Chapter 3

REFLECTION-REFRACTION AT LOOSE CONTACT BETWEEN POROELASTIC SOLID AND CRACKED ELASTIC SOLID

This chapter aims to study the effects of presence of cracks and pores on the phenomenon of reflection and refraction at a loosely bonded interface. The mathematical formulation considers the elastic anisotropy due to the presence of aligned cracks in an elastic solid. Porous solid is in contact with a cracked elastic solid at a plane interface between them. For the presence of vertically aligned microcracks, the elastic solid behaves transversely isotropic to wave propagation. The coefficients of elastic anisotropy depend on the crack density and crack porosity in the medium. A loose bonding is considered between the two solids so that a limiting case could be the welded contact. At the plane interface, the imperfection in welded bonding is represented by tangential slipping and, hence, results in the dissipation of a part of strain energy. Three types of waves propagate in an isotropic fluid-saturated porous medium, which are considered for incidence at the interface. Incidence of a wave results in three reflected waves and two refracted waves. Partition of incident energy among the reflected and refracted waves is studied for each incidence, varying from normal to grazing directions. Numerical example calculates the energy shares of reflected and refracted waves at the plane interface between water-saturated sandstone and basalt. These energy shares are computed and analysed for different values of crack parameters as well as loose bonding parameter.
3.1 Fundamental Relations

3.1.1 Poroelastic solid

Following Biot (1962a,b), a set of differential equations governs the particle motion in an isotropic porous solid frame saturated by a non-viscous fluid. These equations, in the absence of body forces, are given by

\[ \tau_{ij,j} = \rho \ddot{u}_i + \rho_f \ddot{w}_i, \quad (-p_f)_i = \rho_f \ddot{u}_i + m \ddot{w}_i, \tag{3.1} \]

where \( \tau_{ij} \) and \( p_f \) are the stress components in porous aggregate and fluid pressure, respectively. The \( u_i \) are the components of the average displacements for the solid and \( w_i \) are the components of average displacement of fluid relative to the solid. Indices can take the values 1, 2 and 3. Summation convention is valid for repeated indices. The comma (,) before an index represents partial space differentiation and dot over a variable represents partial time derivative. The \( \rho \) and \( \rho_f \) are the densities of porous aggregate and pore fluid, respectively. The inertial parameter \( m \) control the coupling between fluid and solid phases.

The stresses in the isotropic solid matrix of porous aggregate, following Biot (1962a) are defined as

\[ \sigma_{ij} = (\lambda u_{k,k}) \delta_{ij} + \mu (u_{i,j} + u_{j,i}), \tag{3.2} \]

and these are related to \( \tau_{ij} \) by

\[ \tau_{ij} = \sigma_{ij} + \alpha (-p_f) \delta_{ij}, \tag{3.3} \]

through the parameter \( \alpha \) to represent the elastic coupling among the two constituents. \( \delta_{ij} \) is Kronecker symbol. Finally, using the above relations, stresses in porous aggregate and pore-fluid, are expressed as

\[ \tau_{ij} = [(\lambda + \alpha^2 M) u_{k,k} + \alpha M w_{k,k}] \delta_{ij} + \mu (u_{i,j} + u_{j,i}) \]

\[ -p_f = M (\alpha u_{k,k} + w_{k,k}), \tag{3.4} \]

where \( \lambda, \mu, M \) are the elastic constants.
To seek the harmonic solution of (3.1), for the propagation of plane waves, we write

\[ u_j = A_j \exp \{i\omega(s_kx_k - t)\}, \]
\[ w_j = B_j \exp \{i\omega(s_kx_k - t)\}, \]  \hspace{1cm} (3.5)

where, \( \omega \) is angular frequency and \((s_1, s_2, s_3)\) is slowness vector. In terms of phase velocity \( V \), the slowness \((s_1, s_2, s_3) = N/V\). Row matrix \( N = (n_1, n_2, n_3) \) denotes the direction of phase propagation. The vectors \((A_1, A_2, A_3)\) and \((B_1, B_2, B_3)\) define the polarisations for the motions of solid and fluid particles in porous medium. Substituting (3.5) in (3.1), we obtain a system of six equations (Sharma, 2008), given by

\[
\begin{align*}
\{ (\lambda + \mu + \alpha^2 M) n_i n_k + (\mu - \rho V^2) \delta_{ik} \} A_k &= (\rho f V^2 \delta_{ik} - \alpha M n_i n_k) B_k, \\
(\rho f V^2 \delta_{ik} - \alpha M n_i n_k) A_k &= (M n_i n_k - m V^2 \delta_{ik}) B_k. 
\end{align*}
\]  \hspace{1cm} (3.6)

This system is solved into a relation

\[ B_i = \Gamma_{ij} A_j; \quad \Gamma = \frac{\rho f}{m} ( - I + N^T N - \frac{\rho f V^2 - \alpha M}{m V^2 - M} N^T N ), \]  \hspace{1cm} (3.7)

and a subsystem, given by

\[
\begin{align*}
D_{ik} A_k &= 0, \quad D = g N^T N + h (I - N^T N) \\
g &= (\lambda + 2\mu + \alpha^2 M - \rho V^2) + \frac{(\rho f V^2 - \alpha M)^2}{m V^2 - M}, \quad h = \mu - (\rho - \frac{\rho f^2}{m}) V^2, 
\end{align*}
\]  \hspace{1cm} (3.8)

where, \( I \) is a third order identity matrix and \( N^T \) denotes the transpose of row matrix \( N \).

The expression (3.7) relates the displacements \((u, w)\) of two constituent phases in the porous aggregate. The set of equations (3.8) explains the propagation phenomenon in the medium and are called the Christoffel equations for wave propagation in saturated porous solid. Non-trivial solution of the Christoffel system (3.8) is ensured by the equations

\[
\begin{align*}
(\rho m - \rho f^2) V^4 - [m(\lambda + 2\mu + \alpha^2 M) + \rho M - 2\rho f \alpha M]V^2 + (\lambda + 2\mu) M &= 0, \quad (3.9) \\
(\rho f^2 - \rho m) V^2 + \mu m &= 0. \quad (3.10)
\end{align*}
\]
Two roots of quadratic equation (3.9) define the phase velocities (say, \( V_1 \) and \( V_2 \) in descending order) of two harmonic waves in the porous medium. For these velocities, polarization vector \((A_1, A_2, A_3)\) from (3.8) is found to be parallel to \( \mathbf{N} \) and hence represents longitudinal vibration of particles. This implies that the two waves with velocities \( V_1 \) and \( V_2 \) are fast \( P \) (or \( P_f \)) and slow \( P \) (or \( P_s \)) waves of Biot’s theory, respectively. From (3.7), the polarizations of the fluid particles, for longitudinal waves, are given by the relation

\[
(B_1, B_2, B_3) = -\left(\frac{\rho_f V^2 - \alpha M}{mV^2 - \alpha M}\right)(A_1, A_2, A_3), \quad V = V_1, V_2.
\] (3.11)

From equation (3.10), velocity (say, \( V_3 \)) of third harmonic wave in porous medium is given by \( V_3^2 = \mu/(\rho - \rho_f^2/m) \). Corresponding to this velocity, polarization vector \((A_1, A_2, A_3)\) is found to be parallel to a row/column of the singular matrix \((\mathbf{I} - \mathbf{N}^T \mathbf{N})\). This implies that lone transverse wave in porous medium propagates with velocity \( V_3 \). For this wave, the polarization vector \((B_1, B_2, B_3)\) for the fluid particles is calculated from the relation

\[
(B_1, B_2, B_3) = -\left(\frac{\rho_f}{m}\right)(A_1, A_2, A_3).
\] (3.12)

### 3.1.2 Cracked elastic solid

Five elastic constants \( C_{11}, C_{13}, C_{33}, C_{44}, C_{66} \) are used to specify the elastic response of a transversely isotropic elastic solid. The relations \( C_{11} = C_{33} = C_{13} + 2C_{66}, \quad C_{44} = C_{66} \) among these constants reduce the transverse isotropy to isotropy. On the other hand, three anisotropic parameters \((\epsilon, \gamma, \delta)\) measure the deviations of elastic constants from these relations. These parameters are related to elastic constants as

\[
2\epsilon = \frac{C_{11}}{C_{33}} - 1, \quad 2\gamma = \frac{C_{66}}{C_{44}} - 1, \quad 2\delta = \frac{(C_{13} + C_{44})^2}{C_{33}(C_{33} - C_{44})} + \frac{C_{44}}{C_{33}} - 1,
\] (3.13)

so that transverse isotropy can be represented with the values of two elastic constants \((C_{33}, C_{44})\) and three anisotropic parameters \((\epsilon, \gamma, \delta)\).

Wave anisotropy in the medium is resulting from the presence of vertically aligned parallel cracks. Following Thomsen (1986), the anisotropic parameters \((\epsilon, \gamma, \delta)\) relate to crack density
(η) and crack porosity (φ_c) present in the solid. These relations are defined as follows.

\[ 2\varepsilon = \left( \frac{E}{E} - 1 \right)/(1 - \nu^2); \quad 2\gamma = \frac{\mu}{\mu} - 1; \quad 2\delta = \frac{\Delta}{(1 - \nu)(1 - \Delta)}, \]  

(3.14)

with

\[ \Delta = \frac{\mu}{\mu} \left( \frac{E}{E} - 1 \right) \left( \frac{1 - \nu}{1 + \nu} \right) + \left( \frac{\mu}{\mu} - 1 \right)(1 - 2\nu), \]  

(3.15)

where, E and \( \mu \) denote the Young’s modulus and rigidity modulus respectively, for the elastic solid with no cracks. \( \nu \) is Poisson’s ratio for the solid grains. Barred parameters represent the corresponding characteristics in elastic solid with imbedded cracks. Effect of cracks on the elastic constants, used in (3.14)-(3.15), are defined as

\[ \frac{E}{E} = 1 + \eta \frac{16}{3} (1 - \frac{K_f}{K_s})(1 - \nu^2)D_c, \quad \frac{\mu}{\mu} = 1 + \eta \frac{16}{3} \frac{1 - \nu}{2 - \nu}, \quad D_c^{-1} = 1 - \frac{K_f}{K_s} \left( 1 - \frac{16}{9} \frac{\eta}{\phi_c} \frac{1 - \nu^2}{1 - 2\nu} \right), \]  

(3.16)

where \( K_f \) and \( K_s \) denote bulk moduli of the fluid and solid respectively. In relation to aspect ratio \( (c/a) \) of circular cracks (of radius \( a \) and thickness \( c \)), crack porosity, \( \phi_c = \frac{4}{3} \pi \eta \frac{c^2}{a} \).

Finally, the transversely isotropic behaviour of cracked elastic medium can be represented through the values of elastic parameters \( C_{33}, C_{44}, K_f, K_s, \nu \) and crack parameters \( \eta, \phi_c \).

### 3.2 Formulation of the Problem

Objective is to study the effects of presence of cracks on reflection and refraction of plane harmonic waves at the interface between porous solid half-space and cracked elastic solid half-space. This provides a mathematical model to analyse the continuous accumulation of stress around focal region of eventual failure. Dilatancy of cracks is the most direct effect of accumulation of stress before an earthquake. With increasing stress, embedded cracks modify through the changes in their orientation, density and thickness. Response of crust to these crack modifications appear as precursors of an impending earthquake. We propose to consider the possible changes in reflection or refraction coefficients as precursors, during the preparation period of an earthquake. One of the medium is assumed a sedimentary region to be modeled as an isotropic liquid-saturated porous solid. The continuing medium is a
Figure 3.1: Geometry of the medium
transversely isotropic elastic solid half-space. The transverse isotropy in this medium is due to the presence of vertically aligned micro-cracks (Crampin, 1981). Elastic constants for this medium are depending upon the crack density and crack porosity through the relations derived in previous section. Crack modifications due to stress accumulation are translated into the changes in elastic anisotropy.

3.2.1 General solution

In the Cartesian coordinate system (x, y, z), let the plane z=0 define the common boundary, which separates the two solid half-spaces, as shown in Fig. 3.1. The poroelastic solid (say, medium-I) occupies the region $z > 0$. The medium-II, a transversely isotropic elastic solid, occupies the region $z < 0$. A harmonic plane wave travels through the medium-I with velocity $V_0$ and incident at the interface making an angle $\theta_0$ to the z-axis pointing into this medium. For two-dimensional motion in the x-z plane, unit vector $(\sin\theta_0, 0, \cos\theta_0)$ represents phase direction of the incident wave. The incident angle varies from 0 to $\pi/2$. This incidence results in three waves reflected back into the medium-I and two waves refracted to the continuing medium-II. For propagation of incident wave with velocity $V_0$ along direction $(\sin\theta_0, 0, \cos\theta_0)$ in porous medium, $p = \sin\theta_0/V_0$ denotes its horizontal slowness. According to Snell’s law, this quantity remains same for all the waves resulting from this incidence, in either part of the continuum. However, the vertical slowness $q_k$, $(k = 0, 1, 2, 3)$ of different waves in porous medium is defined by $q_k^2 = V_k^{-2} - p^2$. The row vectors $(n_1^{(k)}, 0, n_3^{(k)})$, $(k = 1, 2, 3)$, denote the phase directions of three reflected waves in porous medium.

The displacements in the porous medium are expressed as

$$u_j = A_j^{(0)} \exp \{i \omega (px - q_0 z - t)\} + \sum_{k=1}^3 f_k A_j^{(k)} \exp \{i \omega (px + q_k z - t)\};$$

$$w_j = B_j^{(0)} \exp \{i \omega (px - q_0 z - t)\} + \sum_{k=1}^3 f_k B_j^{(k)} \exp \{i \omega (px + q_k z - t)\}; \quad (j = x, z),$$

(3.17)
where, the values 1 to 3 of index \( k \) represent the reflected \( P_f, P_s, S \) waves, respectively. The \( f_k \) are relative excitation factors for these waves. The slowness vector of wave \( 'k' \) is given by \((p, 0, q_k) = (n_1^{(k)}, 0, n_3^{(k)})/V_k\), such that \((n_1^{(k)})^2 + (n_3^{(k)})^2 = 1\), and from Snell’s law, \( n_1^{(k)}/V_k = \sin \theta_0/V_0, \quad n_2^{(l)} = 0 \). The polarisations \((A_1^{(l)}, 0, A_3^{(l)})\) for \( P_f \) and \( P_s \) waves are same as propagation direction and for \( SV \) wave propagating along \((n_1, 0, n_3)\) it is given by \((1 - n_1^2, 0, -n_1 n_3)\).

The particle motion in \( x-z \) plane of a transversely isotropic medium is represented by the propagation of two coupled waves. These waves identified as quasi-longitudinal (or, qP) wave and quasi-transverse (or, qSV) wave. The displacement \((U_x, 0, U_z)\) for harmonic plane waves in a transversely isotropic elastic solid half-space (medium-II) are written as

\[
U_x = f_4 \exp[i \omega(px - q_4 z - t)] + f_5 \exp[i \omega(px - q_5 z - t)], \\
U_z = R_4 f_4 \exp[i \omega(px - q_4 z - t)] + R_5 f_5 \exp[i \omega(px - q_5 z - t)],
\]

(3.18)

where \( f_4 \) and \( f_5 \) are arbitrary constants. In terms of elastic constants \( C_{11}, C_{13}, C_{33}, C_{44} \) and density \( \rho_c \), the coupling constants \( R_j \) are given by (Sharma, 1999),

\[
R_j = \frac{C_{11} p^2 + C_{44} q_j^2 - \rho_c}{(C_{13} + C_{44})pq_j}, \quad (j = 4, 5).
\]

(3.19)

In order to satisfy Snell’s law, the horizontal slowness \( p \) is same for both the waves. For vertical slowness, we have a quadratic equation, given by

\[
C_{33} C_{44} q^4 + [(C_{11} C_{33} - C_{13}^2 - 2C_{13} C_{44}) p^2 - \rho_c (C_{33} + C_{44})] q^2 + C_{11} C_{44} p^4 - \rho_c (C_{11} + C_{44}) p^2 + \rho_c^2 = 0.
\]

(3.20)

The smaller of the two roots of (3.20) is denoted by \( q_4^2 \) and \( q_5^2 \) denotes its larger root. Then \( q_4 \) and \( q_5 \) define the vertical slowness of qP wave and qSV wave respectively. The stress components on the plane with normal along z-direction are given by

\[
\sigma_{xx} = C_{44} \left( \frac{\partial U_z}{\partial x} + \frac{\partial U_x}{\partial z} \right), \quad \sigma_{zz} = C_{13} \frac{\partial U_x}{\partial x} + C_{33} \frac{\partial U_z}{\partial z}.
\]

(3.21)
3.2.2 Boundary conditions

In general, at the welded interface between liquid saturated porous solid and an elastic solid, the boundary conditions are the continuity of stress and displacement components (Deresiewicz and Skalak, 1963). A condition restricting the flow of fluid from surface pores of porous solid to elastic solid is also considered. When the surface pores are not fully sealed then, at the interface, pore-fluid in porous medium will be in contact with the solid surface of the elastic medium. In aggregate, such a contact will be weaker than the welded contact between two solids. This may be termed as loose contact and is represented by the presence of a thin layer of fluid at the common surface between two porous media. Following Vashisth et al. (1991), for loose contact between porous solid and elastic solid, the appropriate boundary conditions at plane \( z = 0 \) are given by

\[
\begin{align*}
&i) \quad \tau_{zz} = \sigma_{zz}, \quad ii) \quad \tau_{zx} = \sigma_{zx}, \quad iii) \quad \dot{u}_z + \dot{w}_z = \dot{U}_z, \\
&iv) \quad \dot{w}_z = 0, \quad v) \quad \psi \tau_{zx} = (1 - \psi)T(\dot{u}_x - \dot{U}_x).
\end{align*}
\] (3.22)

Parameter \( \psi \) in \((0, 1)\), represents loose bonding as deviation from welded contact \((\psi = 0)\) to smooth contact \((\psi = 1)\). The impedance \( T \) is a non-zero finite positive value and represents resistance to the free discharge of pore-fluid at interface. The boundary conditions \( i) \) and \( ii) \) represent the continuity of normal and tangential stresses between the two media. Boundary condition \( iii) \) is the continuity of vertical displacements and \( iv) \) restricts the discharge of pore-fluid at the interface. The boundary condition \( v) \) implies that tangential stress at the interface is proportional to the tangential slip allowed there due to loose bonding. Such a frictional slip should dissipate a part of energy at the interface. Hence, in deviation from the welded contact, at the loosely-bonded interface \((0 < \psi < 1)\) represented by condition \( v) \), the energy conservation is achieved with its dissipated part. In this case, aggregate energy of reflected and refracted waves at the interface falls short of the incident energy.

3.2.3 Solution of the problem

The general solution (3.17) for displacements in medium-I are used in relations (3.4) to
calculate the stress components \( \tau_{zz} \) and \( \tau_{zx} \). Similarly, the displacement expressions (3.18) are used in relations (3.21) to calculate stresses \( \sigma_{zz} \) and \( \sigma_{zx} \) in medium-II. Now the relevant displacements and stresses are subjected to the boundary conditions (3.22). This yields a system of five linear, non-homogeneous simultaneous equations in five unknowns (i.e., \( f_j, \ j = 1, 2, ..., 5 \)). This system is expressed as

\[
\sum_{j=1}^{5} A_{ij} f_j = b_i, \quad (i = 1, 2, ..., 5),
\]

(3.23)

where, coefficients \( A_{ij} \) and residues \( b_i \) are defined as follows.

\[
a_{1k} = (\lambda + \alpha^2 M)(pA_{x}^{(k)} + q_k A_{z}^{(k)}) + 2\mu q_k A_{z}^{(k)} + \alpha M(pB_{x}^{(k)} + q_k B_{z}^{(k)})^k, \quad a_{2k} = \mu(q_k A_{x}^{(k)} + pA_{z}^{(k)}),
\]

\[
a_{3k} = A_{z}^{(k)}, \quad a_{4k} = B_{z}^{(k)}, \quad a_{5k} = (\psi - 1)TA_{x}^{(k)}; \quad (k = 1, 2, 3).
\]

\[
a_{1k} = C_{33}R_k q_k - C_{13}p, \quad a_{2k} = C_{44}(q_k - pR_k), \quad a_{3k} = -R_k, \quad a_{4k} = 0,
\]

\[
a_{5k} = (1 - \psi)T + \psi C_{44}(q_k - pR_k); \quad (k = 4, 5).
\]

\[
b_1 = -(\lambda + \alpha^2 M)(pA_{x}^{(0)} - q_0 A_{z}^{(0)}) + 2\mu q_0 A_{z}^{(0)} - \alpha M(pB_{x}^{(0)} - q_0 B_{z}^{(0)}), \quad b_{2} = \mu(q_0 A_{x}^{(0)} - pA_{z}^{(0)}),
\]

\[
b_{3} = -A_{z}^{(0)}, \quad b_{4} = -B_{z}^{(0)}, \quad b_{5} = (1 - \psi)TA_{x}^{(0)}.
\]

The algebraically closed system (3.23) is solved numerically through Gauss elimination method. Thus obtained values of \( f_j, \ (j = 1, 2, ..., 5) \), define the ratios of amplitudes of reflected \( P_f, \ P_s, \ SV \) waves and refracted \( qP, \ qSV \) waves to the amplitude of incident wave. These amplitude ratios are used further to calculate the energy shares of reflected and refracted waves, as explained in next section.

### 3.2.4 Energy partition

Distribution of incident energy among reflected and refracted waves is considered across a surface element of unit area at the plane \( z=0 \). The scalar product of surface traction and particle velocity per unit area, denoted by \( P \), represents the rate at which the energy is communicated per unit area of the surface. The time average of \( P \) over a period, denoted by \(< P >\), represents the average intensity of energy transmission. On the surface with normal along \( z \)-direction, the average energy intensities of the waves in a porous medium are defined
by

$$< P > = 0.5 \Re \left[ \tau_{zz} \bar{u}_z + \tau_{xz} \bar{u}_x + (-p_f) \bar{w}_z \right],$$

(3.24)

where bar over a name defines its complex conjugate. The average energy intensities of the refracted waves in elastic medium are defined by

$$< P > = 0.5 \Re \left[ \sigma_{zz} \bar{U}_z + \sigma_{xz} \bar{U}_x \right].$$

(3.25)

In terms of elastic parameters and wave characteristics, we define

$$< P_0 > = -b_1 \bar{A}_z^{(0)} - b_2 \bar{A}_x^{(0)} + M[\alpha(pA_x^{(0)} - q_0 A_z^{(0)}) + pB_x^{(0)} - q_0 B_z^{(0)}]B_z^{(0)};$$

$$< P_k > = \Re \{ a_{1k} \bar{A}_z^{(k)} + a_{2k} \bar{A}_x^{(k)} + M(\alpha(pA_x^{(k)} + q_k A_z^{(k)}) + pB_x^{(k)} + q_k B_z^{(k)})]B_z^{(k)} \} |f_k|^2, \quad (k = 1, 2, 3)$$

$$< P_k > = \Re \{ a_{1k} \bar{R}_k + a_{2k} \} |f_k|^2, \quad (k = 4, 5).$$

(3.26)

With $< P_0 >$ defining the energy intensity of the incident wave, the energy ratios $E_j = < P_j > / < P_0 >, \quad (j = 1, 2, \ldots, 5)$, represent the strengths of three waves reflected back to porous medium and two waves refracted to cracked elastic medium. The energy ratios $E_1, E_2, E_3$ are also called the reflection coefficients of $P_f, P_s, SV$ waves in porous medium respectively. Similarly, $E_4$ and $E_5$ are called the refraction coefficients of $qP, qSV$ waves in elastic solid respectively. In case of welded contact ($\psi = 0$) or smooth interface ($\psi = 1$), the accuracy of the whole reflection/refraction procedure is verified with the relation $\sum_{j=1}^5 |E_j| = 1$, which represents the conservation of incident energy across the interface.

### 3.3 Numerical Example

Purpose of the numerical example is to check the impact of crack modification and loose bonding on the reflected and refracted waves. The medium chosen for the numerical example is the water-saturated sandstone in contact with basalt rock containing uniformly distributed water-saturated vertically aligned circular cracks. The values of relevant coefficients for two adjoining solids are chosen as follows:

**Medium-I**: It is a sedimentary porous solid half-space represented by water saturated sandstone.
The elastic and dynamical constants for this medium are given by (Yew and Jogi, 1976),
\[ \lambda = 2.1 \text{GPa}, \quad \mu = 2.7 \text{GPa}, \quad M = 4.8 \text{GPa} \quad \alpha = 0.8 \]
\[ \rho = 2100 \text{kg/m}^3, \quad \rho_f = 1000 \text{kg/m}^3, \quad m = 3300 \text{kg/m}^3, \quad f = 0.26. \]

**Medium-II:** It is an elastic solid embedded with vertically aligned microcracks. The elastic constants \( C_{33} \) and \( C_{44} \) are derived from density of the medium and speeds \( v_1 \) and \( v_2 \) of P and S waves respectively. For upper pillows of basalt we assume \( v_1 = 5 \text{km/sec}, v_2 = 2.75 \text{km/sec} \) and density \( \rho_c = 2700 \text{kg/m}^3 \). Anisotropic parameters are derived from \( K_f/K_s \) (ratio of bulk moduli of liquid in cracks and solid grains), \( \nu \) (Poisson’s ratio of solid with no cracks), \( \eta \) (crack density) and \( \phi_c \) (crack porosity). It is assumed that \( K_f/K_s = .053 \) (water saturated cracks) and \( \nu = 0.28 \). Surface flow impedance is fixed with a symbolic value \( T = 1 \text{MPa.s/m} \).

Values of \( \eta \) and \( \phi_c \) are varied to check the effects of variations of crack parameters on energy partition.

These numerical values are used to calculate the reflection and refraction coefficients for any wave incident through porous medium. Of the three waves (i.e., \( P_f, P_s, SV \)) in porous half-space, the \( P_s \) wave is found to be very weak as compared to other two waves. Keeping this in mind, incidence of \( P_s \) wave is not considered in this numerical part of study. For the incidence of \( P_f \) or SV wave, the angle of incidence vary from 0 to 90\(^0\). The energy shares \( (E_j, j = 1, 2, 3, 4, 5) \) derived in previous section define the reflection / refraction coefficients of reflected / refracted waves at the interface. Sum of these five energy values is verified to be equal to unity when contact at the interface is either welded (i.e., \( \psi = 0 \)) or smooth (i.e., \( \psi = 1 \)). For intermediate cases of loose contact (i.e., \( 0 < \psi < 1 \)) at the interface, sum of these five energy values falls short of unity. This deficit of energy-sum from unity is defined as the energy dissipated in initiating tangential slip at the imperfectly bonded interface between two solids. Variations in energy shares and dissipated energy with incident direction are plotted in Figs. 3.1 to 3.3 for incident \( P_f \) wave and in Figs. 3.4 to 3.6 for incident SV wave. Details are as follows.
Figure 3.2: Effect of loose bonding on the energy shares of reflected and refracted waves at the interface between water-saturated sandstone and cracked basalt; incident $P_f$ wave
Plots in Fig. 3.1 exhibit variations in the partition of incident energy at the loosely bonded interface between water saturated sandstone and cracked basalt. Three different values of $\psi$ (= 0.01, 0.1, 0.3) represent the magnitude of imperfection in otherwise a welded bonding. Due to this, a part of the incident energy is dissipated in tangential slip at the interface. The values $\eta = 0.2$ and $c/a = 0.01$ define the extent of water saturated cracks present in basalt rock. According to this figure, critical angles for refracted qP and qSV waves appear near incidence at $28^0$ and $57^0$, respectively. These critical angles do not change with any imperfection in bonding at the interface. Reason being that allowing a slip between two media affect only boundary conditions whereas wave velocities are not affected. For pre-critical incidence, the energy share of refracted qP wave decreases slightly with increase of $\psi$. On the other hand, stronger qSV wave refracts from a loosely bonded interface. This is in contrast to the response of reflected $P_f$ and SV waves to loose bonding. However, for post critical incidence, the reflected SV wave may gain strength from loosened bonding. A relatively weaker reflected $P_s$ wave indicates a stronger bonding between two media. More energy is dissipated when value of $\psi$ increases from 0.01 to 0.3. But this increase in dissipation reverses for increasing $\psi$ beyond some value (near 0.5) and the dissipation disappears as $\psi$ approaches to unity. Except near the normal incidence and the grazing incidence, effect of $\psi$ is very much significant on reflected waves, refracted waves and dissipation. In general, this effect on a wave changes gradually with incident direction, except for reflected SV wave from post-critical incidence.

The Fig. 3.2 presents the changes in energy partition at the interface for three different values of crack density $\eta$. Obviously, the value $eta = 0.0001$ denotes the case when cracks are not present in the basalt rock. The uniform thickness of circular cracks is defined by $c/a = 0.01$ and $\psi = 0.2$ defines the loose bonding between two half-spaces. From this figure it is noted that the critical angles for refracted waves do not change with the presence of cracks. The reflected $P_f$ wave gets a jump in its energy share for incidence near the critical
Figure 3.3: Effect of crack density on the energy shares of reflected and refracted waves at imperfect interface between water-saturated sandstone and cracked basalt; incident $P_f$ wave
Figure 3.4: Effect of crack thickness (aspect ratio) on the energy shares of reflected and refracted waves at imperfect interface between water-saturated sandstone and cracked basalt; incident $P_f$ wave.
angle for refracted qP wave. Similarly, reflected SV wave gets a jump in its energy share for incidence near the critical angle for refracted qSV wave. The reflected $P_s$ wave gets a significant gain for post-critical incidence. No SV wave reflects or refracts from the normal incidence of $P_f$ wave. An increase of crack density has a more significant effect on the reflected waves than on the waves refracted to the cracked elastic medium. Change in dissipation with $\eta$ is observed mainly near critical incidence.

In aspect ratio of $(c/a)$ circular cracks with fixed radius $(a)$, $c$ represents their averaged thickness. For incident $P_f$ wave, effect of crack thickness (or $c/a$) on partition of energy at the interface between water-saturated sandstone and cracked basalt is shown in Fig. 3.3. The critical angle for refracted qP wave decreases with increase of crack thickness but critical angle for refracted SV wave remain unchanged. Increase of crack thickness affects the reflected $P_f$ (SV) wave most when incidence takes place near critical angle for refracted qP (qSV) wave. For post-critical incidence, the reflected $P_s$ and SV waves weaken with increase of aspect ratio. Their loss, however, translates into a gain for reflected $P_f$ wave. Dissipated energy remains almost unaffected with a change in $c/a$.

Analogous to the Fig. 3.1, the energy partition of incident SV wave among the reflected and refracted waves is presented in Fig. 3.4. Unlike incident $P_f$ wave, there are three critical angles, i.e., for refracted qP and qSV waves and reflected $P_f$ wave. For post-critical incidence, no wave is refracted and the scattered energy reflects mainly as SV wave. The reflected $P_s$ wave gains enough strength when incidence takes place near critical angle for reflected $P_f$ wave. Significant effect of loose bonding is noted on all the reflected and refracted waves. All the waves at the interface weaken with the increase of imperfection in bonding between two media. The energy lost by these waves is transformed to increase the dissipation. Clearly, more energy will be absorbed to allow a larger slip at the loose interface.
Figure 3.5: Same as Fig. 3.2, but for incident SV wave
Figure 3.6: Same as Fig. 3.3, but for incident SV wave
Figure 3.7: Same as Fig. 3.4, but for incident SV wave
The Fig. 3.5 exhibits the effect of crack density \( \eta \) on the energy shares of reflected and refracted waves due to the incidence of SV wave. Comparing respective plots in Figs. 3.2 and 3.5, it is noted that the scattering of SV wave is less sensitive to crack-dilatancy than \( Pf \) wave. Particularly for post-critical incidence, the presence and modifications of cracks may have no effect on reflected and refracted waves. Else, the effect of \( \eta \) is very significant on refracted waves. To be more specific, refracted \( qP \) (qSV) wave weakens (strengthens) with increase of \( \eta \). Effect of \( \eta \) on reflected \( Pf \) wave is noticed for incidence near critical angle for refracted qSV wave. The effects of crack modifications (i.e., changes in aspect ratio \( c/a \)) on waves scattered due to the incidence of SV wave at loosely bonded interface are shown in Fig. 3.6. Similar to the Fig. 3.5, plots in Fig. 3.6 also imply the little sensitivity of reflection-refraction phenomenon to any change in thickness of cracks imbedded in basalt. It is noted that these crack modifications density are subjected at a fixed crack density (\( \eta = 0.2 \)). However, the increase of crack thickness weakens refracted qP wave a bit. Presence of thick cracks strengthens the reflected \( Pf \) wave but only for incidence near critical angle for refracted qSV wave.

### 3.4 Concluding Summary

The present study considers the problem of reflection and refraction of plane harmonic waves at an interface between fluid-saturated porous solid and cracked elastic solid. The tangential slip allowed at this interface to represent a loose bonding between two solids. Such a slip absorbs a part of incident energy and hence weakens the scattered waves. For the presence of vertically aligned micro-cracks, the elastic solid behaves transversely isotropic to wave propagation. The magnitude of this crack-induced anisotropy is represented by three parameters which are derived from elastic properties and the crack characteristics, viz. crack density and aspect ratio. The numerical results are obtained for a particular example and hence may not be generalised to all such models. However, few conclusions drawn from the analysis of numerical results are explained as follows.
i) Except near the normal incidence and the grazing incidence of $P_f$ wave, effect of loose bonding is very significant on reflected waves, refracted waves and dissipation at boundary. ii) For incident $P_f$ wave, an increase of crack density has a more significant effect on the reflected waves than on refracted waves. Importance of this arises from the fact that refracted waves propagate in the elastic medium which supports the presence of cracks. Change in crack density affects dissipation mainly near critical incidence. iii) In incidence of $P_f$ wave, critical angle for refracted qP wave decreases with the increase of crack thickness while critical angle for refracted SV wave remain unchanged. Any change in thickness (or, aspect ratio) of cracks does not affect dissipation due to loose bonding. iv) Reflected $P_s$ wave gains enough strength when incidence of SV wave takes place near critical angle for reflected $P_f$ wave. Significant effect of loose bonding is noted on all the reflected and refracted waves. With the increase of imperfection in bonding between two media, all the waves at the interface weaken by losing their energies to dissipation at the interface.

v) The phenomenon of reflection and refraction for incident SV wave is less sensitive to crack-dilatancy than for incident $P_f$ wave. Particularly, for post-critical incidence, the presence and modifications of cracks may have no effect on reflected and refracted waves.

vi) Reflection-refraction phenomenon for incident SV wave is not much sensitive to crack modifications restricted by fixed crack density. To some extent, an increase of crack thickness weakens the refracted qP wave but strengthens the reflected $P_f$ wave for incidence near critical angle for refracted qSV wave.

A new field, identified as New Geophysics (Crampin and Gao, 2008), has emerged with the aim of fundamental revision of conventional fluid-rock deformation in in-situ rocks. It has implications for almost all solid-earth geosciences, including earthquake forecasting as well as hydrocarbon exploration and production. The work presented contributes to understand the events of fluid-rock substitution through the modifications in dilatant cracks. The derived expressions define a mathematical model to analyse the continuous accumulation of stress.
around focal region of eventual failure. Moreover, the dissipation of a part of energy due
to wave-induced slip at the loosely bonded interface highlights the importance of porous or
cracked materials in sound / shock absorbing packages.