11)

Let the forces \( f_{\text{fingertip}} \) and moment \( n_{\text{fingertip}} \) be denoted as \((f'_x, f'_y, 0)^T\) and \((0, 0, n'_z)^T\).

\[
\begin{pmatrix}
  f'_x \\
  f'_y \\
  0
\end{pmatrix} =
\begin{bmatrix}
  C_{123} & S_{123} & 0 \\
  -S_{123} & C_{123} & 0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{pmatrix}
  f_x \\
  f_y \\
  0
\end{pmatrix}
\]  \hspace{1cm} (5.12)

and

\[ (0, 0, n'_z)^T = (0, 0, n_z)^T \]  \hspace{1cm} (5.13)
5.2 Kinematics, Statics and Dynamics

Equation 5.13 results from the fact that rotation matrix \( \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \) in equation 5.12 has \((0\ 0\ 1)\) in the last column and row. In order to obtain static forces and moments acting on links of a serial manipulator, when the fingertip (or end effector) is subjected to external forces and moments; the joints of the manipulator are assumed to be locked. The manipulator can be viewed as a structure. Using equations of static equilibrium, the following pair of equations (5.14 and 5.15) are obtained [Please refer to (160, pp. 167) for complete derivation].

\[
\begin{align*}
\mathbf{i} f_i & = \mathbf{i} f_{i+1} [\mathbf{R}_{i+1}]^{-1} \\
\mathbf{i} n_i & = \mathbf{i} f_{i+1} + \mathbf{O}_{i+1} \times \mathbf{i} f_i
\end{align*}
\]

Here \( f_i \) = force exerted on the link \( \{i\} \) by link \( \{i-1\} \), \( n_i \) = moment exerted on link \( \{i\} \) by link \( \{i-1\} \). \( \mathbf{i} O_{i+1} \) is the vector from \( \mathbf{O}_i \) to \( \mathbf{O}_{i+1} \). \( \mathbf{i+1} \) is the \( \{i+1\}^{th} \) frame with respect to \( \{i\}^{th} \) frame. The leading superscript \( i \) signifies that the vectors are described in \( \{i\}^{th} \) frame.

Above equations can be used to compute forces and moments when the forces and moments at the fingertip are known. Applying the above iterative formula at the fingertip going towards the base for the finger shown in Figure 5.2(b).

for \( i = 3 \)

\[
\begin{align*}
\mathbf{3} f_3 & = (f'_x, f'_y, 0)^T \\
\mathbf{3} n_3 & = (0, 0, n'_z + l_3 f'_y)^T
\end{align*}
\]

for \( i = 2 \)

\[
\begin{align*}
\mathbf{2} f_2 & = (C_3 f'_x - S_3 f'_y, S_3 f'_x + C_3 f'_y, 0)^T \\
\mathbf{2} n_2 & = (0, 0, n'_z + l_2 (S_3 f'_x + C_3 f'_y) + l_3 f'_y)^T
\end{align*}
\]

for \( i = 1 \)

\[
\begin{align*}
\mathbf{1} f_1 & = (C_{23} f'_x - S_{23} f'_y, S_{23} f'_x + C_{23} f'_y, 0)^T \\
\mathbf{1} n_1 & = (0, 0, n'_z + l_1 (S_{23} f'_x + C_{23} f'_y) + l_2 (S_3 f'_x + C_3 f'_y) + l_3 f'_y)^T
\end{align*}
\]

The joints can only apply torques about \( \hat{Z} \) axis in order to have static equilibrium. Therefore for a serial link manipulator with rotary joints, the torque required at joint \( i \) can be computed by equation 5.22 (160, pp. 168).

\[
\tau_i = \mathbf{i} n_i \cdot \mathbf{i} \hat{Z}_i
\]
Where $\hat{\mathbf{Z}}_i$ is a vector along $\hat{Z}$ axis at joint $i$.

Torques required at the joints to keep the finger in equilibrium are as follows:

\[
\tau_1 = 1_n^1 \cdot \hat{\mathbf{Z}}_1 = n'_z + f'_x (l_1 S_{23} + l_2 S_3) + f'_y (l_1 C_{23} + l_2 C_3 + l_3) \tag{5.23}
\]

\[
\tau_2 = 2_n^2 \cdot \hat{\mathbf{Z}}_2 = n'_z + f'_x l_2 S_3 + f'_y (l_2 C_3 + l_3) \tag{5.24}
\]

\[
\tau_3 = 3_n^3 \cdot \hat{\mathbf{Z}}_3 = n'_z + f'_y l_3 \tag{5.25}
\]

Rearranging equations 5.23 through 5.25, the joint torques can be expressed as:

\[
\begin{pmatrix}
\tau_1 \\
\tau_2 \\
\tau_3
\end{pmatrix} =
\begin{bmatrix}
L_1 S_{23} + L_2 S_3 & L_1 C_{23} + L_2 C_3 + L_3 & 0 & 0 & 0 & 1 \\
L_2 S_3 & L_2 C_3 + L_3 & 0 & 0 & 0 & 1 \\
0 & L_3 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{pmatrix}
f'_x \\
f'_y \\
0 \\
0 \\
0 \\
n'_z
\end{pmatrix} \tag{5.26}
\]

Substituting from equations 5.12 and 5.13 into equation 5.26:

\[
\begin{pmatrix}
\tau_1 \\
\tau_2 \\
\tau_3
\end{pmatrix} =
\begin{bmatrix}
-L_1 S_1 - L_2 S_{12} - L_3 S_{123} & L_1 C_1 + L_2 C_{12} + L_3 C_{123} & 0 & 0 & 0 & 1 \\
-L_2 S_{12} - L_3 S_{123} & L_2 C_{12} + L_3 C_{123} & 0 & 0 & 0 & 1 \\
-L_3 S_{123} & L_3 C_{123} & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{pmatrix}
f_x \\
f_y \\
0 \\
0 \\
0 \\
n_z
\end{pmatrix} \tag{5.27}
\]

The term in the square bracket is the transpose of the Jacobian $J(\theta)$.

The contact between the object to be grasped and fingertip (of the three link finger) is assumed to be point contact with friction as shown in Figure 5.5. The contact cannot resist any moment applied around its normal. Thus $n_z = 0$ (161).
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Figure 5.5: Schematic of point contact between the object and finger tip

With this assumption, equation 5.27 can be reduced to

\[
\begin{pmatrix}
\tau_1 \\
\tau_2 \\
\tau_3
\end{pmatrix}
= \begin{bmatrix}
-L_1 S_1 - L_2 S_{12} - L_3 S_{123} & L_1 C_1 + L_2 C_{12} + L_3 C_{123} & 0 \\
-L_2 S_{12} - L_3 S_{123} & L_2 C_{12} + L_3 C_{123} & 0 \\
-L_3 S_{123} & L_3 C_{123} & 0
\end{bmatrix}
\begin{pmatrix}
f_x \\
f_y \\
0
\end{pmatrix}
\] (5.28)

From 5.28, the fingertip force \((f_x, f_y)\) in terms of the joint torques can be computed as:

\[
f_x = \frac{(L_2 C_{12}) \tau_1 - (L_1 C_1 + L_2 C_{12}) \tau_2 + L_1 C_1 (\tau_3)}{L_1 L_2 (C_1 S_{12} - S_1 C_{12})}
\] (5.29)

\[
f_y = \frac{(L_2 S_{12}) \tau_1 - (L_1 S_1 + L_2 S_{12}) \tau_2 + L_1 S_1 (\tau_3)}{L_1 L_2 (C_1 S_{12} - S_1 C_{12})}
\] (5.30)

5.2.2.2 Tendon Forces and Joint Torques

Next step is to describe how forces applied at the end of the tendons are related to the torque applied at the joints. Figure 5.2(c) illustrates the flexor \((h_1)\) and extensor \((h_2)\) tendons routing the finger joints. Following (156, pp. 293), the extension function for the flexor and extensor tendons are given as:

\[
h_1(\theta) = l_m + 2 \sqrt{a^2 + b^2 \cos(tan^{-1}(a/b) + \theta_1/2)} - 2b - R_2 \theta_2 - R_3 \theta_3
\] (5.31)

\[
h_2(\theta) = l_m' + R_1 \theta_1 + R_2 \theta_2 + R_3 \theta_3
\] (5.32)
Where \((R_1, R_2, R_3)\) are radius of rotation of the three joints (MCP, PIP and DIP) respectively. \(a\) and \(b\) are half the finger width and distance of the tendon guides form the finger joints as shown in Figure 5.2(c).

The coupling matrix relating the force at the ends of the tendons and the joint torques is as given below.

\[
H_c = \begin{bmatrix}
\frac{\partial h_1}{\partial \theta_1} & \frac{\partial h_2}{\partial \theta_1} \\
\frac{\partial h_1}{\partial \theta_2} & \frac{\partial h_2}{\partial \theta_2} \\
\frac{\partial h_1}{\partial \theta_3} & \frac{\partial h_2}{\partial \theta_3}
\end{bmatrix}
\]  

(5.33)

The joint torque in terms of tendon force is computed as:

\[
\tau = H_c F
\]

(5.34)

\[
= \begin{bmatrix}
-2\sqrt{a^2 + b^2} \sin(tan^{-1}(a/b) + \theta_1/2) & R_1 \\
-R_2 & R_2 \\
-R_3 & R_3
\end{bmatrix} \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}
\]  

(5.35)

Where \(F = [F_1 \ F_2]^T\); \(F_1\) and \(F_2\) are the forces on the flexor and extensor tendons respectively.

Considering the motor torque \(T = [T_1 \ T_2]^T\); \(T_1\) for flexion and \(T_2\) for extension of the finger:

\[
\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{r_1} & 0 \\ 0 & \frac{1}{r_2} \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix}
\]

(5.36)

Where \(r_1\) and \(r_2\); are the radius of the pulleys connected to the flexor and extensor motors.

Using equation 5.35 and 5.36, the joint torques can be expressed as a function of motor torques as follows:

\[
\tau_1 = -2\sqrt{a^2 + b^2} \sin(tan^{-1}(a/b) + \theta_1/2) + \frac{T_1}{r_1} + \frac{T_2}{r_2}
\]

(5.37)

\[
\tau_2 = -\frac{T_1}{r_1} + \frac{T_2}{r_2}
\]

(5.38)

\[
\tau_3 = -\frac{T_1}{r_1} + \frac{T_2}{r_2}
\]

(5.39)

For a tendon network; the tendon forces chosen to exert a given vector of joint torques have the form

\[
F = H_c^+ \tau + F_N
\]

(5.40)
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Where $H_c^+ = (H_c)^T((H_c)(H_c)^T)^{-1}$ is the pseudo-inverse of the coupling matrix. $F_N$ is tendon internal force to ensure that tendons remain taut and chosen as small as possible.

Using equation 5.36 in equation 5.40, the relation between the motor torque and joint torque is obtained as:

$$T = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix} [H_c^+ \tau + F_N] \quad (5.41)$$

Using the values of $\tau_1, \tau_2$ and $\tau_3$ from equation 5.37 through 5.39 into equation 5.29 and 5.30; the fingertip force $(f_x, f_y)$ can be computed in terms of the tendon actuation motor torque as:

$$f_x = \{(L_2C_{12})(-2\sqrt{a^2 + b^2}\sin(tan^{-1}(a/b) + \theta_1/2) \frac{T_1}{r_1} \\
- (L_1C_1 + L_2C_{12})(-R_2 \frac{T_1}{r_1} + R_2 \frac{T_2}{r_2}) + L_1C_1(-R_3 \frac{T_1}{r_1} + R_3 \frac{T_2}{r_2}) \} \\
\{L_1L_2(C_1S_{12} - S_1C_{12})\}^{-1}$$

$$f_y = \{(L_2S_{12})(-2\sqrt{a^2 + b^2}\sin(tan^{-1}(a/b) + \theta_1/2) \frac{T_1}{r_1} + R_1 \frac{T_2}{r_2}) \\
- (L_1S_1 + L_2S_{12})(-R_2 \frac{T_1}{r_1} + R_2 \frac{T_2}{r_2}) + L_1S_1(-R_3 \frac{T_1}{r_1} + R_3 \frac{T_2}{r_2}) \} \\
\{(L_1L_2(C_1S_{12} - S_1C_{12})\}^{-1} \quad (5.42)$$

5.2.3 Dynamic Analysis of a Finger

The goal of dynamic analysis is to determine the motor torque required to be applied such that the finger joints follow the human finger joint trajectories. Finger joint torques for Prototype 1.0 for emulating the natural curling is to be estimated. In order to do this, joint torques as a function of velocity and acceleration are determined for a finger of Prototype 1.0 using equations of motion for a serial link manipulator as detailed in (160, pp. 159).

5.2.3.1 Equations of motion

The equations of motion for a serial link manipulator is given as:

$$[M(\theta)]\ddot{\theta} + [C(\theta, \dot{\theta})]\dot{\theta} + G(\theta) = \tau \quad (5.44)$$
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Where \([M(\theta)]\) is the \(n \times n\) mass matrix. \(C(\theta, \dot{\theta})\) is the \(n \times n\) matrix with \([C(\theta, \dot{\theta})]\dot{\theta}\) representing an \(n \times 1\) vector of centripetal and Coriolis terms. \(G(\theta)\) is an \(n \times 1\) vector containing the gravity terms and \(\tau\) is the \(n \times 1\) vector of joint torques.

\(L_{c1}, L_{c2}\) and \(L_{c3}\) are the distances of the center of mass of the three phalanges (of the finger) from their respective joint origins, viz. \(O_1, O_2\) and \(O_3\). Let the masses of the three links be \(m_1, m_2\) and \(m_3\) and their link inertia about the axis through center of masses are \(I_1, I_2\) and \(I_3\).

The end effector velocity can be computed as a function of the joint velocities \(\dot{\theta} = \dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3, ..., \dot{\theta}_i\) through the Jacobian matrix \(J\). The same methodology can be used to compute the velocity of a generic point of the manipulator, and in particular the velocity \(V_{ci} = \dot{c}_i\) of the center of mass \(p_{ci}\) that results function of the joint velocities \(\dot{\theta}_1, \dot{\theta}_2, \dot{\theta}_3\) only: [Please refer to (162)]

\[
\dot{c}_i = j^i_{c1}\dot{\theta}_1 + j^i_{c2}\dot{\theta}_2 + ... + J^i_{ci}\dot{\theta}_i 
\]

(5.45)

\[
= J^i_{c}\dot{\theta}_i
\]

(5.46)

\[
\omega_i = j^i_{\omega1}\dot{\theta}_1 + j^i_{\omega2}\dot{\theta}_2 + ... + J^i_{\omega i}\dot{\theta}_i
\]

(5.47)

\[
= J^i_{\omega}\dot{\theta}_i
\]

(5.48)

Where

\[
J_{c,i} = [j^i_{c1}, j^i_{c2}, ..., j^i_{ci}, 0...0] 
\]

(5.50)

\[
J_{\omega,i} = [j^i_{\omega1}, j^i_{\omega2}, ..., j^i_{\omega i}, 0...0] 
\]

(5.51)

With

\[
\begin{bmatrix}
    j^i_{cj} \\
    j^i_{\omega j}
\end{bmatrix} = 
\begin{bmatrix}
    Z_{j-1} \times (p_{ci} - p_{j-1}) \\
    Z_{j-1}
\end{bmatrix}
\]

(5.53)

Jacians \(J^i_{\omega,i}\) and \(J^i_{c,i}\) for computing the mass matrix with reference to the base frame \(\{X_o, Y_o\}\) for the three link finger under study are yielded as follows:

for \(i = 1\)

\[
J_{w,1} = 
\begin{bmatrix}
    0 & 0 & 0 \\
    0 & 0 & 0 \\
    1 & 0 & 0
\end{bmatrix}
\]

(5.54)
5.2 Kinematics, Statics and Dynamics

\[ J_{c,1} = \begin{bmatrix} -L_{c1}S_1 & 0 & 0 \\ L_{c1}C_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \] (5.55)

for \( i = 2 \)

\[ J_{w,2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix} \] (5.56)

\[ J_{c,2} = \begin{bmatrix} -L_1S_1 - L_{c2}S_{12} & -L_{c2}S_{12} & 0 \\ L_1C_1 + L_{c2}C_{12} & L_{c2}C_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix} \] (5.57)

for \( i = 3 \)

\[ J_{w,3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \] (5.58)

\[ J_{c,3} = \begin{bmatrix} -L_1S_1 - L_1S_{12} - L_{c3}S_{123} & -L_{c2}S_{12} - L_{c3}S_{123} & -L_{c3}S_{123} \\ L_1C_1 + L_2C_{12} + L_{c3}C_{123} & L_2C_{12} + L_{c3}C_{123} & L_{c3}C_{123} \\ 0 & 0 + L_1L_{c3}C_{23} + 2L_2L_{c3}C_3 \end{bmatrix} \] (5.59)

The equation for mass matrix; which is positive definite and symmetric is given as follows [Please refer to (163, pp. 175)];

\[ M = \sum_{i=1}^{n} M_i \] (5.60)

\[ = \sum_{i=1}^{n} m_i J_{c,i}^T J_{c,i} + J_{w,i}^T M_i J_{w,i} \] (5.61)

For the three link finger under study, the elements of the mass matrix are as follows:

\[ M_{11} = I_1 + I_2 + I_3 + m_1(L_{c1})^2 + m_2(L_1^2 + L_{c2}^2 + 2L_1L_{c2}C_2) + m_3(L_1^2 + L_2^2 + L_{c3}^2 + 2L_1L_2C_2 + 2L_2L_{c3}C_3 + 2L_1L_{c3}C_{23}) \] (5.62)

\[ M_{12} = M_{21} \]

\[ = I_2 + I_3 + m_2(L_{c2}^2 + L_1L_{c2}C_2) + m_3(L_2^2 + L_{c3}^2 + L_1L_2C_2) + L_1L_3C_{23} + 2L_2L_{c3}C_3 \] (5.63)

\[ \text{(5.64)} \]
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\[ M_{13} = M_{31} \]
\[ = I_3 + m_3(L_{c3}^2 + L_1L_{c3}S_{23} + L_2L_{c3}C_{3}) \quad (5.65) \]
\[ M_{22} = I_2 + I_3 + m_2L_{c2}^2 + m_3(L_2)^2 + L_{c2}^2 + 2L_2L_{c3}C_{3} \quad (5.66) \]
\[ M_{23} = M_{32} \]
\[ = I_3 + m_3(L_{c3}^2 + L_2L_{c3}C_{3}) \quad (5.67) \]
\[ M_{33} = I_3 + m_3L_{c3}^2 \quad (5.68) \]

The \( [C(\theta, \dot{\theta})] \dot{\theta} \) is a \( n \times 1 \) vector \( V \) whose elements are quadratic functions of joint velocities \( \dot{\theta} \). The \( k^{th} \) element of this vector is given as [Please refer to (162)]

\[ V_k = \sum_{j=1}^{n} C_{kj} \dot{\theta}_j \quad (5.69) \]

Where the elements \( C_{kj} \) are computed as

\[ C_{kj} = \sum_{i=1}^{n} C_{ijk} \dot{\theta}_i \quad (5.70) \]

With

\[ C_{ijk} = \frac{1}{2} \left( \frac{\partial M_{ki}}{\partial \theta_i} + \frac{\partial M_{kj}}{\partial \theta_j} - \frac{\partial M_{ij}}{\partial \theta_k} \right); \text{which is known as Christoffel symbols} \quad (5.71) \]

Following the above formulation, the elements of \( [C(\theta, \dot{\theta})] \) for the three link finger shown in Figure 5.2 are

\[ C_{11} = -\left( m_2L_1L_{c3}S_2 + m_3L_1L_2S_2 + m_3L_1L_{c3}S_{23} \right) \dot{\theta}_2 \]
\[ -\left( m_3L_2L_{c3}S_3 + m_3L_1L_{c3}S_{23} \right) \dot{\theta}_3 \quad (5.72) \]
\[ C_{12} = -\left( m_2L_1L_{c2}S_2 + m_3L_1L_2S_2 + m_3L_1L_{c3}S_{23} \right) \dot{\theta}_1 - \left( m_2L_1L_{c2}S_2 \right) \]
\[ +m_3L_1L_2S_2 + m_3L_1L_{c3}S_{23} \right) \dot{\theta}_2 - \left( m_3L_1L_{c3}S_{23} + m_3L_2L_{c3}S_3 \right) \dot{\theta}_3 \quad (5.73) \]
\[ C_{13} = -\left( m_3L_2L_{c3}S_3 + m_3L_1L_{c3}S_{23} \right) \dot{\theta}_1 - \left( m_3L_1L_{c3}S_{23} + m_3L_2L_{c3}S_3 \right) \dot{\theta}_2 \]
\[ -\left( m_3L_1L_{c3}S_{23} + m_3L_2L_{c3}S_3 \right) \dot{\theta}_3 \quad (5.74) \]
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\[ C_{21} = -(m_2 L_1 c_2 S_2 + m_3 L_1 L_2 S_2 + m_3 L_1 L_3 S_{23}) \dot{\theta}_1 - (m_3 L_2 L_3 S_3) \dot{\theta}_3 \] (5.75)

\[ C_{22} = -(m_2 L_2 L_3 S_3) \dot{\theta}_3 \] (5.76)

\[ C_{23} = -(m_3 L_2 L_3 S_3) \dot{\theta}_1 - (m_3 L_2 L_3 S_3) \dot{\theta}_2 - (m_3 L_2 L_3 S_3) \dot{\theta}_3 \] (5.77)

\[ C_{31} = (m_3 L_2 L_3 S_3 + m_2 L_1 L_3 S_{23}) \dot{\theta}_1 + (m_3 L_2 L_3 S_3) \dot{\theta}_2 \] (5.78)

\[ C_{32} = (m_3 L_2 L_3 S_3) \dot{\theta}_1 + (m_3 L_2 L_3 S_3) \dot{\theta}_2 \] (5.79)

\[ C_{33} = 0 \] (5.80)

Finally the gravity terms \( G_i \) of the gravity vector \( G(\theta) \) are computed from the expression [Please refer to (163, pp. 177)]:

\[ G_i = - \sum_{j=1}^{n} m_j g^T J_{i,j} \] (5.81)

Where

\[ g = [0 \quad -g \quad 0]^T; g \text{ being the acceleration due to gravity} \]

Which results into the gravity terms as follows:

\[ G_1 = m_1 g L_1 C_1 + m_2 g (L_1 + L_2 C_1) + m_3 g (L_1 C_1 + L_2 C_2 + L_3 C_{123}) \] (5.82)

\[ G_2 = m_2 g L_2 C_{12} + m_3 g (L_2 C_{12} + L_3 C_{123}) \] (5.83)

\[ G_3 = m_3 g L_3 C_{123} \] (5.84)

Using the mass matrix from equation 5.60, centripetal and coriolis matrix from equation 5.70 and gravity terms from 5.81, the joint torques are computed as follows:

\[ \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_3 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{bmatrix} + \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \\ G_3 \end{bmatrix} \] (5.85)

5.2.3.2 Torques for Natural Curling

In order to find the motor torques to be applied to the flexor and extensor motors for natural curling of the finger, the velocity and acceleration corresponding to the human finger joints during natural curling are obtained. These velocities and accelerations are used for finding the joint torques through the equations of motion which in turn is used for determining the flexor and extensor motor torques. The steps followed for determination of the finger joint torques are enlisted below:
5.2 Kinematics, Statics and Dynamics

- The human finger joint trajectories have been adopted from the Figure 3 reported in (159) and are shown in Figure 5.6.

![Figure 5.6](image_url)

**Figure 5.6:** Human Finger Joint Trajectories. The trajectories in red are for human fingers and in blue are for the dynamic model of human hand proposed in (159). Digit 2, 3, 4, 5 in the figure represents the index, middle, ring and little finger. The x-axis is the flexion angle in degree and y-axis is the time in sec.

- The trajectories in Figure 5.6 are digitized using Engauge Digitizer 5.1 (164) that converts an image file showing a graph into numbers.
  
  - The equation representing the digitized finger joint trajectories are obtained through polynomial curve fitting. Figure 5.7 and Table 5.7 shows the digitized finger joint trajectories and representative equations respectively. The choice of a particular polynomial order was based on absolute value error between the digitized and curve fitted trajectories.
Figure 5.7: Digitized and Curve Fitted Finger Joint Trajectories. The x-axis is the flexion angle in degree and y-axis is the time in sec.
### Table 5.7: Curve Fitted Finger Joint Trajectories for Natural Curling

<table>
<thead>
<tr>
<th>Finger</th>
<th>Joints</th>
<th>Curve Fitted Trajectories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index</td>
<td>MCP</td>
<td>$1.1e + 004x^9 - 2.5e + 004x^5 + 1.9e + 004x^4 - 6.7e + 003x^3$ +1.1e + 003x^2 - 64x + 0.85</td>
</tr>
<tr>
<td></td>
<td>PIP</td>
<td>$-7.3e + 004x^7 + 2.1e + 005x^6 - 2.4e + 005x^5 + 1.3e + 005x^4$ -3.7e + 004x^3 + 4.7e + 003x^2 - 2.3e + 002x + 21</td>
</tr>
<tr>
<td></td>
<td>DIP</td>
<td>$-1.5e + 004x^9 + 3.9e + 004x^5 - 3.9e + 004x^4 + 1.7e + 004x^3$ -3e + 003x^2 + 1.8e + 002x - 21</td>
</tr>
<tr>
<td>Middle</td>
<td>MCP</td>
<td>$1.2e + 005x^7 - 3.6e + 005x^6 + 4.1e + 005x^5 - 2.3e + 005x^4$ +6.3e + 004x^3 - 7.2e + 003x^2 + 2.6e + 002x + 1.4</td>
</tr>
<tr>
<td></td>
<td>PIP</td>
<td>$9.6e + 005x^8 - 2.1e + 006x^7 + 1.8e + 006x^6 - 6.7e$ +005x^5 + 9.3e + 004x^4 + 3.9e + 003x^3 - 1.3e</td>
</tr>
<tr>
<td></td>
<td>DIP</td>
<td>$2.1e + 005x^7 - 4.2e + 005x^6 + 3.2e + 005x^5 - 1e + 005x^4$ +8.7e + 003x^3 + 1.7e + 003x^2 - 66x - 8.1</td>
</tr>
<tr>
<td>Ring</td>
<td>MCP</td>
<td>$3.9e + 005x^8 - 1.3e + 006x^7 + 1.8e + 006x^6 - 1.3e$ ++006x^5 + 5.1e + 005x^4 - 1e + 005x^3 + 1e + 004x^2 - 4.3e</td>
</tr>
<tr>
<td></td>
<td>PIP</td>
<td>$5e + 006x^{10} + 2e + 007x^9 - 3.4e + 07x^8 + 3.1e + 07x^7$ -1.7e + 07x^6 + 5.9e + 06x^5 - 1.3e + 06x^4 + 1.7e + 05x^3</td>
</tr>
<tr>
<td></td>
<td>DIP</td>
<td>$2.9e + 006x^{10} - 1.2e + 007x^9 + 2.3e + 007x^8 - 2.3e + 007x^7$ +1.4e + 007x^6 - 5.5e + 06x^5 + 1.3e + 06x^4 - 1.6e + 05x^3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$+1.1e + 04x^2 + 5.1e + 02x + 6.7$</td>
</tr>
<tr>
<td></td>
<td>Little</td>
<td>MCP</td>
</tr>
<tr>
<td></td>
<td></td>
<td>PIP</td>
</tr>
<tr>
<td></td>
<td></td>
<td>DIP</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
• The finger joint velocity and acceleration corresponding to the equation representing the finger joint trajectories are determined.

• Velocity and acceleration values corresponding to the natural curling operations of the human finger along with the finger specification of Prototype 1.0 were fed into equation 5.85 to find the corresponding joint torques.

• The corresponding average joint torques required for replicating the finger joint angles are presented in Table 5.8. The simulation results for finger joint trajectories, velocity and acceleration are presented in Figure III.1 through Figure III.12 in Appendix-III.

• Using Equation 5.41, motor torques required corresponding to the finger joint torques are determined.

<p>| Table 5.8: Average Finger Joint Torques for Natural Curling Operations |
|-------------------------|-----------------|----------------|</p>
<table>
<thead>
<tr>
<th>Finger Joints</th>
<th>Torque (Nm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index MCP</td>
<td>0.090</td>
</tr>
<tr>
<td>Index PIP</td>
<td>0.051</td>
</tr>
<tr>
<td>Index DIP</td>
<td>0.020</td>
</tr>
<tr>
<td>Middle MCP</td>
<td>0.110</td>
</tr>
<tr>
<td>Middle PIP</td>
<td>0.071</td>
</tr>
<tr>
<td>Middle DIP</td>
<td>0.021</td>
</tr>
<tr>
<td>Ring MCP</td>
<td>0.081</td>
</tr>
<tr>
<td>Ring PIP</td>
<td>0.061</td>
</tr>
<tr>
<td>Ring DIP</td>
<td>0.022</td>
</tr>
<tr>
<td>Little MCP</td>
<td>0.071</td>
</tr>
<tr>
<td>Little PIP</td>
<td>0.041</td>
</tr>
<tr>
<td>Little DIP</td>
<td>0.011</td>
</tr>
<tr>
<td>Thumb MCP</td>
<td>0.070</td>
</tr>
<tr>
<td>Thumb DIP</td>
<td>0.031</td>
</tr>
</tbody>
</table>

• The fingertip forces corresponding to the motor torques are determined through Equation 5.42 and 5.43. Table 5.9 shows the average fingertip forces corresponding to the joint torques.
### Table 5.9: Average Fingertip Forces corresponding to the Joint Torques

<table>
<thead>
<tr>
<th>Finger</th>
<th>Average Fingertip Force</th>
</tr>
</thead>
<tbody>
<tr>
<td>Index</td>
<td>1.12 N</td>
</tr>
<tr>
<td>Middle</td>
<td>1.30 N</td>
</tr>
<tr>
<td>Ring</td>
<td>1.00 N</td>
</tr>
<tr>
<td>Little</td>
<td>0.91 N</td>
</tr>
<tr>
<td>Thumb</td>
<td>1.32 N</td>
</tr>
</tbody>
</table>

In the preceding sections, the methods for computing the joint torques required by Prototype 1.0 to replicate the natural curling operations were presented. Given these joint torques, equation to determine the motor torques for a tendon driven system is presented. Finally fingertip forces corresponding to the motor torques for natural curling are determined. In the next section we discuss a feedback control based on the fingertip forces.

**5.2.4 PID Control**

This section reports the simulation of a PID controller for emulating the grasp types by Prototype 1.0. The goal of the PID controller is to inhibit the fingertip force from exceeding the desired force. The selection of the PID controller is based on the fact that it has better static and dynamic performance (165). The input to the actuator required for generating the fingertip force as in Table 5.9 is applied through a PID controller. The difference of the required force and the actual actual force is fed as error into the PID controller. The steps for design and simulation of the PID controller is detailed in the following sections.

**5.2.4.1 System Linearity**

The system linearity is checked by applying a pulse width modulated (PWM) voltage to the driver circuit of the motor and finding the corresponding output of the fingertip force. Figure 5.8 represents the relationship of the input voltage versus the fingertip force sensor output determining the linearity range of the system. It has been found that the system is linear within a range of input voltage 1.75 volt (V) to 10.00 V with an output fingertip force in the range of 0.6 N to 45 N.
5.2.4.2 System Transfer Function

Once the linear operating range of the system is obtained, the next step is to find the transfer function for the system. The transfer function relating the input voltage \( v_a \) and output angular velocity \( \omega \) of the motor in Laplace domain is as follows (163):

\[
\frac{\omega(s)}{v_a(s)} = \frac{K_t}{(Ls + R)(Js + b) + K_t K_e} \quad (5.86)
\]

Where

- Moment of inertia of the rotor \( (J) = 0.01 \)
- Motor viscous friction constant \( (b) = 0.1 \) Nmsec
- Electromotive force constant \( (K_e) = 0.01 \) V/radian/sec
- Motor torque constant \( (K_t) = 3.3 \) Nm/ Amp
- Electric resistance \( (R) = 1 \) ohm
- Electric inductance \( (L) = 0.5 \) Henry

The transfer function of the used DC geared motor used determined using the above specifications (166) as follows:

\[
G(s) = \frac{3.3}{0.005s^2 + 0.06s + 0.1001} \quad (5.87)
\]
5.2.4.3 PID Controller Design

Following design criteria are set for design of the PID controller.

- Overshoot < 5%
- Settling Time < 0.5 sec
- Steady state error < 2%

The transfer function of a PID controller (165) is given as follows:

\[
T(s) = K_p + K_I/s + K_Ds
\]  \hspace{1cm} (5.88)

where

- \(K_P\) = Proportional Gain
- \(K_D\) = Differential Gain
- \(K_I\) = Integral Gain

The values of the controller gains have been selected through manual tuning using proportional gain \(K_P\) to decrease the rise time, differential gain \(K_D\) to reduce the overshoot and settling time and integral gain \(K_I\) to eliminate the steady-state error.

The process is followed in line with (167) and is tabulated in Table 5.10. The response of the PID controller with different values of the gains is shown in Figure 5.9. A set of values of \(K_P = 1.7\), \(K_D = 5.35\) and \(K_I = 0.0085\) are selected for almost zero steady state error, zero overshoot and 0.2 sec setting time.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Rise Time</th>
<th>Overshoot</th>
<th>Small Change</th>
<th>Steady State Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_P)</td>
<td>Decrease</td>
<td>Increase</td>
<td>Small Change</td>
<td>Decrease</td>
</tr>
<tr>
<td>(K_I)</td>
<td>Decrease</td>
<td>Increase</td>
<td>Increase</td>
<td>Eliminate</td>
</tr>
<tr>
<td>(K_D)</td>
<td>Small Change</td>
<td>Decrease</td>
<td>Decrease</td>
<td>No Change</td>
</tr>
</tbody>
</table>

5.2.4.4 Simulation Results

The simulation of the designed PID controller was carried out in MATLAB. The schematic of the simulation model is shown in Figure 5.10.

The fingertip force corresponding to the joint torques for the desired finger joint trajectories obtained from the dynamic and static analysis is fed to the PID controller. The gains of the PID controller were set as obtained in section 5.2.4.3. The PID control
5.2 Kinematics, Statics and Dynamics

![Figure 5.9: Response of the PID controller with different values of the Gains. The arrow shows the response selected for the Experiment](image)

**Figure 5.9:** Response of the PID controller with different values of the Gains. The arrow shows the response selected for the Experiment

![Figure 5.10: Schematic of the Simulation Model](image)

**Figure 5.10:** Schematic of the Simulation Model

The signal is converted into a PWM signal through a PWM generator. The PWM signal drive the motor actuating the fingers on Prototype 1.0. The torques measured at the output of the motor was fed back through the feedback path. The force corresponding to the resulting torque i.e. actual fingertip force obtained through the feedback transfer
function is fed back to the PID controller. The controller minimizes the difference of the actual fingertip force and desired fingertip force. In the simulation, the corresponding joint torques are computed as a function of the motor torque using Equation 5.41. Resulting finger joint trajectories are shown in Figure 5.11.
5.3 Graspability of Prototype 1.0

Figure 5.12 shows Prototype 1.0 performing the six grasp types under study. The

![Prototype 1.0 performing grasp types](image)

Figure 5.12: Prototype 1.0 performing grasp types: a. Power b. Palm-up c. Precision d. Hook e. Pinch and f. Oblique

posture and/or number of fingers involved in each grasps are different as detailed in section 2.5.1.1 in Chapter 2. Prototype 1.0 can grasp a the objects like cricket tennis ball, square bar, circular bar and table tennis ball stably. The video of Prototype 1.0 performing six grasp types through off-line EMG controlled is available in

http://www.tezu.ernet.in/ bcr/video.html.

While grasping, Prototype 1.0 is subjected to the limitation of grasping an object with the thumb mimicking the thumb of human hand. This is mainly because of its lesser DoF as discussed towards the end of the section 5.1. The thumb being only of two DoF, can not grasp an object with its tip. Further, for the absence of abduction-adduction of four fingers, Prototype 1.0 can not grasp objects of larger size. However, it can emulate all the six grasp under study.
5.4 Summary

This chapter presented the development of a biomimetic hand prototype: Prototype 1.0. The prototype has been developed following a biomimetic approach inspired by human hand anatomy. It mimics the human hand both in geometry and function. The prototype exhibits all the functionality except abduction/adduction movement of the digits. During grasping operations by human hand, objects are held firmly because of the palm prehension; which is achieved in the prototype by making the palm a two piece structure in order to have proper grasp modes. The human wrist is a complex structure with eight carpel bones of semicircular surface giving three DoF. This is achieved in the prototype by arranging three motors in mutually perpendicular axes. The thumb mechanism in the prototype is a simplified version of the human thumb. Further, the dynamic constraints have been considered for tendon actuation in the hand as stated in section 5.2.1.1. This is subsequently used in the control of the hand.

Following a two layered control architecture, the prototype reproduce the grasping operations involved during 70% of dla with 97.5% accuracy. Control is two layered: a SHC recognizes grasp type attempted by the user based on EMG signals; a LHC was implemented to control the finger joint torques and angles in the prosthesis for the grasp attempted. SHC recognized six grasp types used during 70% of dla. Grasp recognition was through RBF kernel SVM using PCA of TFD features. The control in the LHC was based on kinematic, static and dynamic analysis. Prototype 1.0 emulates the grasping operations following the joint angle trajectories of the human finger during natural curling operations.