CHAPTER 1

INTRODUCTION
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1.1. LITERATURE REVIEW:

Inventory can be considered as an accumulation of physical commodity that can be used to satisfy some future demand for that commodity. The main and foremost reason for maintaining inventory level is to shorten the gap between demand and supply for the commodity under consideration. Any inventory system consists of an input process and output process. The input process refers to supply either by means of production or purchase while the output process refers to demand due to which depletion of inventory occurs. Thus, supply is a replenishment process, whereas demand is a depletion process.

Though the inventories are essential and provide an alternative to production or purchase in future, they also mean lock up capital of an enterprise. Maintenance of inventories also costs money by way of expenses on stores, equipment, personnel, insurance etc. Thus excess of inventories are undesirable. This calls for controlling the inventories in the most profitable way. Hence inventory theory deals with the determination of the optimal level of such ideal resources. An excellent survey on research in inventory management in a single product, single location inventory environment is provided by Lee and Nahmias (1993).

The conventional inventory models can be placed in two categories: deterministic and stochastic. In deterministic models, all input data are assumed deterministic and are given and based upon the known data, a mathematical model is applied to minimize the total inventory costs. In stochastic models, a probabilistic distribution of the input data is specified, and a mathematical model is used to minimize the total expected inventory costs. Important contributions in the development of mathematical theory of inventories include the landmark articles by Wilson (1934), Arrow, Harris and Marshak (1951), Dvoretzky, Kiefer and Wolfowitz (1952a, 1952b), Bellman, Glicksberg and Gross (1955), Arrow, Karlin and Scarf (1958) and Wagner and Whitin (1958).
The original economic order inventory model (EOQ) assumes that inventory items are unaffected by time and replenishment is done instantaneously. However, this ideal case is not always applicable. Inventories are often replenished periodically at certain production rate, which is seldom infinite. Even for purchase items, when supply varies at the warehouse, it may take days for receiving department to completely transfer the supply into storage room.

Perishable inventory forms a large portion of total inventory and include virtually all foodstuffs, pharmaceuticals, fashion goods, electronic items, periodicals (magazines/Newspapers), digital goods (computer software, video games, DVD) and many more as they lose value with time due to deterioration and/or obsolescence. Perishable goods can be broadly classified into two main categories based on: (i) Deterioration (ii) Obsolescence. Deterioration refers to damage, spoilage, vaporization, depletion, decay (e.g. radioactive substances), degradation (e.g. electronic components) and loss of potency (e.g. chemicals and pharmaceuticals) of goods. Obsolescence is loss of value of a product, due arrival of new and better product. Perishable goods have continuous or discrete loss of utility and therefore can have either fixed life or random life. Fixed life perishable products have a deterministic, known and definite shelf life and examples of such goods are pharmaceuticals, consumer packed goods and photographic films. On the other hand, random life perishable products have a shelf life that is not known in advance and vary depending on variety factors including storage atmosphere. Items are discarded when they spoil and the time to spoilage is uncertain. For example, fruits, vegetables, dairy products, bakery products etc., have random life.

A large number of researchers developed the models in the area of deteriorating inventories. At first Whitin (1957) considered an inventory model for fashion goods deteriorating at the end of a prescribed storage period. Various types of inventory models for items deteriorating at a constant rate were discussed by Roy, Chowdhury and Choudhuri (1983). A complete survey of the published literature in mathematical modeling of deteriorating inventory systems is given by Raafat (1991). Goyal and Giri (2001) developed recent trends of inventory models for deteriorating items. Teng and
Chang (2005) determined economic production quantity in an inventory model for deteriorating items.

In most inventory models it is implicitly assumed that the product to be ordered is always available (i.e. continuous supply availability), that is when an order is placed it is either received immediately or after a deterministic or perhaps random lead time. However if the product is purchased from another company (as in the JIT deliveries of parts and components), the supply of the product may sometimes be interrupted due to the supplier’s equipment breakdowns, labor strikes or other unpredictable circumstances. Silver (1981) appears to be first author to discuss the need for models that deal with supplier uncertainty. Supply uncertainty can have a drastic impact on firms who fail to protect against it. Supply uncertainty has become a major topic in the field of inventory management in recent years. Notable events such as the 9/11terrorist attacks or major natural disasters have generated significant interest in supply chain disruption studies. Not only have modern events demonstrated the relevance and impact of uncertainty in supply, but analytical studies have also demonstrated the harm that can come from ignoring it. Supply disruptions can be caused by factors other than major catastrophes. More common incidents such as snow storms, customs delays, fires, strikes, slow shipments, etc. can halt production and/or transportation capability, causing lead time delays that disrupt material flow. Improved transportation and communication capabilities allow companies to expand globally, which also puts them at higher risk of supply variability.

There appears to be very few articles dealing with the issue of supply uncertainty phenomenon. One of the earliest articles to analyze the problem with supply interruptions was by Meyer, Rothkopf and Smith (1979). They investigated a system consisting of production facility subject to random failure and repair processes and developed expressions for the average inventory level and the fraction of time demand is met. Chao (1987) has used dynamic programming to find optimal policies for electric utility companies that may face market disruptions. Groenevelt, Pintelon and Seidmann (1992) modeled a deterministic demand production lot sizing problem where the effects
of machine breakdowns and corrective maintenance on the economic lot sizing decisions are analyzed. Articles by Parlar and Berkin (1991) consider the supply uncertainty problem, for a class of EOQ model with a single supplier where the availability and unavailability periods constitute an alternating Poisson process. Parlar and Berkin (1991) assume that at any time the decision maker is aware of the availability status of the product although he does not know when the ON (available) and OFF (unavailable) periods will start and end. When the inventory level reaches the reorder point of zero and the status is ON, the order is received; otherwise the decision maker must wait until the product becomes available. Parlar and Perry (1996) generalized the formulation of Parlar and Berkin (1991) by first assuming that the reorder point \( r \) is a non negative decision variable instead of being equal to zero. In stochastic inventory models fixing the reorder point at zero usually results in sub optimal solutions. Hence, making \( r \) as another decision variable and solving the more general problem eliminates the above mentioned sub optimality. Kandpal and Gujarathi (2003,2006) has extended the model of Parlar & Perry (1996) by considering demand rate greater than one and for deteriorating items for single supplier.

The supplier’s market may not always be monopolistic as competitive spirit in the business has increased especially after induction of multinational companies. The duopolistic case can be explained as follows where the decision maker deals with two suppliers who may be ON or OFF. Here there are three states that correspond to the availability of at least one supplier that is states 0, 1 and 2 whereas state 3 denotes the non-availability of either of them. Status of both the suppliers is explained as below.

<table>
<thead>
<tr>
<th>State</th>
<th>Status of supplier 1</th>
<th>Status of supplier 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>ON</td>
<td>ON</td>
</tr>
<tr>
<td>1</td>
<td>ON</td>
<td>OFF</td>
</tr>
<tr>
<td>2</td>
<td>OFF</td>
<td>ON</td>
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<tr>
<td>3</td>
<td>OFF</td>
<td>OFF</td>
</tr>
</tbody>
</table>
Here it is assumed that one may place order to either one of the two suppliers or partly to both when both suppliers are available (i.e. state 0 of the system).

In the classical, EOQ it is assumed that the payments for the goods received should be made immediately after the receipt of the order. Furthermore, while developing a mathematical model in inventory control, it is assumed that the payment will be made to the suppliers for the goods immediately after receiving the consignment. However, in day-to-day dealing it is found that a supplier allows a certain fixed period to settle the account. During this fixed period the supplier charges no interest, but beyond this period interest is charged by the supplier under the terms and conditions agreed upon, since inventories are usually financed through debt or equity. In case of debt financing, it is often a short term financing. Thus, interest paid here is nothing but the cost of capital or opportunity cost. Also, short-term loans can be thought of as having been taken from the suppliers on the expiry of the credit period. However, before the account has been settled, the customer can sell goods and continues to accumulate revenue and earn interest instead of paying the overdraft that is necessary if the supplier requires settlement of the account after replenishment. Interest earned can be thought of as a return on investment since the money generated through revenue can be ploughed back into the business. Therefore, it makes economic sense of the customer to delay the settlement of the replenishment account up to the last day of the credit period allowed by the supplier. If the credit period is less than the cycle length, the customer continues to accumulate revenue and earn interest on it for the rest of the period in the cycle, from the stock remaining beyond the credit period. The primary benefit of taking trade credit is that one can have savings in purchase cost and opportunity cost, which becomes quite relevant for deteriorating items. In such cases one has to procure more units than required in the given cycle to account for the deteriorating effects. In particular, when the unit purchase cost is high and decay is continuous, the saving due to delayed payment appears to be more significant than when the decay is continuous without delayed payments.
Goyal (1985) has studied an EOQ system with deterministic demands and delay in payments is permissible which was reinvestigated by Chand and Ward (1987). Mandal and Phaujdar (1989) extended Goyal’s (1985) model to the case of shortages. Shah (1991) studied the same problem with uncertainty in the quantity received, resulting in a random duration between successive orders. Using the first two moments of the distribution of random quantities received, Shah (1991) arrived at modified results of Goyal (1985) by using probabilistic demand. The aspect of admissible delay in payments has been extended to the case of two level of storage by Shah and Shah (1992). Shah (1993) developed model for exponentially deteriorating items when delay in payments is permissible by assuming deterministic demand. Thereafter, under the same situation, Shah (1993) developed model for probabilistic demand. Chung (1998) studied the same model as Goyal (1985) and presented an alternative approach to find a theorem to determine the EOQ under condition of permissible delay in payments. In Goyal’s (1985) model, it is assumed that no deterioration is allowed to occur and the capacity of the warehouse is unlimited. Aggarwal and Jaggi (1995) extended Goyal’s model to the case of deterioration under permissible delay in payment. Jamal et al. (1997) developed EOQ model with constant deterioration rate and permissible delay in payments. An EOQ model for inventory control in the presence of trade credit is presented by Chung and Huang (2005). The optimal replenishment policy for EOQ models under permissible delay in payments is also discussed by Chung et al. (2002) and Chung and Huang (2003). An EOQ model under conditionally permissible delay in payments was developed by Huang (2007) and obtained the retailer’s optimal replenishment policy under permissible delay in payments. Jaggi et al. (2007) developed an inventory model under two levels of trade credit policy by assuming the demand is a function of credit period offered by the retailer to the customers using discounted cash-flow (DCF) approach. Jaggi et al.(2008) developed a model retailer’s optimal replenishment decisions with credit- linked demand under permissible delay in payments. Hou and Lin (2009) developed a cash flow oriented EOQ model with deteriorating items under permissible delay in payments and the minimum total present value of the costs is obtained. Tripathi and Misra (2010) developed EOQ model credit
financing in economic ordering policies of non-deteriorating items with time-dependent demand rate in the presence of trade credit using a discounted cash-flow (DCF) approach.

From a financial standpoint, an inventory represents a capital investment and must compete with other assets for a firm’s limited capital funds. The effects of inflation are not usually considered when an inventory system is analyzed because most people think that the inflation would not influence the inventory policy to any significant degree. Due to high inflation, the financial situation has changed in many developing countries, especially in politically turmoil countries such as united Germany, Russia and Iraq; so it is necessary to consider the effects of inflation on the inventory system. Following Buzacott (1975) and Misra (1979), Bierman and Thomas (1977) investigated the inventory decisions under an inflationary condition in a standard EOQ model. Misra (1979) developed a discount cost model in which the effects of both inflation and time value of money are considered. Chandra and Bahner (1985) developed models to investigate the effects of inflation and time value of money on optimal order policies. An inventory model with deteriorating items under inflation when a delay in payment is permissible is analyzed by Liao et al. (2000). Bhahmbhatt (1982) developed an EOQ model under a variable inflation rate and marked-up price. Ray and Chaudhuri (1997) presented an EOQ model under inflation and time discounting allowing shortages. B.R. Sarker, A.M.M. Jamal, Shajunwang (2000) developed Supply Chain Models for Perishable products under inflation and permissible delay in payment. Agrawal et al. (2009) developed a model on integrated inventory system with the effect of inflation and credit period. In this model the demand rate is assumed to be a function of inflation. This EOQ model is applicable when the inventory contains trade credit that supplier give to the retailer. Tripathi et al. (2010) developed an inventory model for non-deteriorating items and time-dependent demand under inflation when delay in payment is permissible.

Inventory model for non-deteriorating and deteriorating items with future supply uncertainty considering demand rate as d for single supplier was developed by Gujarathi and Kandpal (2003). We have developed stochastic inventory models for perishable products where the effect of inflation and permissible delay in payment was considered.
1.2. SUMMARY OF THESIS:

Chapter 1 gives a detailed introduction of stochastic inventory models for perishable items under inflation and permissible delay in payment and its need. An exhaustive literature survey on various models is discussed. Various real life examples and their application areas are discussed in this chapter.

Our contribution is divided into 7 chapters from chapter 2 to 8. Its brief account of the work done is as follows:

The thesis “Perishable Products Stochastic Inventory Models under Inflation and Permissible delay in payment” is divided into two parts.

Part I

Single supplier stochastic inventory models

Part II

Two suppliers stochastic inventory models

Part I deals with inventory models of future supply uncertainty with single supplier. Our contribution is divided into 2 chapters: chapter 2 and chapter 3. Every chapter is followed by an illustrative example clarifying the results obtained and their practical utility. Also sensitivity analysis is done with respect to some important parameters. This is in support of verification of the usual behaviour of economics variables and hence to exemplify the scope of their real life applications.

In chapter 2 a stochastic inventory model for deteriorating items under inflation and permissible delay in payments is developed. The inventory model is developed for a supplier that allows some credit period $T_0$ for settling the accounts of purchased quantity. The credit period is a known constant as it is settled between a supplier and a buyer at the time of the deal. The effect of inflation and time value of money was investigated under given sets of inflation and discount rates. Expressions are derived for obtaining the optimum order quantity and reorder quantity and hence the optimum cycle time. (Paper based on this chapter is published in an International Journal of Probability and Statistical Science (JPSS) in August 2009 issue of the journal.)
Chapter 3 discusses the impact of permissible delay in payment by introducing a provision for part payment, which is a practical aspect. It is a common practice that an instalment of payment is made during the period of the admitted delay in payments. The part to be paid and the time at which it is to be paid are mutually settled between the supplier and the buyer at the same time of purchase of goods. (Paper based on this chapter is **published** in the journal of Indian Association for Productivity, Quality and Reliability (IAPQR) in November 2011 issue of the journal.)

**Part II** deals with inventory models of future supply uncertainty with two suppliers. Here there are three states that correspond to the availability of at least one supplier, that is, states 0, 1 and 2 whereas state 3 denotes non-availability of either of them. Here it is assumed that one may place order to either one of the two supplier or partly to both when both the suppliers are available (i.e. state 0 of the system.) In case of two suppliers, spectral theory is used to derive explicit expressions for the transition probabilities of a four states continuous time Markov Chain representing the status of the system. These probabilities are used to compute the exact form of the average cost expression. Optimal solutions are obtained by Newton-Rapson method in R programming.

Our contribution is divided into 4 chapters: **chapter 4 to chapter 7**. In support of the results developed, in each chapter an illustrative example is given and sensitivity analysis is done with respect to some important parameters.

In **chapter 4** stochastic inventory model for two suppliers under trade credit is developed. From this model we have concluded that cost is minimum when the account is settled at credit time given by both the suppliers. Sensitivity analysis is also carried out. (Paper based on this chapter is **published** in the journal of Calcutta Statistical Association Bulletin (CSA) in September 2010 issue of the journal.)

In **chapter 5** stochastic inventory model for two suppliers under inflation and permissible delay in payment is developed. From this model we have concluded that as inflation rate increases, average cost also increases and cost is minimum when account is settled at credit time given by both the suppliers. (Paper based on this chapter is
published in the journal of Indian Statistical Association (JISA) in December 2010
issue)

In chapter 6 stochastic inventory model for two suppliers under permissible
delay in payment allowing partial payment is developed. From this model we have
concluded that cost is minimum if part payment is not done at \( T_{ii} \) but account is cleared
at \( T_{i} \) and the cost is maximum if part payment is done at \( T_{ii} \) but account is not cleared at
\( T_{i} \), this implies that we encourage the small businessmen to do the business by allowing
partial payment and simultaneously we want to discourage them for not clearing the
account at the end of credit period. (Paper based on this chapter is published in an
international journal of Engineering and Management Sciences (IJEEMS) in April 2013
issue of the journal.)

In chapter 7 stochastic inventory model for two suppliers under inflation and
trade credit allowing partial payment is developed. From this model we have concluded
that as inflation rate increases, average cost also increases and cost is minimum if part
payment is not done at \( T_{ii} \) but account is cleared at \( T_{i} \) and the cost is maximum if part
payment is done at \( T_{ii} \) but account is not cleared at \( T_{i} \), this implies that we encourage the
small businessmen to do the business by allowing partial payment and simultaneously
we want to discourage them for not clearing the account at the end of credit period. (Paper based on this chapter is accepted for publication in the journal of Indian Society
for Probability and Statistics (ISPS)).

In chapter 8 stochastic inventory model for multiple suppliers is developed.
From this model we have concluded that when the number of suppliers becomes large,
the objective function of the multiple supplier problem reduces to that of the classical
EOQ model. (Paper based on this chapter is published in an international journal of
Science, Engineering and Technology Research (IJSETR) in September 2013 issue of
the journal.)

We have also thought of following problems for our post Ph.D. work.

(i) Problem of diversification when both the suppliers are available.
(ii) Demand can be considered probabilistic.