4 TSUNAMI WAVE PROPAGATION MODELS BASED ON HEXAGONAL CELLULAR AUTOMATA

4.1 INTRODUCTION

Nowadays, assessment of hazard conditions related to complex natural phenomena increasingly take advantage of computer-assisted analyses and simulations. To study the tsunami waves, oceans area is divided into a matrix of identical square cells, with side length $L$, and it is represented by a cellular automaton with this value, for a more clarity of the wave details. The mathematical models for the wave spread can be classified into two types: vector models and cellular automata models. If wave spreading conditions are uniform, a single shape can be used to determine the wave size, the perimeter over time, and the area by means of the use of fractals.

In this chapter, a new cellular automata model has been proposed based on the transfer of fractional traversed area. A simulation tool that can be used to mimic the real time wave spread using hexagonal cellular automata under spatially variable topographic, slope, and meteorological conditions is given. This model introduces factor of propagation from nearest and adjacent cells and includes, in a detailed form, the rate of wave spread. This will add lot of value in the testing and calibration of tsunami computer models and in improving tsunami early warning systems [179].

Working models of tsunami wave visual representation are obtained using the java programming language. It is used for finding the rate of spread of the tsunami wave under two types of ocean, eight topological and wave conditions.

In the first section, various existing approaches for the rate of spread of tsunami wave propagation modeling are examined. In section 2, the basic theory of cellular automata and hexagonal cellular automata is presented. In section 3, the new model is proposed. In section 4, homogeneous and non
homogeneous ocean modules are checked and their simulation results are shown. In section 5, graphic representation of hexagonal cellular automata is presented. Section 6 discusses the results obtained.

4.2 THEORY OF CELLULAR AUTOMATA AND HEXAGONAL CA MODELS

In particular, Cellular Automata (CA) is a powerful tool for modelling natural and artificial systems that can be specified in terms of local interactions among their constituent parts. Cellular automata are arrays of cells that interact with their neighbors. These arrays can take on any number of dimensions, starting from a one dimensional string of cells. Each cell has its own state that can be a variable, property or other information. At the beginning of the simulation, cell states are initialized by means of input matrices. Model parameters have also to be assigned in this phase, by taking into consideration their physical/empirical meaning. By simultaneously applying the transition function to all the cells, at discrete steps, states are changed and the evolution of the phenomenon can be simulated.

4.2.1 Hexagonal Cellular Automata and their Features

The hexagonal model of a tsunami wave is investigated, along with the mathematical equation and the physical properties of examining the wave characteristics. The hexagonal model and physical properties of a tsunami wave are involved in this research for several representative conditions, and a representative real world tsunami is simulated through computer modeling. The graphs, equation, and simulation results are represented to understand and create a typical model of a tsunami wave.

In order to fix the values of such essential global parameters, further points must be considered, especially when the phenomenon is complex and involves time and/or space heterogeneity. The present approach differs in its use of discrete space (cells), and discrete time increments (steps): accordingly, continuum limit operations are not required. The Two-dimensional cellular automata model (discussed in chapter 3) does not have accuracy in the shape of the output obtained. Further, the rate of spread is
high, which decreases the efficiency. Hence, the hexagonal cellular automata are used to accurately study the rate of spread and the shape of the output as in real tsunamis. As the state of the cell can be decomposed into sub states, the transition function may also be split into local interactions: the elementary processes. Different elementary processes may involve different neighborhoods; the CA neighborhood is given by the union of all the neighborhoods associated to each process.

Let us focus on simplicity of a single CA cell of the two-dimensional space: it is considered limited to the universe of its neighborhood, which consists of m cells (the central cell and its adjacent cells). Indexes are utilized to indicate the central cell O and the adjacent ones (1, 2 . . . m − 1), respectively. The states considered are 0 if the cell is not traversed or partially traversed and 1 if the cell is fully traversed [8]. Using the fractional spread area traversed 2D-CA are finite dynamic systems using discrete number of uniform cells, arranged in a space. The state of the cell changes at every time step by a specific rule CA can be represented by a 4-uplet $U = (R, N, Q, k)$ Where R is the cellular space formed by a two-dimensional array of $q \times b$ cells: $\{ (x, y), 1 \leq x \leq q, 1 \leq y \leq b \}$, Here each cell assume a state.

In the two dimensional case, the cells are expressed as hexagonal areas as shown in Figure 4.1 and Figure 4.2 of tessellation of the plane. This representation gives a more practical simulation of 2D-CA. The state of each cell is an element of a finite or infinite state set, N. Moreover, the state of the cell $(x, y)$ is finite and denoted by $N^{(t)}_{x,y}$ at time t.

The indices of CA forms a subset $Q \subset \mathbb{S} \times \mathbb{S}, |Q| = c$, here every cell$(x, y)$ has a neighborhood $Q_{(x,y)}$:

$$Q_{(x,y)} = \{(x + \lambda_1,y + \delta_1), \ldots, (x + \lambda_c,y + \delta_c) : (\lambda_r,\delta_r) \in \mathbb{Q}\} \quad (4.1)$$

The appropriate neighborhood can be chosen depending on the model. In this work, the neighborhood of a cell $O=(x, y)$ is given by the set
Figure 4.3. The two types of neighbor cells of O have been distinguished, depending on whether the neighbor seen in Figure 4.3.

Figure 4.1: Square cellular space

Figure 4.2: Hexagonal cellular space

The two types of neighbor cells of O have been distinguished, depending on whether the neighbor of the cell O is an adjacent cell or not as seen in Figure 4.3.

Figure 4.3: Hexagonal model and Tessellation of the plane.

Here adjacent and diagonal neighbors are considered.

Cells are the set

\[ Q_0 = \{ O, N, NE, SE, S, SW, NW, NNE, E, SSE, SSW, W, NNW \} \] as it is shown in Figure 4.3.
$Q_n = \{N, NE, SE, S, SW, NW\}$, whereas the distant neighbor cells are given by the set $Q_d = \{NNE, E, SSE, SSW, W, NNW\}$. Furthermore, each distant neighbor cell has two near neighbor cells associated, those with common sides:

![Hexagonal model neighbor cells - Associated near neighbor cells and distant neighbor cell](image)

**Figure 4.4:** Hexagonal model neighbor cells - Associated near neighbor cells and distant neighbor cell

In this way, if $(\lambda, \delta) \in Q_d$ then the associated neighbor cells of $(x + \lambda, y + \delta)$ are denoted by $(x + \lambda^+, y + \delta^+)$ and $(x + \lambda^-, y + \delta^-)$.

The neighborhood depends on the cell to be, $Q$ depends on $O=(x,y)$ considered. In this sense, if $y$ is odd, then

$$Q^{odd} = \{(0,0), (-1,0), (0,1), (1,1), (1,0), (1, -1), (0, -1), (-1,1), (0,2), (2,1), (2, -1), (0, -2), (-1, -1)\}$$

Consequently, the neighborhood of the cell $(x,y)$, with $y$ odd, is:

$$Q_{(x,y)}^{odd} = \{(x,y), (x - 1, y), (x, y + 1), (x + 1, y + 1), (x - 1, y + 1), (x, y + 2), (x + 2, y + 1), (x + 2, y - 1), (x, y - 2), (x - 1, y - 1)\} \quad (4.2)$$

On the other hand, if $y$ is even, then

$$Q^{even} = \{(0,0), (-1,0), (-1,1), (0,1), (1,0), (0, -1), (-1, -1)\}$$ \{ (-2,1), (0,2), (1,1), (1, -1), (0, -2), (-2, -1) \}

and, Further, the neighborhood of the cell $(x,y)$, with $y$ even, is (Figure 4.5):
\[
Q_{(x,y)}^{even} = \{(x,y), (x - 1, y), (x - 1, y + 1), (x, y + 1), \\
(x + 1, y), (x, y - 1), (x - 1, y - 1), \\
(x - 2, y + 1), (x, y + 2), (x + 1, y + 1), \\
(x + 1, y - 1), (x, y - 2), (x - 2, y - 1)\}
\]

(4.3)

As these labels do not affect to the calculus below, a generic neighbor \( Q \) has been worked for the sake of simplicity as seen in Figure 4.5.

![Figure 4.5: Even neighbor cells and Odd neighbor cells](image)

Hence CA determine evolves in transition of cell states by function \( f : S^{13} \rightarrow S \). The states of the cell \((a,b)\) change depending the state of the thirteen neighborhood cells, that is

\[
N_{xy}^{(t+1)} = k \left( N_{x+\lambda_1,y+\delta_1}^{(t)}, \ldots, N_{x+\lambda_{13},y+\delta_{13}}^{(t)} \right)
\]

(4.4)

Moreover, the matrix

\[
R^{(t)} = \left( N_{xy}^{(t)} \right), 1 \leq x \leq q, 1 \leq y \leq b, \text{ is state of the CA at time } t \quad B^{(0)}
\]

is the initial position. Boundary conditions are to be considered for the dynamics of finite CA. In this work, the null boundary conditions have been considered, that is

If \((x, y) \notin \{(i,j), 1 \leq i \leq q, 1 \leq j \leq b\} \Rightarrow N_{xy}^{(t)} = 0

A very important type of CA is linear CA, whose local transition function is as follows:
\[ N_{xy}^{(t+1)} = f \left( \sum_{(\lambda, \delta) \in Q} \mu_{\lambda\delta} N_{x+\lambda, y+\delta}^{(t)} \right), \mu_{\lambda\delta} \in \mathbb{Z} \]

(4.5)

where \( f: \mathbb{Z} \to \mathbb{R} \) is a suitable discretization function.

### 4.3 THE HEXAGONAL CELLULAR AUTOMATA BASED MODEL FOR SPREADING OF OCEAN WAVES

In this section, the model for predicting a tsunami wave spreading based on two-dimensional linear cellular automata with hexagonal cellular space is proposed.

#### 4.3.1 The Model

Here spreading of tsunami works using hexagonal cellular automata is proposed.

The ocean area can be represented as hexagonal cellular automata of cells with length L. Obviously, each one of these region stands for a cell of the CA.

The state of a cell \((x, y)\) at a time \(t\), is defined as follows:

\[ N_{xy}^{(t)} \text{= traversed area of } (x,y) \text{ at a time } t \text{ / total area of } (x,y). \]

It can be observed then area of the hexagonal cell \((x,y)\) is \(3\sqrt{3}L^2/2\).

If \( N_{xy}^{(t)} = 0 \), then the cell \((x,y)\) is said to be untraversed at time \(t\);

If \( 0 < N_{xy}^{(t)} < 1 \), then the cell \((x,y)\) is partially traversed out at time \(t\), and finally

If \( N_{xy}^{(t)} = 1 \), the cell is completely traversed out at time \(t\). It can be observed that \( N_{xy}^{(t)} \) may be greater than 1. In that case, the state of the cell \((x,y)\) at time \(t\) is taken as 1.

The properties of CA assume that the cell \((x, y)\) state at time \(t+1\) on the neighbor cells at time \(t\). In particular;
\[
N_{xy}^{(t+1)} = f(N_{xy}^{(t)} + \sum_{(\lambda,\delta)\in Q_n} \mu_{\lambda\delta}^{(xy)} N_{x+\lambda,y+\delta}^{(t)} + \sum_{(\lambda,\delta)\in Q_d} \mu_{\lambda\delta}^{(xy)} N_{x+\lambda,y+\delta}^{(t)}) \tag{4.6}
\]

where \( \mu_{\lambda\delta}^{(xy)} \in \mathbb{Z} \) are integers based on properties of cells. The transfer function \( f \) is given by

\[
f: [0,1] \rightarrow N
\]

\[
a \mapsto f(a) = \frac{[10a]}{10}, \text{ where } [c] \text{ is the nearest integer to } c. \text{ Each hexagonal cell } (x,y) \text{ is endowed with three parameters:}
\]

The rate of wave spread \( RA_{(x,y)} \), the wave speed \( WA_{(x,y)} \), and the depth \( DE_{(x,y)} \) of the cell.

Thus, parameter \( \mu_{\lambda\delta}^{(xy)} \) expressed as:

\[
\mu_{\lambda\delta}^{(xy)} = \omega a_{\lambda\delta}^{(xy)} . de_{\lambda\delta}^{(xy)} . ra_{\lambda\delta}^{(xy)} \tag{4.7}
\]

where \( WA_{(a,b)} \) stands for the wave influence of the neighbor cell \( (x + \lambda, y + \delta) \) on \( (x,y) \), such that \( WA_{(a,b)} = \{ \omega a_{\lambda\delta}^{(xy)} , (\lambda,\delta) \in Q \} \);

\( de_{\lambda\delta}^{(xy)} \) represents the height influence and, as is shown below, it is a function of \( DE_{(x,y)} - DE_{(x-\lambda,y-\delta)} \) where \( DE_{(x,y)} \) is the height in the central point of the hexagonal area which is represented by the cell \( (x,y) \).

It is assumed that every point of cell has same height. Here, \( ra_{\lambda\delta}^{(xy)} \) represents the influence of the different rates of wave spread.

### 4.3.2 The Size of Discrete Time Step

It is necessary to determine the size of time step \( \tilde{t} \). \( RA_{(x,y)} \) represents the rate of wave spread of cell \( (x,y) \) and depends on positioning the cell. This calculates the time for the cell \( (x,y) \), \( RA_{(x,y)} \), to be completely traversed

If \( RA_{(x,y)} = 0 \) and \( N_{xy}^{(t)} = 0 \), then the cell \( (x,y) \) is untraversed. Using this parameter the size of time step \( \tilde{t} \) can be calculated.
For homogeneous model the rate of wave spread is uniform for all cells:

\[ RA_{(x,y)} = RA, 1 \leq x \leq q, 1 \leq y \leq b \]

If N is only traversed cell at time t in the neighborhood of O, then the time taken for O to be completely traversed is

\[ \tilde{t} = \sqrt{3}L/R \]

If only one adjacent cell of \( O = (x, y) \) are traversed and all other cells in the neighbor of O are uniform, then at time \( t + 1 \), the cell \((x, y)\) is completely traversed: \( N_{xy}^{(t+1)} = 1 \). But most of real oceans are non-homogeneous. Therefore, the step size is taken to be largest rate spread for all the cells to be completely traversed out. That is

\[ \tilde{t} = \sqrt{3} \frac{L}{RA} \]  \hspace{1cm} (4.8)

where \( RA = \max \{ RA_{(x,y)}, 1 \leq x \leq q, 1 \leq y \leq b \} \)

If \( NNE \), a distant neighbor cell of \( O = (x, y)\), is the only completely traversed out cell at time t, then \( N_{xy}^{(t+1)} = \gamma < 1 \). In a step time \( \tilde{t} \), the cells N and NE that are neighbor cells of NNE and a circular sector of the cell O will be traversed. As the distance covered by the tsunami wave spread of speed R in time \( \tilde{t} \) is \( \sqrt{3}L \), the radius of the circular sector is \( \sqrt{3}L - L \), as seen in Figure 4.6.

\[ Figure 4.6: \text{Determining the size of the discrete time step} \]

As a consequence, the traversed out area of the cell O will be
\[ \frac{\pi(\sqrt{3}-1)^2L^2}{2\pi} = \frac{4-2\sqrt{3}}{3} \pi L^2, \] if only a distance neighbor is traversed and other neighbor cells are not traversed at time at time t, then

\[ N_{xy}^{(t+1)} = \gamma = \frac{4-2\sqrt{3}}{3} \frac{\pi L^2}{\frac{3\sqrt{3}}{2} L^2} = \frac{8\sqrt{3}-12}{27} \pi \approx 0.21 \] (4.9)

### 4.3.3 The Influence of Tsunami Wave

The wave speed and direction are influence the tsunami wave spreading. As seen earlier, their effect in the cell O is

\[ WA_{(x,y)} = \{ \omega a^{(x,y)}_{\lambda \delta}, (\lambda, \delta) \in Q \} \] (4.10)

where \( \omega a^{(x,y)}_{\lambda \delta} > 0 \), in such a way that if no wave is traversed on \( O = (x, y) \) then \( \omega a^{(x,y)}_{\lambda \delta} = 1 \) for every \( (\lambda, \delta) \in Q \); if the wave is traversed from North to South, then the coefficients \( \omega a^0_{NW}, \omega a^0_{NNE}, \omega a^0_{N}, \omega a^0_{NE} \) must be larger than the rest of coefficients, and so on. The value of such coefficients stand for the magnitude of the wave as seen in Figure 4.7

![Figure 4.7: The calculus of γ](image)

### 4.3.4 The Influence of Topography

Further the differences in heights at various points in ocean also effects spreading of wave. The difference between height of two neighbor cells \((x + \lambda, y + \delta)\), on a cell \( O = (x, y) \) is given by \( de_{\lambda \delta}^{(x,y)} \), which depends on the difference of height between each pair of cells considered, that is

\[ de_{\lambda \delta}^{(x,y)} = \phi(HA_{(x,y)} - HA_{(x+\lambda,y+\delta)}) \] (4.11)
The function $\phi(a)$, where $a$ stands for the height difference, must be determined according to the characteristic of the tsunami, and, also, it has to satisfy the following conditions:

If $a > 0$, then $\phi(a) < 1$

If $a = 0$, then $\phi(a) = 1$

If $a < 0$, then $\phi(a) > 1$

It is observed

If $DE_{(x,y)} > DE_{(x+\lambda,y+\delta)}$, the rate of spread increases; the second condition states that

If $DE_{(x,y)} = DE_{(x+\lambda,y+\delta)}$, there is no change of wave spread

If $DE_{(x,y)} < DE_{(x+\lambda,y+\delta)}$ the rate of spread is restrained.

If NNE is the distance neighbor cell of O. Using the height difference of neighbor cell on

$$de^{0}_{NNE} = \frac{1}{4}\left[ \phi(DE_{O} - DE_{N}) + \phi(DE_{N} - DE_{NNE}) + \phi(DE_{O} - DE_{NE}) + \phi(DE_{NE} - DE_{NNE}) \right]$$

and so on.

It is noted that for all cells with equal height $\phi(a) \leq 1$ and consequently $de^{(x,y)}_{\lambda\delta} = 1$ for all cell $(x,y)$ and any $\lambda, \delta \in Q$.

4.3.5 The Spread Rate of Waves

For non homogeneous ocean, Let $R$ is the maximum spread rate of wave Let $d_{\lambda\delta}^{(x,y)}$ denotes the influence of the different spread rates of neighbors. If only one near neighbor cell N of O is traversed and other neighbor cells are untraversed, then after a time step, $t$ the traversed space by the wave is:

$$q = RA_{O}t = \sqrt{3} \frac{RA_{O}}{RA} L$$

(4.13)
Here two cases arise:

when \( q \leq L \), and when \( q > L \).

If \( q \leq L \), then \( \sqrt{3} \frac{R_{A_0}}{R_A} L \leq L \), and \( \frac{R_{A_0}}{R_A} \leq \frac{\sqrt{3}}{3} \approx 0.57735 \)

Consequently, the area traversed by cell \( O = (x,y) \) after time step \( \bar{t} \) is

\[
L_q + 2 \frac{q^2 \pi}{6} = (\sqrt{3} + \frac{\pi}{2} \frac{R_{A_0}}{R_A}) \frac{R_{A_0}}{R_A} L^2
\]

Further,

\[ q_N^0 = \frac{1}{2} \sqrt{3} L^2 \]

If \( q > L \), then \( L < \sqrt{3} \frac{R_{A_0}}{R_A} L \), and \( 0.57735 \approx \frac{\sqrt{3}}{3} < \frac{R_{A_0}}{R_A} \leq 1 \)

After time step \( \bar{t} \), the area traversed by cell \( O = (x,y) \) can be calculated as

\[
\left[ 1 + \sin\left(\frac{\pi}{6} - \lambda\right) + \sqrt{3} \lambda \frac{R_{A_0}}{R_A} \right] \sqrt{3} \frac{R_{A_0}}{R_A} L^2
\]

Figure 4.8: The rate of spread (near cell) \( r \leq L \) and (near cell) \( r > L \)

where,

\[
\lambda = \frac{\pi}{6} - \arccos\left(\frac{\sqrt{3}}{4} \frac{R_A}{R_{A_0}} + \frac{\sqrt{3}}{12} \sqrt{12 - 3 \left( \frac{R_{A_0}^2}{R_A^2} \right)} \right), \quad 0 \leq \lambda < \frac{\pi}{6}
\]

As a consequence:
\[ q_N^O = \frac{1 + \sin(\pi/6 - \lambda) + \sqrt{3} \lambda RAO}{3\sqrt{3}L^2} \times \frac{\sqrt{3} RAO L^2}{RA} = \frac{2}{3} \left[ 1 + \sin(\pi/6 - \lambda) \right] \frac{RAO}{RA} + \frac{2\sqrt{3}}{3} \lambda \frac{RAO^2}{RA^2} \] (4.16)

For homogeneous ocean, then \( R_{ab} = R \) for every cell \((x,y)\), and

\[ \lambda = \frac{\pi}{6} - \arccos\left(\frac{\sqrt{3}}{2}\right) = 0 \] (4.17)

So, \( q_N^O = 1 \) as was expected.

If NNE is the neighbor distance where cell of O that is traversed after time steps \( \tilde{t} \) and other neighbor cells are untraversed then the wave spread reaches the border of neighbor N and NE and thus effects cell O.

Here, The border line between N and NE is traversed in \( \tilde{t}_0 = L/\max\{RA_N, RA_{NE}\} \) Moreover, the traversed spread along O is:

\[ RA_O(\tilde{t} - \tilde{t}_0) = \left( \sqrt{3} \frac{1}{RA} - \frac{1}{\max\{RA_N, RA_{NNE}\}} \right) RA_O L \]

As a consequence, the traversed out area of O after a time step \( \tilde{t} \) is:

\[ \frac{\pi}{3} \left( \sqrt{3} \frac{1}{RA} - \frac{1}{\max\{RA_N, RA_{NNE}\}} \right)^2 RA_O^2 L^2 = \frac{\pi}{3} \left( \sqrt{3} \frac{1}{RA} - \frac{1}{\max\{RA_N, RA_{NNE}\}} \right)^2 RA_O^2 L^2 \] (4.18)

Obviously, the state of the cell O is

\[ q_{NNE}^O = \frac{\pi}{3} \left( \sqrt{3} \frac{1}{RA} - \frac{1}{\max\{RA_N, RA_{NNE}\}} \right)^2 RA_O^2 L^2 = \frac{2\pi\sqrt{3}}{27} \left( \sqrt{3} \frac{RAO}{RA} - \frac{RAO}{\max\{RA_N, RA_{NNE}\}} \right)^2 \]

Note that if the ocean is homogeneous, then

\[ q_{NNE}^O = \frac{2\pi\sqrt{3}}{27} (\sqrt{3} - 1)^2 = \frac{8\sqrt{3} - 12}{27} \] (4.19)
4.4 SIMULATION RESULTS OF HEXAGONAL CELLULAR AUTOMATA MODEL

Figure 4.9: Hexagonal cellular automata simulation results for Tsunami wave propagation with different topographic conditions

4.4.1 Homogeneous Ocean, Rate of Spread Calculation

- **Near Cells:**

  If \( q \leq L \), then,

  \[
  q_{N}^{O} = \frac{2\sqrt{3}}{9} \left( \sqrt{3} + \frac{\pi}{2} \frac{R_{A_{O}}}{R_{A}} \right) \frac{R_{A_{O}}}{R_{A}}
  \]

  If \( q > L \), then,

  \[
  q_{N}^{O} = 1 + \sin(\frac{\pi}{6} - \lambda) + \sqrt{3} \lambda \frac{R_{A_{O}}}{R_{A}} \sqrt{3} \frac{R_{A_{O}}}{R_{A}} L^{2} \]; where, \( \lambda = 0 \).

- **Distant Cells:**

  \[
  q_{NNE}^{O} = \frac{2\pi\sqrt{3}}{27} (\sqrt{3} - 1)^2 = \frac{8\sqrt{3} - 12}{27} = \gamma
  \]

4.4.1.1 Homogeneous Ocean, Horizontal Wave Motion with Primary Wave Front

In the case of horizontal wave motion, the spread is even and with primary wave spread, the spread is even in all directions as shown in the Figure 4.10
4.4.1.2  Homogeneous Ocean, Horizontal Wave Motion with Secondary Wave Front

In this case, from the central cell, the neighboring cells are partially traversed, and then they are completely traversed, in the specified direction. The spread is even and with secondary wave direction, the spread is more in the specified direction as shown in Figure 4.11.

Figure 4.11: Homogeneous Ocean, Horizontal Wave Motion with Secondary Wave Front and direction in North

4.4.1.3  Homogeneous Ocean, Vertical Wave Motion and Primary Wave Front

In the case of vertical wave motion, the spread is very fast in the trans-ocean tsunami direction, moderate on the sides is the slope, and very slow down the slope. With primary wave spread, the spread is even as shown in Figure 4.12.

Figure 4.12: Homogeneous Ocean, Vertical Wave Motion with Primary Wave Front
4.4.1.4 Homogeneous Ocean, Vertical Wave Motion with Secondary Wave Front

In this case, from the central cell, the neighboring cells are partially traversed, and then they are completely traversed, in the specified direction. In vertical wave motion, the spread is very fast in the trans-ocean tsunami direction, moderate in the sides of the slope, and very slow down the slope. With secondary wave directions, the spread is more on the specified direction; the spread is shown in Figure 4.13.

Figure 4.13: Homogenous Ocean, Vertical Wave Motion with Secondary Wave Front in North

4.4.2 Non Homogeneous Ocean, Rate of Spread Calculation

► Near Cells:

If \( q \leq L \) then,

\[
q_N^O = \frac{2\sqrt{3}}{9} \left( \sqrt{3} + \frac{\pi}{2} \frac{R_A O}{R_A} \right) \frac{R_A O}{R_A}
\]

If \( q > L \), then,

\[
q_N^O = \frac{2}{3} \left( 1 + \sin \left( \frac{\pi}{6} - \lambda \right) \right) \frac{R_A O}{R_A} + \frac{2\sqrt{3}}{3} \lambda \frac{R_A O}{R_A^2}
\]

where, \( \lambda = \frac{\pi}{6} - \arccos \left( \frac{R_A}{4 \frac{R_A O}{(x,y)}} \right) + \frac{\sqrt{3}}{12} \sqrt{12 - 3 \frac{R_A^2}{R_A O^2 (x,y)}} \) \( 0 \leq \lambda \leq \pi/6 \)

► Distant Cells:

\[
q_{NNE}^O = \frac{\pi}{3} \left( \frac{\sqrt{3}}{R_A} - \frac{1}{\max[R_{A N}, R_{ANNE}]} \right)^2 \frac{R_A O}{R_A^2 L^2}
\]
4.4.2.1 Non Homogeneous Ocean, Horizontal Wave Motion with Primary Wave Front

In the case of horizontal wave motion, the spread is even and with primary wave spread, the spread is even in all directions which is shown in Figure 4.14.

![Figure 4.14: Non homogenous Ocean, Horizontal Wave Motion with Primary Wave Front](image)

4.4.2.2 Non Homogeneous Ocean, Horizontal Wave Motion with Secondary Wave Front

In the case of horizontal wave motion, the spread is even and with secondary wave direction, the spread is more in the specified direction, which is shown in Figure 4.15.

![Figure 4.15: Non Homogenous Ocean, Horizontal Wave Motion with Secondary Wave spread in North-West](image)
4.4.2.3 Non Homogeneous Ocean, Vertical Wave Motion with Primary Wave Front

In the case of vertical wave motion, the spread is very fast in the trans-ocean tsunami direction, moderate in the sides of the slope and very slow down the slope. With primary wave spread, the spread is even, which is shown in Figure 4.16.

![Figure 4.16](image)

Figure 4.16: Non-homogenous Ocean, Vertical Wave Motion with Primary Wave Front

4.4.2.4 Non Homogeneous Ocean, Vertical Wave Motion and Secondary Wave Direction

In the case of vertical wave motion, the spread is very fast in the trans-ocean tsunami direction, moderate in the sides of the slope, and very slow down the slope. With secondary wave direction, the spread is more in the specified direction, which is shown in Figure 4.17.

![Figure 4.17](image)

Figure 4.17: Non Homogenous Ocean, Vertical Wave Motion with Secondary Wave Front in North-West
4.5 GRAPHIC REPRESENTATION OF RATE OF SPREAD IN HEXAGONAL CA

4.5.1 Rate of Spread in Homogeneous Oceans

The rate of spread of hexagonal cellular automata for homogeneous ocean [9] with horizontal wave motion and primary wave is shown in the following graph. It is found that the rate of spread is constant for both the cases, as there are same types of waves throughout the ocean as shown in Figure 4.18 and Figure 4.19.

Figure 4.18: The rate of spread by hexagonal CA in homogeneous Ocean

Figure 4.19: The rate of spread by hexagonal CA in homogeneous Ocean with different Richter values
4.5.2 Number of Cells Traversed

The number of cells traversed in hexagonal cellular automata for homogeneous ocean with horizontal wave motion and primary waves are shown in the following Figure 4.20.

![Figure 4.20: Number of cells traversed by hexagonal CA homogeneous Ocean](image)

4.5.3 Rate of Spread in Non-Homogeneous Oceans

The rate of spread of hexagonal cellular automata for non-homogeneous oceans with horizontal wave motion and primary wave are shown below. The rate of spread is uneven in the results, which is more compatible with the rate of spread in non homogeneous oceans as shown in Figure 4.21.

![Figure 4.21: The Rate of spread by Hexagonal CA in Non homogeneous Ocean](image)
4.5.4 Number of Cells Traversed

The number of cells traversed in hexagonal cellular automata for non-homogeneous oceans with horizontal wave motion and primary wave is shown below. In our result, the area of the cells is more and the number of cells traversed is less, which is more accurate as shown in Figure 4.22.

![Figure 4.22: Number of cells traversed by hexagonal cellular automata in non-homogeneous Ocean](image)

4.6 DISCUSSION

A hexagonal cellular automata model of tsunami wave propagation has been presented for independent geographical areas. The hexagonal cellular automata model determines the dynamic of the wave front in both the homogeneous and non-homogeneous ocean with eight cases depending on the wave based topography conditions to determine the proposed models. From these results it is evident that the hexagonal models are a suitable approach to tsunami wave modeling. This method of modeling tsunami wave can also be easily recreated and used if one wants to experiment with different tsunami wave models. The research also explained how the tsunami wave propagation are visualized and tested. Shallow water wave equation may be used to make the model more accurate. However, some changes in the notation of the state of the cell can be studied. In this case, a similar cellular automata model will be designed in which the states of the cell will be defined by means of transfer energy, instead of transfer of fractional traversed area.